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THE TEMPERATURE OF PLASMA ELECTRONS IN A VARIABLE ELECTRIC FIELD

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The heating of electrons in a plasma in a variable electric field is considered. It is shown that the electron gas can exist in two stable states with different temperatures; the transition from one state to the other takes place at certain critical values of the field and is accompanied by an appreciable change in the electron temperature. A peculiar type of hysteresis takes place in the dependence of the electron temperature on the field amplitude and frequency. The influence of a constant magnetic field on this effect is also taken into account. An expression is obtained for the complex conductivity of the plasma in variable electric and constant magnetic fields (with account of interelectronic collisions).

1. We consider an unbounded plasma placed in a spatially homogeneous electric field. We assume that the plasma is sufficiently strongly ionized that the principal influence on the distribution of the electrons is that due to collisions between electrons and between electrons and ions; we shall con-

sider these to be elastic.

In this case, as is shown in the Appendix, if the condition

$$dT_e/dt \ll \nu_{\text{eff}} T_e, \tag{1}$$

is satisfied, the symmetric part of the electron distribution function is Maxwellian; the electron temperature T_2 is defined by the equation

$$\frac{dT_e}{dt} + \delta\nu_{\text{eff}\,i}(T_e)(T_e - T) = \frac{2e\mathbf{E}}{3kN}\,\mathbf{j}(T_e). \tag{2}$$

Here, as usual, k is Boltzmann's constant, T the ion temperature, e the electronic charge, N the density of the electrons, and E the electric field intensity. Furthermore, $\delta = 2m/M$ is the average fraction of the energy given by an electron to an ion in a single collision (m is the electron mass and M the ion mass), and ν_{eff} is a parameter which it is reasonable to call the effective collision frequency of electrons with ions:*

$$\nu_{\text{eff}\,i} = \frac{4}{3}\sqrt{2\pi}\,\frac{e^4 N_i}{\sqrt{m}\,(kT_e)^{3/2}}\ln\left(\frac{kT_e}{e^2}p_{\max}\right), \tag{3}$$

where $p_{\max}$ is the maximum of the impact parameter, which must be taken as equal to the Debye radius [but sometimes another, somewhat different quantity is used (see reference 2, Sec. 82; also reference 4)]. We note that the quantity appearing in (3) under the logarithm sign is always much larger than unity. As a consequence, even for large changes in the electron temperature, the logarithm does not change appreciably, and it can be taken approximately that

$$\nu_{\text{eff}\,i} = \nu_0 (T_e/T)^{-3/2}, \tag{3'}$$

where ν_0 is the effective collision frequency of electrons with ions in a weak field (when $T_e = T$).

x	K_σ	K_ε
0	1.950	4.595
0.05	1.921	4.512
0.1	1.859	4.340
0.2	1.652	3.792
0.5	1.075	2.297
1.0	0.721	1.405
2.0	0.623	1.048
4.0	0.727	0.972
6.0	0.824	0.976
10.0	0.917	0.987
35.0	0.992	0.997

*We note that the effective collision frequency of the electron thus introduced coincides with that considered in reference 1. In reference 2 (see Secs. 61 ff) some other quantity is used as $\nu_{\text{eff}\,i}$ (its use leads to a series of difficulties which are pointed out in references 1 and 3).

Finally, j is the electron current density

$$\mathbf{j} = \left(\sigma + i\omega\frac{\varepsilon - 1}{4\pi}\right)\mathbf{E} \tag{4}$$
$$= \frac{e^2N}{m}\left\{\frac{\nu_{\text{eff}\,i}}{\omega^2 + \nu_{\text{eff}\,i}^2}K_\sigma\left(\frac{\omega}{\nu_{\text{eff}\,i}}\right) - i\frac{\omega}{\omega^2 + \nu_{\text{eff}\,i}^2}K_\varepsilon\left(\frac{\omega}{\nu_{\text{eff}\,i}}\right)\right\}\mathbf{E}.$$

Here ν_{eff_i} is the effective collision frequency of the electrons, defined by (3), (3′), while $K_\sigma(x)$ and $K_\epsilon(x)$ are certain functions the computed values of which are given in the table.*

It is seen from the table that the functions K_σ and K_ϵ do not change very markedly; therefore, for qualitative estimates, we can set them equal to unity, i.e., we can use the simple ("elementary") formulas† for σ and ϵ.

2. We now turn to the analysis of the solution of Eq. (2). In this case we assume that the temperature of the heavy particles in the plasma is constant (this case, which was considered in Sec. 3, is of interest in a methodological way, since it allows us to clarify some peculiarities of the heating of an electron gas in a plasma).

We first assume that the electric field $\mathbf{E} = \mathbf{E}_0 \cos \omega t$ changes rapidly ($\omega \gg \delta\nu_0$). In this

*If we neglect the coefficients K_σ and K_ε in Eq. (4), then it coincides with the well known "elementary" expression which is obtained in the hydrodynamic approximation (see, e.g., reference 2, Sec. 57). As is well known,[3] the latter is strictly valid for electrons in a plasma for any frequency of the electric field, only if the condition is satisfied that the frequency of collision of the electron with heavy particles is independent of their velocity. In the case of collision with ions, this condition is naturally not satisfied ($\nu \sim v^{-3}$). Therefore, the kinetic consideration given in the Appendix is also necessary. It leads to the appearance of corrected coefficients K_σ and K_ε which reflect the dispersion of the electron collision frequency. For $\omega \gg \nu_{\text{eff}\,i}$, the coefficients K_σ and K_ε tend to unity, which is in accord with reference 5. In the case of a constant electric field ($\omega = 0$), the conductivity computed by Eq. (4) [see also Eq. (10)] coincides with that obtained in reference 6, as it must.

†It should be noted that the values of the functions K_σ and K_ε given in the table were computed for the case in which the electron density in the plasma is equal to the ion density. The functions K_σ and K_ε in the case in which the ion density is much greater than that of electrons (i.e., when there are more negative ions in the plasma and, therefore, collisions between the electrons are not important) were obtained in reference 1 (Fig. 7). The same functions K_σ and K_ε are also valid for multiply ionized plasma, when $Z \gg 1$ (it must be kept in mind that the effective collision frequency of the electron is increased Z-fold in this case). The functions K_σ and K_ε for doubly, triply, . . . , ionized plasma lie between the limiting functions for $Z = 1$ (the table) and for $Z \gg 1$; it is easy to compute them by making use of the results of Landshoff[6] (comparing the results of reference 6 with Eq. (10) for the case $\omega = 0$).

case the field changes more rapidly with time than the electron temperature can be changed. Therefore, this temperature must be established at some mean value of T_{e0} that is time independent; deviations from this value are small. Actually, the solution of Eq. (2) in the case under consideration has the following form:

$$T_e = T_{e0} + \frac{\delta\nu_0}{2\omega}\frac{T_{e0}-T}{(T_{e0}/T)^{3/2}}\sin 2\omega t + O\left[\left(\frac{\delta\nu_0}{\omega}\right)^2\right], \qquad (5)$$

where the temperature T_{e0} is determined from the transcendental equation

$$\frac{T_{e0}}{T}-1 = \frac{e^2E_0^2}{3kTm\delta\nu_0^2}\,\frac{K_\sigma[(\omega/\nu_0)(T_{e0}/T)^{3/2}]}{(\omega/\nu_0)^2+(T_{e0}/T)^{-3}}\,. \qquad (6)$$

The condition (1) for establishing the solution (5) is always satisfied.

Analysis of the solution of the latter equation shows that at high frequencies ($\omega > \nu_0$) there corresponds to each value of the field a particular value of the temperature T_{e0} as, generally speaking, ought to be the case. However, at low frequencies ($\omega \ll \nu_0$), an interesting peculiarity arises: to a definite value of the field (in the region $E_k^{\mathrm{II}} < E_0 < E_k^{\mathrm{I}}$) there corresponds not one but three values of the temperature T_{e0} (see Fig. 1).* The reason for this is that the electron gas in the plasma is not a closed system: in the particular model it absorbs energy from the electric field and transfers it to the heavy particles of the plasma; its stationary temperature is determined, naturally, by the condition of energy balance between that

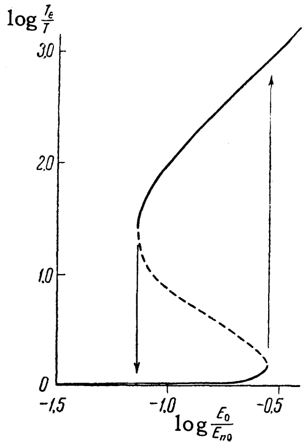

FIG. 1. Dependence of T_e/T on E_0/E_{n0} for $\omega = 0.01\nu_0$.

absorbed from the field and that given to the heavy particles [Eq. (6)]. In this case it is shown that for low frequencies the amount of energy absorbed and transmitted by the electron gas is linearly independent of its temperature and, thanks to this fact, Eq. (6) can be satisfied simultaneously for several different values of the electron temperature. However, only two of them, which correspond to the upper and lower curve (the solid line plotted in Fig. 1), are stable; the state corresponding to the middle curve (the dotted line in Fig. 1) is not stable. Moreover, the solution corresponding to the upper and lower curves becomes unstable at points where they join to the middle curve, i.e., at $E_0 = E_k^{\mathrm{I}}$ for the lower curve and for $E_0 = E_k^{\mathrm{II}}$ for the upper curve.*

In the following, we restrict ourselves to the state corresponding to the lower curve, called state I, and to the upper, state II. The value of the electron temperature in state I at $\omega \ll \nu_0$ is given approximately by the following expression:

$$T_{e0}^{\mathrm{I}} = T\left(1+\frac{K_\sigma(0)\,(E_0/E_{n0})^2}{1-4.5K_\sigma(0)\,(E_0/E_{n0})^2}\right),$$

and in state II by

$$T_{e0}^{\mathrm{II}} = T\left(\frac{E_0}{E_{n0}}\right)^2\left(\frac{\nu_0}{\omega}\right)^2\left[1+\left(\frac{\omega}{\nu_0}\right)^4\Big/\left(\frac{E_0}{E_{n0}}\right)^6\right]^{-1}. \qquad (7)$$

Here E_{n0} is the characteristic field for the plasma:[3]

$$E_{n0} = \sqrt{3kTm\delta\nu_0^2}/e \approx 2\cdot 10^{-8}\frac{N_i}{T}\sqrt{\delta}\ \mathrm{V/cm}. \qquad (8)$$

The first critical value of the field $E_k^{\mathrm{I}} \approx 0.28\,E_{n0}$; the corresponding temperature is $T_k^{\mathrm{I}} = 1.5\,T$ in state I and T_k^{II}; $0.076\,(\nu_0/\omega)^2\,T$ in state II. For a sufficiently low frequency ω, the temperature T_k^{II} is many times larger than T_k^{I} (for example, in the case shown in Fig. 1 ($\omega = 0.01\,\nu_0$), $T_k^{\mathrm{I}} \approx 1.5\,T$, and $T_k^{\mathrm{II}} \approx 760\,T$). The second critical value for the field is $E_k^{\mathrm{II}} \approx 1.7\,(\omega/\nu_0)^{2/3}\,E_{n0}$; the corresponding value of the temperature in state I is $T_k^{\mathrm{I}} \approx T$, and in state II, $T_k^{\mathrm{II}} \approx 1.2\,(\nu_0/\omega)^{2/3}\,T$.

We now consider how the heating of the electron gas in the plasma will take place at slow changes in the amplitude of the intensity of the variable electric field. At small values of E_0 the electron gas is located in state I and the temperature of the electrons differs slightly from the ion temperature (see Fig. 1). However, when the field E_0 reaches its own first critical value E_k^{I}, state I becomes unstable (since the electrons can no longer transfer all their absorbed energy to the heavy particles). Therefore the electron gas at $E_0 = E_k^{\mathrm{I}}$ is heated,

*In constructing the dependence of T_{e0}/T on E_0/E_{n0}, it is particularly useful to inverse the problem and construct the graph of the dependence of E_0/E_{n0} on T_{e0}/T.

*For analysis of the stability of a stationary value of T_{e0}, it is necessary to investigate the behavior of the derivative dT_e/dt close to T_{e0}. This can be done both analytically (by considering the change in time of a small perturbation ΔT_e) and graphically [by constructing the dependence of dT_e/dt on T_e, with the help of Eq. (2)].

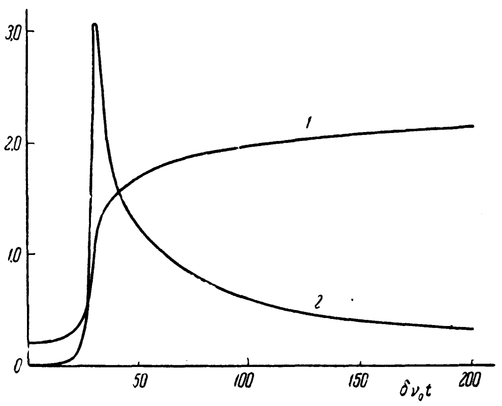

FIG. 2. Time dependence of the electron temperature and its derivative for the transition I→ II (for $\omega = 0.01\nu_0$); curve 1 – for log (T_e/T), 2 – for $(1/T\delta\nu_0)dT_e/dt$.

and its temperature rises to the corresponding value in state II (this transition is shown by the dashed curve in Fig. 1).

If the field E_0 now begins to fall below the value E_k^I, then T_{e0} takes on still higher values, corresponding to state II. However, when the field decreases to its second critical value E_k^{II}, state II becomes unstable (the electrons transfer more energy to the heavy particles than the field gives to them). Therefore, at $E_0 = E_k^{II}$, the electron gas is cooled and its temperature drops to the value corresponding to state I, i.e., to T (this transition is also shown by the dashed curve in Fig. 1). Thus, in the case under consideration, an unusual hysteresis takes place in the dependence of the electron temperature on the amplitude of the intensity of the variable electric field.

The process of transition from state I to state II, and vice versa, is described by Eq. (2).* The corresponding solution for the transition (for $\omega = 0.01\,\nu_0$) is shown in Fig. 2. It is seen from the figure that the rate of increase of the electron temperature increases rapidly at first, reaches a maximum, and then falls off slowly; its maximum value is rather high:

$$(dT_e/dt)_{\max} \approx 0.03\,(\nu_0/\omega)\,\delta\nu_0 T. \tag{9}$$

The change in the electron temperature for the transitions is naturally accompanied by corresponding changes in the effective collision frequency of the electrons (3) and, consequently, of the kinetic coefficients, too.

The dependence of the temperature T_{e0} on the frequency ω is completely analogous to the dependence of T_{e0} on E_0. For large values of E_0, only one stationary value of electron temperature corresponds to each value of the frequency; for small E_0, two values of T_{e0} can correspond to a single value of ω (see Fig. 3). In this case, however, transitions from state I (lower curve) to state II (upper curve) are possible only for E_0 larger than E_k^I; therefore state I at $E_0 < E_k^I$ is stable for arbitrary values of the frequency ω.

The critical frequency ω_k and the critical field E_k (above which only a single stationary state exists) are determined from the conditions

$$dT_{e0}/d\,[(E_0/E_{n0})^2] \to \infty,\ d^2T_{e0}/d^2\,[(E_0/E_{n0})^2] \to \infty,$$

from which it follows that $\omega_k \approx 0.2\,\nu_0$; $E_k \approx 0.4\,E_{n0}$. Hysteresis of the dependence of T_{e0} on E_0 is possible only for $\omega < \omega_k$, while hysteresis of the dependence of T_{e0} on ω is possible for $E_k^I < E_0 < E_k$.

The external electric field is assumed to be rapidly changing ($\omega \gg \delta\nu_0$). Quite analogous effects take place also in the opposite case, when $\omega \lesssim \delta\nu_0$. Their analysis is, generally speaking, more complicated. However, we can note that if the amplitude of E_0 is larger than $E_k^I/\sqrt{2}$, the electron temperature executes a complete hysteresis cycle during each period, similar to that described above.* In this case the temperature corresponding to state II increases with decrease in ω as $1/\omega^2$ (this is seen, for example, from Eq. (7); see also Fig. 3). For a constant field ($\omega = 0$) no stationary state II exists in general: in this case, if only the field E_0 is larger than $E_k^I/\sqrt{2}$, the energy of the electrons increase continuously with time. Condition (1) is violated here; the electron velocity distribution function takes on a sharply directional character.†

3. It is not difficult to generalize these results to the case where a constant magnetic field **H** is present in the plasma. Equation (2) for the electron temperature and Eq. (3) for the effective collision frequency are preserved in this case; only the expression for the electron current is changed: the conductivity and the dielectric constant of the plasma become tensors.

The tensor components of σ and ϵ are expressed by means of the functions K_σ and K_ϵ,

*Although dT_e/dt increases appreciably in the transition, the condition (1) is seen to be satisfied as previously (for arbitrary $\omega > \delta\nu$).

*It should be noted, however, that condition (1) in this case is not always satisfied; the electron velocity distribution function in this case is definitely not Maxwellian.

†It should be emphasized that, as is seen from the results of this section, the stationary low frequency state ($\omega \ll \nu$) can exist only in the case of a weak electric field ($E_0 \ll E_{n0}$). Therefore the expressions for the complex conductivity and thermoelectric coefficient, obtained in reference 6 (see also references 4 and 7), are also valid in the present case.

developed above. For the components of the tensors σ and ϵ in the direction parallel to the magnetic field, the same expressions (4) are still valid; in a plane perpendicular to **H** (the plane xy), we have:

$$\sigma_{xx}=\sigma_{yy}=\frac{e^2N}{m}\frac{\nu_{\text{eff}\,i}}{2}\left\{\frac{K_\sigma[|\omega-\omega_H|/\nu_{\text{eff}\,i}]}{(\omega-\omega_H)^2+\nu^2_{\text{eff}\,i}}+\frac{K_\sigma[(\omega+\omega_H)/\nu_{\text{eff}\,i}]}{(\omega+\omega_H)^2+\nu^2_{\text{eff}\,i}}\right\},$$

$$\sigma_{xy}=-\sigma_{yx}=-i\frac{e^2N}{m}\frac{\nu_{\text{eff}\,i}}{2}\left\{\frac{K_\sigma[|\omega-\omega_H|/\nu_{\text{eff}\,i}]}{(\omega-\omega_H^2)+\nu^2_{\text{eff}\,i}}-\frac{K_\sigma[(\omega+\omega_H)/\nu_{\text{eff}\,i}]}{(\omega+\omega_H)^2+\nu^2_{\text{eff}\,i}}\right\}, \quad (10)$$

$$\varepsilon_{xx}=\varepsilon_{yy}=1-\frac{4\pi e^2N}{m}\frac{1}{2\omega}\left\{\frac{(\omega-\omega_H)K_\varepsilon[|\omega-\omega_H|/\nu_{\text{eff}\,i}]}{(\omega-\omega_H)^2+\nu^2_{\text{eff}\,i}}+\frac{(\omega+\omega_H)K_\varepsilon[(\omega+\omega_H)/\nu_{\text{eff}\,i}]}{(\omega+\omega_H)^2+\nu^2_{\text{eff}\,i}}\right\},$$

$$\varepsilon_{xy}=-\varepsilon_{yx}=i\frac{4\pi e^2N}{m}\frac{1}{2\omega}\left\{\frac{(\omega-\omega_H)K_\varepsilon[|\omega-\omega_H|/\nu_{\text{eff}\,i}]}{(\omega-\omega_H)^2+\nu^2_{\text{eff}\,i}}-\frac{(\omega+\omega_H)K_\varepsilon[(\omega+\omega_H)/\nu_{\text{eff}\,i}]}{(\omega+\omega_H)^2+\nu^2_{\text{eff}\,i}}\right\}.$$

Here $\omega_H = eH/mc$ is the gyromagnetic frequency.

Now, substituting the expression for the electron current in (2), we get an equation through which the temperature of the electrons is also determined. In a rapidly varying field, it is stationary and is equal to T_{e0}; the latter satisfies the following transcendental equation (see reference 1):

$$\frac{T_{e0}}{T}-1=\left(\frac{E_0}{E_{n0}}\right)^2\left\{\frac{\cos^2\beta K_\sigma[(\omega/\nu_0)(T_{e0}/T)^{3/2}]}{(\omega/\nu_0)^2+(T_{e0}/T)^{-3}}+\frac{\sin^2\beta K_\sigma[(|\omega-\omega_H|/\nu_0)(T_{e0}/T)^{3/2}]}{2[(\omega-\omega_H)^2/\nu_0^2+(T_{e0}/T)^{-3}]}+\frac{\sin^2\beta K_\sigma[((\omega+\omega_H)/\nu_0)(T_{e0}/T)^{3/2}]}{2[(\omega+\omega_H)^2/\nu_0^2+(T_{e0}/T)^{-3}]}\right\}. \quad (11)$$

Here β is the angle between **E** and **H**. For H = 0 (and also if $\beta = 0$) Eq. (11) coincides with (6), as it should.

Analysis of the dependence of the temperature T_{e0} on the amplitude of the field intensity E_0 is not difficult to carry out, in the same fashion as was done above in the case H = 0. For a high frequency ω_H ($\omega_H \gg \nu_0$), effects similar to those considered above (two stationary states with different electron temperatures) take place in the gyro-resonance frequency region, when $|\omega-\omega_H| \ll \nu_0$, $\sin\beta \neq 0$. It is interesting to observe that in this case, for small values of the amplitude of the field ($E_0 < \sqrt{2}\,E_k^I/\sin\beta$) there is no sort of resonance increase in the electron temperature in the vicinity of the gyro-frequency (state I, for $E_0 < \sqrt{2}\,E_k^I/\sin\beta$, is stable for an arbitrary frequency ω). On the other hand, in the case in which $E_0 \geq \sqrt{2}\,E_k^I/\sin\beta$, a strong resonance effect appears close to the gyro-frequency.

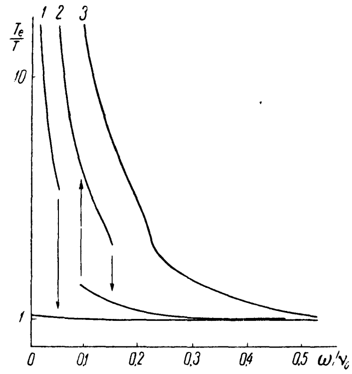

FIG. 3. Dependence of T_e/T on ω/ν_0 for various values of E_0: curve 1 – $E_0 < E_k^I$; curve 2 – $E_k^I < E_0 < E_k$; curve 3 – $E_k < E_0$.

4. The heating of the plasmas by the heavy particles was not considered above. However, it is entirely understood that in the adiabatic, isolated plasma, under conditions for which the gas is heated more slowly by the heavy particles, than the electron gas,* the consideration carried out is valid; it suffices only to assume that the temperature of the plasma T increases slowly with time. This leads to a situation in which the relative magnitude of the variable electric field E_0/E_{n0} increases with time (i.e., the point describing the state of the system on Fig. 1 is shifted to the right). When the ratio E_0/E_{n0} is equal to E_0/E_k^I, a transition takes place from state I to state II. It is essential that the temperature of the electrons in this transition changes rather sharply [see Eq. (9)]; therefore, even in the case in which the heating of the heavy particles, generally speaking, cannot be regarded as slow (for example, in a completely ionized gas, when $\alpha \approx 1$, see footnote below), the process of the transition of the electron gas from state I to state II at sufficiently low frequency is expressed explicitly, as before.

In this connection it should also be noted that the entire argument above was based on the assumption that the electric field is homogeneous in space. In a real system the field is usually in-

*The heavy particles in the plasma are heated through the work of the ion current and the energy transferred to it by electrons in collisions. The ion current in the cases of interest to us is always much weaker than the electron current; therefore, the condition of quasi-stationarity for the plasma heating due to the work of the ion current is always satisfied. The heating of the heavy particles due to collisions with electrons can be regarded as slow (quasi-stationary) for the case that $\alpha = N/(N_i + N_m) \ll 1$ (here N, N_i, N_m are the densities of electrons, ions, and molecules, respectively). This condition is well satisfied, for example, in the upper ionosphere.

homogeneous. Therefore, the critical value E_k^I is achieved initially only in a certain region of the plasma (and not throughout its volume); in this region the transition of the electron gas also begins from state I to state II.* A sharp electron temperature rise (and, consequently, in the pressure) in this region of the plasma during the transition ought to lead to the excitation of essentially nonstationary effects.

The author expresses his thanks to V. L. Ginzburg for interest in the research.

APPENDIX

ELECTRON-VELOCITY DISTRIBUTION FUNCTION IN A STRONGLY-IONIZED PLASMA IN A VARIABLE ELECTRIC FIELD†

As is well known,[8] we can write the electron velocity distribution function for the plasma in the form

$$f(\mathbf{v}, t) = f_0(v, t) + \frac{\mathbf{v}}{v}\mathbf{f}_1(v, t). \tag{A.1}$$

The Boltzmann equation reduces in this case to the following system of equations for the functions f_0 and $\mathbf{f}_1$:

$$\frac{\partial f_0}{\partial t} + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\cdot\mathbf{f}_1) - \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\nu v^2\frac{kT}{M}\frac{\partial f_0}{\partial v} + \nu\frac{m}{M}v^3 f_0\right\}$$
$$= -\int \sigma_{ee}(w, \theta')\, w\, \{f_0(v,t) f_0(v_1, t) - f_0(v', t) f_0(v_1', t)\}\, d\mathbf{v}_1 d\Omega'; \tag{A.2}$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + \frac{e}{mc}[\mathbf{H}\times\mathbf{f}_1] + \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \nu\mathbf{f}_1$$
$$= -\frac{3}{4\pi}\int \sigma_{ee}(w, \theta')\, w \left\{\frac{\mathbf{v}\cdot\mathbf{f}_1(v, t)}{v} f_0(v_1, t) + \frac{\mathbf{v}_1\cdot\mathbf{f}_1(v_1, t)}{v_1} f_0(v, t) - \frac{\mathbf{v}'\cdot\mathbf{f}_1(v', t)}{v'} f_0(v_1', t) - \frac{\mathbf{v}_1'\cdot\mathbf{f}_1(v_1', t)}{v_1'} f_0(v_1, t)\right\} d\mathbf{v}_1 d\Omega' d\Omega. \tag{A.3}$$

These equations differ from those obtained by Davydov[8] by the fact that interelectronic collisions are considered in them. These collisions are described by the integral terms on the right-hand side. Here $\mathbf{v}$, $\mathbf{v}_1$ and $\mathbf{v}'$, $\mathbf{v}_1'$ are, respectively, the velocities of the two electrons before and after their collisions; $\mathbf{w} = \mathbf{v} - \mathbf{v}_1$, θ' is the angle between $\mathbf{w}$ and $\mathbf{w}'$, $d\Omega' = \sin\theta'\, d\theta'\, d\varphi'$ and $\sigma_{ee}(w, \theta')$ is the effective differential scattering cross section for collisions between electrons; $\nu = \nu(v)$, as usual, is the collision frequency of the electron with heavy particles (see, for example, reference 2).

The expansion (A.1), and consequently Eqs. (A.2) and (A.3), are valid under the conditions*

$$\delta = 2m/M \ll 1, \quad \partial f_0/\partial t \ll \nu_{\rm eff} f_0. \tag{A.4}$$

Here $\nu_{\rm eff}$ is the effective collision frequency of the electrons with the heavy particles.

We shall seek a solution of Eqs. (A.2) and (A.3) by the method of successive approximations $f_0 = f_{00} + f_{01} + \ldots$, $\mathbf{f}_1 = \mathbf{f}_{10} + \mathbf{f}_{11} + \ldots$. We assume that the form of the symmetric part of the distribution function (f_{00}) in the zeroth approximation is determined by the integral term (i.e., that interelectronic collisions play a dominant role†). The solution of Eq. (A.2) in this approximation is, of course, the Maxwell distribution function. However, the latter is determined only with accuracy up to some arbitrary function of time $T_e(t)$ (the electron temperature) which must be found from the equation for the subsequent approximation (see below).

In order to find the zero approximation for the directional part of the distribution function $\mathbf{f}_{10}$, it is necessary to solve the integro-differential equation (A.3). This problem is generally very complicated. It was solved by Landshoff[8] for the case of a constant electric field (see also reference 7). The results of this work can be generalized to the case of a variable electric field.‡ In fact, if use is made of the condition (A.4), it is not difficult to show (similarly to what was done in reference 9) that the dependence of the function f_{00} on time does not have to be taken into account in the integration of Eq. (A.3). In this approximation, Eq. (A.3) is a linear integro-differential equation with time independent coefficients; obviously, the time variation in such an equation is easily

*Heat exchange with the other parts of the plasma can be reduced by eliminating the magnetic field. This also leads to a weakening of the effect of boundaries which, generally speaking, is very important (especially under conditions where the characteristic dimensions of the plasma are less than or comparable with $l\sqrt{\delta}$, where l is the electron mean free path).

†The corresponding calculation was carried out by Landshoff[6] (see also references 4 and 7) only for the case of a quasistationary ($\omega \ll \nu$) electric field. The case of a variable field of arbitrary frequency was considered for a weakly-ionized plasma, in which one could neglect collisions between electrons (see references 1 and 10 to 12).

*We note that the condition $\partial f_0/\partial t \ll \nu_{\rm eff} f_0$ was not noted by Davydov.

†The opposite extreme, in which the integral term is not important, was considered in references 1 and 8 to 12.

‡The possibility of such a generalization is based on the physically evident equivalence of the motion of the electrons in the direction of a weak electric field in the variable field case and in a constant field perpendicular to the magnetic (if only $\omega_H = \omega$).

separated, after which the equation becomes quite analogous to the one considered by Landshoff. The solution of this equation was found in reference 6 in the form of a series of Laguerre polynomials of order $3/2$. The same expression for the function $\mathbf{f}_{10}$ is valid even in the case of a variable electric field. It is only necessary to replace the gyrofrequency ω_H by $\omega \pm \omega_H$). Making use of this, it is not difficult to obtain expressions for the conductivity and dielectric constant of the plasma [which were obtained above, Eqs. (4) and (10)], and also for the other coefficients of transfer in a variable electric field.

Now substituting f_{00} and $\mathbf{f}_{10}$ in (A.2), we obtain the following equation for the first approximation (f_{01}):

$$\int \sigma_{ee}\cdot w\,\{f_{01}(v,\ t)\,f_{00}(v_1,\ t) + f_{01}(v_1,\ t)\,f_{00}(v,\ t) - f_{01}(v',\ t)\,f_{00}(v_1',\ t) - f_{01}(v_1',\ t)\,f_{00}(v',\ t)\}\,dv_1\,d\Omega' = -\frac{1}{v^2}\frac{\partial}{\partial v}\left\{\frac{v^3}{2T_e}f_{00}(v,\ t)\left[\frac{dT_e}{dt} - \delta\nu\,(T_e - T)\right] - v^2\frac{eE}{3m}\,\mathbf{f}_{10}(v,\ t)\right\}. \quad \text{(A.5)}$$

This integral equation is degenerate: if we multiply it by v^4 and integrate over dv, the integral term vanishes identically. It is natural that the right hand side of the equation must also vanish in this case [in the opposite case Eq. (A.5) has no solution]. This condition reduces to Eq. (2), (as can easily be seen) which also determines the temperature of the electrons $T_e(t)$.

If we estimate the value of terms of the second approximation and compare them with the zero approximation, we find that we can neglect them if the conditions

$$\delta\nu_{\mathrm{eff}}/\nu_e \ll 1, \quad dT_e/dt \ll \nu_e T_e, \quad \text{(A.6)}$$

are satisfied, where ν_e is the mean collision frequency between electrons.

In a strongly ionized plasma ν_e and ν_{eff} are of the same order. Consequently, conditions (A.6) and (A.4) are identical in this case. This means that in the case of a strongly ionized plasma, the solution of Eqs. (A.2) and (A.3), obtained above as a zeroth approximation, is the complete solution; it is valid with the same degree of accuracy with which Eqs. (A.2) and (A.3) are themselves valid.

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