SHORT RANGE INTERACTION BETWEEN THE ELECTRON AND OTHER PARTICLES

N. N. KOLESNIKOV and G. A. JACOBI

Moscow State University

Submitted to JETP editor February 13, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 381-391 (August, 1958)

The possibility is discussed of the existence of an electron structure and its description from the viewpoint of the linear theory of extended particles, and also from the viewpoint of nonlinear theory. Both variants of the theory lead to similar results for the interaction of the electron with the proton, the neutron and light nucleí; however, according to the nonlineary theory, the interaction of two electrons at very short ranges must be essentially different from the interaction of the electron with the positron.

RECENTLY, experiments on the scattering of fast electrons on protons[1,3] have revealed a deviation from the usual formula of Mott[3] which is valid for two point particles with strong Coulomb interaction. The character of these deviations point to a failure of the purely Coulomb law of interaction at very short ranges. Hofstadter and others[1,2,4] have shown that to explain the experimental results, it is enough to assume that, in contrast to the electron, the proton is not a point particle, but that its charge and magnetic moment are distributed in space according to some law.[2] However, although contemporary theory does not suggest sufficient arguments in support of the extended electron, still it is not excluded that part of the Hofstadter effect might be connected with the structure of the electron (which a number of authors assume[2,5]). The difficulty of infinite density is avoided in the theory of the extended electron and the so-called classical radius of the electron[6] $r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ cm takes on a physical significance.

On the other hand, experiments on the scattering of fast electrons on protons could be explained by the hypothesis that there are nonlinear effects of the electromagnetic field at short ranges. We shall consider both possibilities and shall attempt to explain how future experiments could clarify the actual nature of the short-range interactions.

1. LINEAR THEORY OF EXTENDED PARTICLES

The Interaction of the Electron and Proton

We shall consider throughout only the interaction of electrical charges which under the experimental conditions of Hofstadter[3] play an important role at small angles of scattering. Furthermore, we shall consider the charge distribution in the proton $\rho_p(r_1)$ and in the electron $\rho_e(r_2)$ to be spherically symmetric (r_1 and r_2 are distances from the center of the proton and electron, respectively).

We can then write the interaction energy of the extended proton with the extended electron, whose centers are separated a distance r, in the form

$$V(r) = -e^2 \iint \frac{\rho_p(r_1)\rho_e(r_2)\,dv_1 dv_2}{|\mathbf{r}-\mathbf{r}_1-\mathbf{r}_2|} \tag{1}$$

where $\rho_p(r_1)$ and $\rho_e(r_2)$ are normalized to unity.

Integrating over the angles in (1), we get:

$$V(r) = -\frac{e^2}{r} + \frac{4\pi e^2}{r}\int_r^\infty \rho_p(r_1)\,r_1(r_1-r)\,dr_1 + \frac{8\pi^2 e^2}{r}\int_0^\infty \rho_p(r_1)\,r_1 dr_1 \int_{|r-r_1|}^{r+r_1} dr_2 \int_{r_2}^\infty \rho_e(r_2')\,r_2'(r_2'-r_2)\,dr_2'. \tag{2}$$

For the case of scattering of fast electrons on protons, the relativistic Born approximation[3,4,7]

$$d\sigma = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\,|f(\theta)|^2\, 2\pi \sin\theta\, d\theta$$

gives results only slightly different from the values of the effective cross sections given by the exact formula,[4,8] while the dependence on the special features of the interaction at short ranges, i.e., on the structure of the particles, is determined by the form factor[2,7]

$$f(\theta) = -\frac{2m}{\hbar^2}\int_0^\infty \frac{\sin Kr}{Kr}\,V(r)\,r^2\,dr. \tag{3}$$

Substituting (2) in (3) and integrating by parts, we get

$$f(\theta) = \frac{2me^2}{\hbar^2 K^2}\left\{4\pi\int_0^\infty \frac{\sin Kr}{Kr}\rho_p(r)\,r^2\,dr - 8\pi^2 K^2\int_0^\infty \frac{\sin Kr}{Kr}\,r\,dr\int_0^\infty \rho_p(r_1)\,r_1 dr_1 \times \int_{|r-r_1|}^{r+r_1} dr_2 \int_{r_2}^\infty \rho_e(r_2')\,r_2'(r_2'-r_2)\,dr_2'\right\}. \tag{4}$$

We now assume that only the region $Kr \ll 1$ is significant in these integrals, because of the rapid fall-off ρ_p and ρ_e. That is, the momenta $p = \hbar K$ at large distances are not large, which, in any case, is satisfied for energies that are not too large. Then, substituting both terms of the expansion in the first term of (4), we obtain

$$\sin Kr / Kr \approx 1 - K^2 r^2/6, \tag{5}$$

In the second term of (4), limiting ourselves to the first term of the expansion, we get

$$f(\theta) = \frac{2a_0}{(a_0K)^2}\left\{1 - \frac{K^2}{6}\left(\langle r_p^2\rangle + \langle r_e^2\rangle\right)\right\}, \tag{6}$$

where $a_0 = \hbar^2/me^2$ is the Bohr radius,

$$\langle r_p^2\rangle \equiv \int \rho_p(r)\,r^2\,dv, \tag{7}$$

$$\langle r_e^2\rangle \equiv \int \rho_e(r)\,r^2 dv = 6\cdot 8\pi^2\int_0^\infty r\,dr\int_0^\infty \rho_p(r_1)\,r_1 dr_1 \times \int_{|r-r_1|}^{r+r_1} dr_2\int_{r_2}^\infty \rho_e(r_2')\,r_2'(r_2'-r_2)\,dr_2'. \tag{8}$$

The latter equation is obtained by integration of the right side of (7) (by parts) with account of the normalization condition for ρ_p.

The difference between (5) and the corresponding formula that does not take the dimensions of the electron into account lies in the presence of an additional term proportional to $\langle r_e^2\rangle$. Thus, if we assume the non-point character of the electron then the experimental value for the rms radius of the proton[2] ($R_0 = 0.8 \times 10^{-13}$ cm) must be the square root of the sum of the mean squares of the radii of the proton and electron: $R_0 = \sqrt{\langle r_p^2\rangle + \langle r_e^2\rangle}$.

Furthermore, in order to agree with experiment at much higher energies, when the expansion (5) is no longer a good approximation, it is necessary that the extended electron and proton interact exactly the same as, under the assumption of a point nature for the electron, the electron interacts with an extended proton with charge distribution ρ_0 (according to recent data, ρ_0 is best represented by an exponential[2]), i.e., that

$$\int_r^\infty (\rho_0 - \rho_p)\,r_1(r_1 - r)\,dr = 2\pi\int_0^\infty \rho_p(r_1)\,r_1\,dr_1\int_{|r-r_1|}^{r+r_1} dr_2\int_{r_2}^\infty \rho_e(r_2')\,r_2'(r_2'-r_2)\,dr_2.$$

Similarly, the effect of structure of the electron would manifest itself in the displacement of the electron levels in the hydrogen atom.[9] The displacement of the electron levels connected with the structure of the proton and electron, in first approximation, without consideration of the deformation of the electronic wave functions and relativistic corrections which are not appreciable in the case under discussion,[10] is equal to

$$\Delta E = -\int |\psi_e(r)|^2\,\{V + e^2/r\}\,dv. \tag{9}$$

Inasmuch as the sum $V + e^2/r$ falls off rapidly at distances much smaller than the radius of the Bohr orbit, we can remove the square of the wave function of the electron for the n-th state of the electron ψ_c from under the integral sign setting it equal to its value at zero. Substituting in (9) the value of V according to (2), and integrating by parts, similarly to what was done previously,[9] we get

$$\Delta E = \frac{\alpha}{3}|\psi_e(0)|^2\{\langle r_p^2\rangle + \langle r_e^2\rangle\}, \tag{10}$$

where ΔE is expressed in cm^{-1}.

Interaction of the Electron with Other Particles

Employing a method similar to the above, we can show that at relatively low energies the scattering cross section of neutrons on electrons (through electrostatic interaction) depends only on the rms radius of the charge distribution in the neutron, namely:

$$f(\theta) = -2a_0\langle r_n^2\rangle / a_0^2. \tag{11}$$

The elastic scattering cross section of electrons on deuterons at relatively low energies depends not only on $\langle r_p^2\rangle$, $\langle r_e^2\rangle$ and $\langle r_n^2\rangle$, but also on $\langle r_D^2\rangle = \int |\psi_p|^2 r^2\,dv$,[9] where ψ_p is the wave function of the proton in the nucleus of deuterium:

$$f_D(\theta) = \frac{2a_0}{(a_0K)^2}\left\{1 - \frac{K^2}{6}\left(\langle r_D^2\rangle + \langle r_p^2\rangle + \langle r_e^2\rangle - \langle r_n^2\rangle\right)\right\}. \tag{12}$$

The displacement of the electrons of the n-th level in deuterium is determined by the same form factors, namely

$$\Delta E_D = \frac{\alpha}{3}|\psi_e(0)|^2\{\langle r_D^2\rangle + \langle r_p^2\rangle + \langle r_e^2\rangle - \langle r_n^2\rangle\}. \tag{13}$$

It follows from (13) and (10) that the volume part of the Lamb shift,[11] equal to the energy difference between (13) and (10), does not depend on the dimensions of the electron and the proton:

$$\Delta E_D - \Delta E_n = \frac{\alpha}{3} |\psi_e(0)|^2 \{\langle r_D^2\rangle - \langle r_n^2\rangle\}. \quad (14)$$

The difference in the elastic scattering cross section of electrons on deuterons and on protons depends approximately on the same difference $\langle r_D^2\rangle - \langle r_n^2\rangle$. If the charge distribution in the positron were the same as for the electron, the effects of structure in its interaction with other particles would be the same as for the electron.

Characteristic Energy and the Dimensions of the Electron

The electrostatic energy $\mathcal{H}$ of the electron with charge distribution ρ_e can be written in the form

$$\mathcal{H} = \frac{1}{8\pi}\int E^2 dv = e^2 / {}_{-1}^{\ 1}R_e, \quad (15)$$

where we have used Gauss' theorem and the notation.

$$\frac{1}{{}_{-1}^{\ 1}R_e} = \int_0^\infty \frac{\rho_e(r)\, dv}{r} \int_0^r \rho_e(r_1)\, dv_1. \quad (16)$$

Further, assuming that

$$\mathcal{H} = m_0 c^2 \eta, \quad (17)$$

where m_0 is the rest mass of the electron, η is the ratio of the field mass of the electron to its total mass, we have, by (15),

$${}_{-1}^{\ 1}R_e = r_0/\eta,$$

where r_0 is the classical radius of the electron.

We shall show that for any non-alternating $\rho_e(r)$, the rms radius ${}_2R_e = \sqrt{\langle r_e^2\rangle}$ ought to be larger than ${}_{-1}^{\ 1}R_e/2$. Above all it can be established that for fixed values of ${}_{-1}^{+1}R_e$ the rms radius ${}_2R_e$ has a minimum value for uniform charge distribution over the volume.* But the Coulomb energy of the charge, uniformly distributed over the volume of a sphere of radius a, is equal to $3e^2/5a$, and, consequently, ${}_{-1}^{\ 1}R_e = 5a/3$, whereas ${}_2R_e = \sqrt{3/5}a$. Consequently, ${}_2R_e \geq (3/5)^{3/2}\ {}_{-1}^{\ 1}R_e$, or

$${}_2R_e > {}_{-1}^{\ 1}R_e/2 = r_0/2\eta. \quad (18)$$

If neither the charge distribution of the proton ρ_p nor the charge distribution of the electron ρ_e are alternating, then the rms radius of the electron ${}_2R_e$ should in every case be not larger than the value of $R_0 = 0.8 \times 10^{-13}$ cm obtained by Hofstadter. But then the inequality (18) can be satisfied for $\eta > 1.75$; η must be > 1 for other considerations also. The extended electron cannot be stable and undeformed in strong external fields in the absence of additional internal forces that are large in magnitude, forces of attraction of a non-electromagnetic character; therefore the non-field mass of the electron must be considerable.

2. NONLINEAR THEORY

General Properties of a Nonlinear Field

In the nonlinear theory there is no need of considering the electron to be an extended particle. If the electromagnetic field has a nonlinear character[6,13-16] that appears at short ranges, then this is equivalent, in a well known sense, to the presence of structured particles. We shall first attempt to make clear in what measure the requirements of the theory and the experimental data limit the choice of the Lagrangian. In order that the Lagrangian of the field L be invariant, it must be a function of the invariants of the electromagnetic field, consisting of the derivatives of the potentials of the electromagnetic field.[6,16] An invariant which consists of the components of the potentials A_μ must not be contained in the Lagrangian; otherwise we would not arrive at equations of the Maxwell type. We shall further consider L to be a function of only a single invariant $I = f_{\mu\nu}f_{\mu\nu}$ consisting of the components of the antisymmetric tensor

$$f_{\mu\nu} = \partial A_\nu/\partial x_\mu - \partial A_\mu/\partial x_\nu. \quad (19)$$

Furthermore, by forming the action integral and varying it with respect to the variable A_ν and $A_{\mu,\nu} = \partial A_\mu/\partial x_\nu$, then equating to zero the variant of the action integral, we obtain Euler's equations:[16]

$$\partial p_{\mu\nu}/\partial x_\mu = 0, \quad (20)$$

where

$$p_{\mu\nu} = \partial L/\partial A_{\mu,\nu} = -\partial L/\partial f_{\mu\nu}. \quad (21)$$

On the other hand, as a consequence of the antisymmetry of the tensor $f_{\mu\nu}$ we get

$$dL = \frac{1}{2}\frac{\partial L}{\partial f_{\mu\nu}}\, df_{\mu\nu} = -\frac{1}{2} p_{\mu\nu} df_{\mu\nu}. \quad (22)$$

---

* For proof, we write on the one hand,

$$\frac{1}{{}_{-1}^{\ 1}R} = \frac{\mathcal{H}}{e^2} = \frac{1}{8\pi}\int E^2 dv = \frac{1}{2}\int\left(\frac{d\varphi}{dr}\right)^2 r^2 dr,$$

since $E = -\operatorname{grad}\varphi$.[12] On the other hand, according to the foregoing [see the derivation of Eq. (10)],

$${}_2R^2 = \langle r^2\rangle = 6\int_0^\infty \left(\varphi - \frac{1}{r}\right) r^2 dr.$$

Now, constructing the function ${}_2R^2 - \lambda/{}_{-1}^{\ 1}R$ and varying it in φ, we obtain

$$\delta J = \int_0^\infty \left\{6r^2 - \frac{1}{2\lambda}\frac{d}{dr}\left(2r^2\frac{d\varphi}{dr}\right)\right\}\delta\varphi\, dr.$$

Then, setting the variation equal to zero, we find $\nabla^2\varphi = 6/\lambda$ = const; this means that the density of charge distribution must be constant.

The invariant properties of the Lagrangian relative to a transformation of the coordinates allows us to write it in a much simpler form. In order to show this, we carry out an infinitely small trans formation of the coordinates:[16,17]

$$\bar{x}_\mu = x_\mu + \varepsilon\xi_\mu(\bar{x}_\nu),$$

for which the potentials A_μ and their derivatives are equal to

$$\bar{A}_\mu = A_\mu - \varepsilon \frac{\partial \xi_\nu}{\partial \bar{x}_\mu} A_\nu,$$

$$\bar{A}_{\mu,\nu} = \frac{\partial \bar{A}_\nu}{\partial \bar{x}_\mu} = \frac{\partial}{\partial x_\sigma}\left(A_\nu - \varepsilon \frac{\partial \xi_\tau}{\partial x_\nu} A_\tau\right)\frac{\partial x_\sigma}{\partial \bar{x}_\mu}$$

We also find that

$$\delta L = \frac{1}{2} p_{\mu\nu}\,\delta A_{\mu\nu} = -\frac{\varepsilon}{2}(p_{\mu\nu}A_{\sigma\nu} + p_{\nu\mu}A_{\nu\sigma}) \times \frac{\partial \xi_\sigma}{\partial x_\mu} + \frac{1}{2} p_{\sigma\mu}A_\nu \frac{\partial^2 \xi_\nu}{\partial x_\sigma \partial x_\mu}$$

and the Jacobian of the transformation is equal to

$$\partial(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)/\partial(x_1, x_2, x_3, x_4) = 1 + \varepsilon\partial\xi_\mu/\partial x_\mu.$$

From the invariance of the Lagrangian under a coordinate transformation, it follows that

$$\int (L + \delta L)\,dx = \int L\,dx \tag{23}$$

(the integration is carried out over all four variables), whence, taking it into account that $p_{\sigma\mu}A_\nu\partial^2\xi_\nu/\partial x_\sigma\partial x_\mu = 0$ by virtue of the antisymmetry of $p_{\sigma\mu}$, we get (by equating tne expressions for $d\xi_\sigma/dx_\mu$):

$$L = \frac{1}{2}(p_{\mu\nu}A_{\mu\nu} + p_{\nu\mu}A_{\nu\mu}) = \frac{1}{2} p_{\mu\nu}f_{\mu\nu}. \tag{24}$$

Furthermore, the energy-momentum tensor of the electromagnetic field can be determined in the usual fashion; it can be shown that in the case of nonlinear electrodynamics the same conservation laws will be satisfied as in the case of the linear theory.[6,16] Moreover, Laue's theorem is valid. The Hamiltonian of the nonlinear field, which is of interest to us, can be written in the following form:

$$H_{44} = {}^1/_2\, p_{\mu\nu}f_{\mu\nu} + p_{\nu 4}f_{\nu 4}. \tag{25}$$

We now proceed to the usual vector description of electrodynamics, defining the electric field intensity $\mathbf{E}$ and magnetic field intensity $\mathbf{H}$, and the corresponding inductions $\mathbf{D}$ and $\mathbf{B}$, as

$$\mathbf{E} = i\sqrt{4\pi}\{f_{14}, f_{24}, f_{34}\}, \quad \mathbf{D} = i\sqrt{4\pi}\{p_{14}, p_{24}, p_{34}\},$$
$$\mathbf{B} = \sqrt{4\pi}\{f_{23}, f_{31}, f_{12}\}, \quad \mathbf{H} = \sqrt{4\pi}\{p_{23}, p_{31}, p_{12}\}. \tag{26}$$

Then the Euler equations (20) and the integrability conditions

$$\partial f_{\mu\nu}/\partial x_\sigma + \partial f_{\nu\sigma}/\partial x_\mu + \partial f_{\sigma\mu}/\partial x_\nu = 0$$

(which follow from the special choice of the functions $f_{\mu\nu}$) give the Maxwell equations in their usual form.[6,14,16] However, while in the linear theorem, for the case of a vacuum, $\mathbf{E} = \mathbf{D}$, $\mathbf{B} = \mathbf{H}$, these relations are no longer preserved in the nonlinear theory. In fact,

$$p_{\mu\nu} = -\frac{dL}{dI}\frac{\partial I}{\partial f_{\mu\nu}} = \varepsilon(I) f_{\mu\nu}$$

or,

$$\mathbf{D} = \varepsilon(I)\mathbf{E};\ \mathbf{H} = \varepsilon(I)\mathbf{B}. \tag{27}$$

In the usual notation, the expressions for I, L and H_{44} take the following form

$$I = -(E^2 - B^2)/8\pi; \tag{28}$$

$$L = (1/8\pi)(-DE + BH); \tag{29}$$

$$H_{44} = (1/8\pi)(DE + BH). \tag{30}$$

The Equivalent Charge Distribution and the Effective Radius of Nonlinearity

We shall show that the Lagrangian of the nonlinear field can also be given with the aid of the density distribution of the equivalent charge of the linear field. For this purpose, we consider a special static case. Let there be a point charge e. The solution of the equation $\operatorname{div}\mathbf{D} = 4\pi e\delta(\mathbf{r})$ is $\mathbf{D} = e\mathbf{r}/r^3$. We now compare the nonlinear field $\mathbf{E} = \mathbf{D}/\epsilon(I)$ with the linear field $\mathbf{E}_{lin}$ which is equal to it at all points of space. It is obvious that the source of $\mathbf{E}_{lin}$ could be any distribution ρ'_e of the same charge e (generally speaking, not a point distribution), while, in accord with Gauss' theorem:

$$\mathbf{E}_{lin} = \frac{e\mathbf{r}}{r^3}\int_0^r \rho'_e dv.$$

But since, by assumption, $\mathbf{E} = \mathbf{E}_{lin}$ at each point of space, we can write

$$\mathbf{E} = \mathbf{D}\int_0^r \rho'_e dv = \mathbf{D}\int_0^{(e^2/D^2)^{1/4}} \rho'_e dv, \tag{31}$$

since $r = (e^2/D^2)^{1/4}$. But then, by (29),

$$L = \frac{D^2}{8\pi}\int_0^{\sqrt{e/D}} \rho'_e dv \tag{32a}$$

in the static case, and

$$L = I\int_0^{(e^2/8\pi I_1)^{1/4}} \rho'_e dv \tag{32b}$$

in the general case, in which $I_1 = (D^2 - H^2)/8\pi$ and consequently, L is invariant. As is seen from the latter equations, the Lagrangian can actually be written with the aid of functions of the equivalent distribution ρ_e'.[18] Such a form of the solution turns out to be more useful for subsequent analysis. We express the total energy of the field $\mathcal{H} = \int H_{44}\,dv$ as a function of the equivalent distribution of charge ρ_e', making use of Eqs. (30) and (31), and carrying out the integration by parts. We obtain

$$\mathcal{H} = e^2/2\,{}_{-1}R_e, \quad 1/{}_{-1}R_e = \int_0^\infty \frac{\rho_e' dv}{r}. \tag{33}$$

In the case of the nonlinear theory, it is natural to assume that the mass of the electron has a purely electromagnetic origin i.e., that $\mathcal{H} = m_0c^2$. But it then follows from (33) that

$${}_{-1}R_e = r_0/2. \tag{34}$$

We shall show that for an arbitrary $\rho_e'(r)$ which changes sign nowhere the rms radius of distribution of the effective charge is larger than ${}_{-1}R_e$.[18] For this purpose, we write ${}_2R^2 \equiv \langle r_e^2 \rangle$ in the form

$$\langle r_e^2 \rangle = \int_0^\infty \rho_e' r^4 dr \Bigg/ \int_0^\infty \rho_e' r^2 dr$$

$$= \frac{1}{{}_{-1}R^2} \int_0^\infty \rho_e' r^4 dr \left[\int_0^\infty \rho_e' r\, dr\right]^2 \left[\int_0^\infty \rho_e' r^2 dr\right]^{-3}$$

and we show that the factor in front of ${}_{-1}R^{-2}$ is not smaller than unity or, otherwise, that

$$F = \int_0^\infty \rho_e' r^4 dr \left[\int_0^\infty \rho_e' r\, dr\right]^2 - \left[\int_0^\infty \rho_e' r^2 dr\right]^3 \geqslant 0.$$

To do this, we introduce the notation $\varphi(r) = \rho_e' r^2$, write F in the form of the limit of the sum

$$F = \lim\left\{\sum_{i=1}^\infty \varphi(r_i) r_i^2 \Delta r_i \left[\sum_{j=1}^\infty \varphi(r_j)\frac{\Delta r_j}{r_j}\right] - \left[\sum_{i=1}^\infty \varphi(r_i)\,\Delta r_i\right]^3\right\}$$

$$= \lim \sum_{i>j>k} \varphi(r_i)\varphi(r_j)\varphi(r_k)\,\Delta r_i \Delta r_j \Delta r_k$$

$$\times \left\{\frac{r_i^2}{r_j r_k} + \frac{r_j^2}{r_k r_i} + \frac{r_k^2}{r_i r_j} - 3\right\}.$$

But, inasmuch as

$$(r_i^3 + r_j^3 + r_k^3 - 3r_i r_j r_k) = \frac{1}{2}(r_i + r_j + r_k)$$
$$\times [(r_i - r_j)^2 + (r_j - r_k)^2 + (r_k - r_i)^2] \geqslant 0,$$

then $F \geq 0$, and consequently,

$${}_2R_e \geqslant {}_{-1}R_e = r_0/2. \tag{35}$$

The latter equation is similar to Eq. (18) of the linear theory of the extended electron. If the distribution ρ_p is not alternating in sign, then, as can be shown, ${}_2R_e$ should not be larger than the rms radius of Hofstadter, R_0. But this would contradict the inequality (35), since $R_0 < r_0/2$. Thus, either ρ_e' or ρ_p (which is less probable from the viewpoint of meson theory[19,4]) must change sign. This is the essential result of the nonlinear theory.

Particle Interaction in the Nonlinear Theory

The problem of particle interaction in the nonlinear theory is more difficult than in the linear theory. In the nonlinear theory it is difficult to separate the characteristic energy from the energy of interaction. The interaction energy between two (for example, point) particles is no longer equal to the product of the charge of the first particle by the potential created and the charge of the second particle at the location of the first particle (in its absence). In a similar fashion, the usual expression for the Lorentz forces can be shown to be unsatisfactory. For interpretation of the experimental data, it is important to know the law of interaction between the particles. Therefore, we consider the interaction between two charged particles, in the general case of attraction, with the charge distributions ρ_1 and ρ_2. We make use of the fact that in the nonlinear theory the induction vectors add linearly, as before: $\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2$, which is not valid for the fields. In this case,

$$\mathbf{D}_1 = \frac{e_1\mathbf{r}_1}{r_1^3}\int_0^{r_1} \rho_1 dv_1, \qquad \mathbf{D}_2 = \frac{e_2\mathbf{r}_2}{r_2^3}\int_0^{r_2} \rho_2 dv_1. \tag{36}$$

Furthermore, in accord with Eqs. (30) and (31) we can write down the expression for the total field energy in the following form:

$$\mathcal{H} = \int H_{44} dv = \frac{1}{8\pi}\int (D_1^2 + D_2^2 + 2\mathbf{D}_1\cdot\mathbf{D}_2)\,dv \int_0^{\sqrt{e/D}} \rho' dv',$$

where ρ' is the density of the equivalent charge distribution. Then, computing the energy of the free field (the sum of the characteristic energies of the isolated noninteracting particles)

$$\mathcal{H}_0 = \frac{1}{8\pi}\int D_1^2 dv \int_0^{\sqrt{e/D_1}} \rho' dv' + \frac{1}{8\pi}\int D_2^2 dv \int_0^{\sqrt{e/D_2}} \rho' dv,$$

we get the "pure" interaction energy:

$$W \doteq \mathcal{H} - \mathcal{H}_0 = W_{11} + W_{12} + W_{22}. \tag{37}$$

Here

$$W_{11} = \frac{1}{8\pi}\int D_1^2 dv \int_{r_1'}^{r_3'} \rho' dv', \tag{38}$$

$$W_{12} = \frac{1}{4\pi}\int \mathbf{D}_1 \cdot \mathbf{D}_2\, dv \int\limits_0^{r_3'} \rho' dv', \tag{39}$$

$$W_{22} = \frac{1}{8\pi}\int D_2^2 dv \int\limits_{r_2'}^{r_3'} \rho' dv', \tag{40}$$

where

$$r_1' = \sqrt{\frac{e}{D_1}} = r_1 \Big/ \left(\int\limits_0^{r_1} \rho_1\, dv_1\right)^{1/2};$$
$$r_2' = \sqrt{\frac{e}{D_2}} = r_2 \Big/ \left(\int\limits_0^{r_2} \rho_2 dv_1\right)^{1/2}; \tag{41}$$
$$r_3' = \{(r_2')^{-4} + (r_1')^{-4} + 2\,(r_1')^{-2}\,(r_2')^{-2} \cos\chi\}^{-1/4};$$

χ is the angle between the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

It is easy to establish the fact that in the transition to ordinary electrodynamics and to point particles, we obtain $W = W_{12} = e_1 e_2/r_{12}$. We now consider the special case of the interaction of the proton with the electron. We shall assume that the dimensions of the proton (for example, as a consequence of the interaction with the meson field) are much larger than the dimensions of the electron, which we shall consider to be a point (this means that $r_2' = r_2$). Let the nonlinearity ap-appear only in the small region r^0, so that $\int_0^r \rho' dv' \approx 1$ for all $r > r^0$, while r^0 is less than the radius of the proton. We shall show that in this case the expression for the interaction energy of the particles (37) is materially simplified. From the locations of the first particle (the center of the extended proton) and the second particle (the electron) we draw spheres of radius r^0. The following two cases are then possible, depending on the distance between the particles: (a) the spheres do not overlap or (b) they do overlap. We consider the first case, in which $r_0 \leq r/2$ (r = distance between the particles). In the region $r_2 < r^0$, $r_3' \approx r_2$ [see (41)], since $r_2 > r_1'$, inasmuch as r_1' [as follows from its definition, see Eq. (41)] cannot be smaller than the radius of the proton r_p, which in turn is larger (by assumption) than r^0. In the region $r_2 > r_0$, r_3' is not smaller than $1/(r_2^{-1} + r_p^{-1})$, which is in any case larger than r^0, and since $r_2 > r_0$, then, not changing the value of the integral $\int_0^{r_3'} \rho' dv'$, we can substitute the upper limit for r_2.

In similar fashion, we can consider the case (b) and show that for one reason or another we can replace r_3' by r_2. We then have approximately

$$W_{12} \approx \frac{1}{4\pi}\int \mathbf{D}_1 \cdot \mathbf{D}_2\, dv \int\limits_0^{r_2} \rho\, dv'.$$

Substituting the values for $\mathbf{D}_1$ and $\mathbf{D}_2$ from (36) (and taking it into account that now $\mathbf{D}_2 = e\mathbf{r}_2/r_2^3$) we have

$$W_{12} \approx \frac{1}{4\pi}\int dv \left\{\frac{e r_1}{r_1^3}\int\limits_0^{r_1} \rho_p dv_1\right\}\left\{\frac{e r_2}{r_2^3}\int\limits_0^{r_2} \rho' dv'\right\}. \tag{42}$$

But this coincides in form with the interaction of two extended particles in the linear theory: the proton with a charge distribution ρ_p and the electron with the distribution ρ'. In other words, Eq. (1) and the consequences obtained in Secs. 1 and 2, for the effective cross section and displacement of the electronic levels in the atoms, remain valid in this case.

An estimate shows that the correction term to (42) is $\frac{1}{2}(r^0/r_p)^2$ times smaller than the principal term; the term W_{11} amounts to less than $\frac{1}{3}(r^0/r_p)^3$ part of W_{12}, while W_{22} is $\frac{1}{4}(r^0/r_p)$ part.

In the case of the interaction of two electrons, the terms W_{11} and W_{22} are still quite substantial at distances of the order of the effective radius of nonlinearity. An important peculiarity of the nonlinear theory is the difference of the scattering cross section of positrons on electrons from the scattering of electrons on electrons, since in the case of the interaction of particles of identical charge sign, W_{12} is added to W_{11} and W_{22}, while for the interaction of differently charged particles, W_{12} is subtracted from the sum $W_{11} + W_{22}$. A similar effect ought to appear in the scattering of positrons on a proton, by comparison with the scattering of electrons. However, if the radius of nonlinearity is smaller than the larger of the radii of the proton and electron, the effect of nonlinearity would show itself to be negligible. On the other hand, inasmuch as from the viewpoint of the nonlinear theory ρ_e' (or ρ_p) must be charge alternating, this would necessarily lead to peculiarities in the angular distribution.

The possible corrections can be deduced with the aid of the dynamic effects, although in principle the interaction of the electric charges under the experimental conditions[1,2] must be distinguished from the two effects.[4] A more precise calculation of the radiative corrections must also be carried out, both for the case of the linear theory of the extended electron and in particular for the nonlinear theory. However, improving the precision of the calculations of the radiative corrections, which are relatively small in magnitude,[20] almost never leads to a serious change in the results obtained.

We take this opportunity to acknowledge our deep gratitude to Prof. Louis de Broglie and D. D. Ivanenko, who maintained constant interest in the research, and also to J. P. Vigier, D. Bohm and T. Takabayashi, who made a number of valuable comments.

[1] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).

[2] F. Bumiller and R. Hofstadter, Bull. Am. Phys. Soc., Ser. 2. **2**, 390 (1957); M. R. Yearian and R. Hofstadter, Bull. Am. Phys. Soc. Ser. 2, **2**, 389 (1957).

[3] N. F. Mott, Proc. Roy. Soc. (London) **A124**, 425 (1929).

[4] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956); Yennil, Levy and Ravenhall, Revs. Modern Phys. **29**, 144 (1957).

[5] V. A. Petukhov, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 379 (1957); Soviet Phys. JETP **5**, 317 (1957).

[6] D. D. Ivanenko and A. A. Sokolov, Классическая теория поля (Classical Theory of Fields), GITTL, 1951.

[7] H. Mott and G. Massey, Theory of Atomic Collisions (Oxford, 1933).

[8] H. Feshbach, Phys. Rev. **84**, 1206 (1951). A. Bodmer, Proc. Phys. Soc. (London) **66**, 1041 (1953).

[9] N. N. Kolesnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 819 (1957); Soviet Phys. JETP **6**, 631 (1958).

[10] D. D. Ivanenko and N. N. Kolesnikov, Dokl. Akad. Nauk SSSR **91**, 47 (1953).

[11] Lamb, Triebwasser, and Dayhoff, Phys. Rev. **89**, 98 (1953).

[12] L. Cooper and E. Henley, Phys. Rev. **92**, 801 (1953).

[13] D. I. Blokhintsev, Usp. Fiz. Nauk **61**, 137 (1957).

[14] M. Born, Proc. Roy. Soc. (London) **A143**, 410 (1934).

[15] M. Born and L. Infeld, Proc. Roy. Soc. **A144**, 425 (1934); L. Infeld, Proc. Camb. Phil. Soc. **32**, 127 (1936); **33**, 70 (1937).

[16] M. Born, Annales de l'institut Henri Poincaré, v. 7, Paris, 1937.

[17] W. Pauli, Relativitatstheorie, Encylcop. d. Math. Wiss. v. 19.

[18] G. Jacobi and N. Kolesnikov, Comp. rend. **245**, 285 (1957).

[19] I. E. Tamm, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 178 (1957); Soviet Phys. JETP **5**, 154 (1957).

[20] J. Schwinger, Phys. Rev. **75**, 898 (1949); H. Suura, Phys. Rev. **99**, 1020 (1955).

Translated by R. T. Beyer
70

SOVIET PHYSICS JETP VOLUME 35(8), NUMBER 2 FEBRUARY, 1959

THE TEMPERATURE OF PLASMA ELECTRONS IN A VARIABLE ELECTRIC FIELD

A. V. GUREVICH

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor April 22, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 392-400 (August, 1958)

The heating of electrons in a plasma in a variable electric field is considered. It is shown that the electron gas can exist in two stable states with different temperatures; the transition from one state to the other takes place at certain critical values of the field and is accompanied by an appreciable change in the electron temperature. A peculiar type of hysteresis takes place in the dependence of the electron temperature on the field amplitude and frequency. The influence of a constant magnetic field on this effect is also taken into account. An expression is obtained for the complex conductivity of the plasma in variable electric and constant magnetic fields (with account of interelectronic collisions).

1. We consider an unbounded plasma placed in a spatially homogeneous electric field. We assume that the plasma is sufficiently strongly ionized that the principal influence on the distribution of the electrons is that due to collisions between electrons and between electrons and ions; we shall con-