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HYPERON BETA DECAY

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The probability for hyperon decay into nucleons (hyperons) and leptons is computed. The energy distribution, correlations, polarization, and asymmetry of emission of the particles are determined. Numerical calculations are given for the case of a universal V-A interaction.[1] The probabilities for the leptonic decay modes of Σ^- and Λ^0 are found to exceed considerably the corresponding experimental upper limits.

1. INTRODUCTION

RECENTLY Feynman and Gell-Mann,[1] as well as Marshak and Sudershan,[2] proposed the V-A covariants as a universal four-fermion interaction and listed a number of points in favor of this proposal. As was shown in reference 1 the general character of the interaction was such that the following β decays of hyperons, thus far not observed, should be possible

$$\begin{array}{ll} \Lambda^0 \to p + e^- + \tilde{\nu}, & \Lambda^0 \to p + \mu^- + \tilde{\nu}, \\ \Sigma^- \to n + e^- + \tilde{\nu}, & \Sigma^- \to n + \mu^- + \tilde{\nu}, \\ \Sigma^- \to \Lambda^0 + e^- + \tilde{\nu}, & \Sigma^+ \to \Lambda^0 + e^+ + \nu, \\ \Xi^- \to \Lambda^0 + e^- - \tilde{\nu}, & \Xi^- \to \Lambda^0 + \mu^- + \tilde{\nu}, \\ \Xi^- \to \Sigma^0 + e^- + \tilde{\nu}, & \Xi^- \to \Sigma^0 \to \mu^- + \tilde{\nu}. \end{array} \quad (1)$$

The probabilities of the electron decay modes of Λ^0 and Σ^- were computed in reference 1. In this work we compute, in addition to total probabilities, the energy distribution, correlation, polarization, and asymmetry of emission of particles formed in the decay of a polarized hyperon. Since the coupling constant of the axial-vector interaction may be renormalized by strong interactions in a manner different from the vector constant, one must take in general $C_A \neq -C_V$. In the computation of polarization and asymmetry it was found convenient to calculate traces and integrate over unobserved variables in a coordinate system in which the momentum of the particle under study was equal to the momentum of the decaying hyperon, and only afterwards to transform to the rest system of the hyperon.[3] The assumption of nonzero rest mass for all four fermions participating in the decay process does not complicate the calculations and permits application to processes involving neutral currents such as $\Lambda^0 \to n + e^- + \mu^+$, if it should turn out that these exist. The total decay probability is given in terms of an integral which is given explicitly when the mass of one of the decay products equals zero.

Numerical values for the probability of hyperon β decay and emission asymmetry are obtained for the V-A universal interaction without renormalization effects ($C_A = -C_V$). In Appendix B we discuss possible types of universal parity-nonconserving interactions, and outline simple methods for deriving the S + P − T interaction results from the V-A (and vice versa) for processes involving free particles as well as for the usual β-decay process.

2. ENERGY CORRELATIONS

The various processes given by Eq. (1) can be written in a unified form as

$$Y \to \left\{ {N \atop Y'} \right\} + \left\{ {e^- \atop \mu^-} \right\} + \tilde{\nu} \quad (2)$$

where Y stands for a hyperon and N for a nucleon. The universal four-fermion interaction responsible for these processes can be written as follows (we consider the V and A covariants only):

$$H_{\text{int}} = (\bar{\psi}_N \gamma_\mu (C_V - C_A \gamma_5) \psi_Y)(\bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu). \quad (3)$$

(From now on we shall write all formulas for the decay of a hyperon into a nucleon and electron only. The corresponding expressions for the hyperonic or μ-mesonic modes of decay are obtained by replacing the index N by Y′ or e by μ.)

The transition probability [calculated using the Hamiltonian (3)] for the decay of a hyperon Y at rest with polarization ζ_Y accompanied by the emission of an electron of energy E_e into the solid angle element $d\Omega_e$ and a nucleon into the solid angle element determined by its energy E_N and the azimuth angle φ_N about the axis $\mathbf{p}_e$ (measured from the $\zeta_Y \mathbf{p}_e$ plane) is given by (E_i, m_i, $\mathbf{p}_i$, ζ_i stand for the energy, mass, momentum, and polarization respectively of the i-th particle in its rest frame):

$$dW(\zeta_Y;\ E_e, E_N, \Omega_e, \varphi_N) = \frac{1}{2\pi^3}\frac{d\Omega_e}{4\pi}\frac{d\varphi_N}{2\pi} dE_e dE_N \{\eta^2[(m_Y - E_N - E_e) - \zeta_Y(\mathbf{p}_N + \mathbf{p}_e)][E_N E_e - s_{Ne}] \tag{4}$$
$$+ \xi^2[E_e - (\zeta_Y\cdot\mathbf{p}_e)][E_N(m_Y - E_e) - m_N^2 + s_{Ne}] + \eta\xi m_N[E_e(m_Y - E_N) - m_e^2 + s_{Ne} - (m_Y - E_N)(\zeta_Y\cdot\mathbf{p}_e) + E_e(\zeta_Y\cdot\mathbf{p}_N)]\}.$$

Here

$$\eta = C_V - C_A, \quad \xi \equiv C_V + C_A; \tag{5}$$

$$s_{Ne} \equiv (\mathbf{p}_N\cdot\mathbf{p}_e) = {}^1/_2[m_Y^2 + m_N^2 + m_e^2 - m_\nu^2 - 2m_Y(E_N + E_e) + 2E_N E_e], \tag{6}$$

so that the direction of the vector $\mathbf{p}_N$ in $\zeta_Y\cdot\mathbf{p}_N$ is free to vary only in the angle φ_N. Integration over this angle yields

$$(\zeta_Y\cdot\mathbf{p}_N) \to (\zeta_Y\cdot\mathbf{p}_e)\, s_{Ne}/(E_e^2 - m_e^2). \tag{7}$$

The electron-antineutrino correlation is given by a more symmetric expression (we continue to write simply ν instead of $\tilde{\nu}$):

$$dW(\zeta_Y;\ E_e, E_\nu, \Omega_e, \varphi_\nu) = \frac{1}{2\pi^3}\frac{d\Omega_e}{4\pi}\frac{d\varphi_\nu}{2\pi} dE_e dE_\nu \{\eta^2[E_\nu + (\zeta_Y\cdot\mathbf{p}_\nu)][E_e(m_Y - E_\nu) - m_e^2 + s_{e\nu}]$$
$$+ \xi^2[E_e - (\zeta_Y\cdot\mathbf{p}_e)][E_\nu(m_Y - E_e) - m_\nu^2 + s_{e\nu}] + \eta\xi m_N[E_e(\zeta_Y\cdot\mathbf{p}_\nu) - E_\nu(\zeta_Y\mathbf{p}_e)]\}, \tag{8}$$

where, as in Eq. (6)

$$s_{e\nu} \equiv (\mathbf{p}_e\cdot\mathbf{p}_\nu) = {}^1/_2[m_Y^2 - m_N^2 + m_e^2 + m_\nu^2 - 2m_Y(E_e + E_\nu) + 2E_e E_\nu]. \tag{9}$$

The nucleon-antineutrino correlation can be obtained from Eqs. (4) to (7) by interchanging the indices e and ν and by using the substitution

$$\eta \rightleftarrows \xi \ \text{(i.e., } C_A \to -C_A), \quad \zeta_Y \to -\zeta_Y. \tag{10}$$

3. ENERGY DISTRIBUTION, POLARIZATION, AND EMISSION ASYMMETRY OF PARTICLES PRODUCED IN THE DECAY OF A POLARIZED HYPERON

In order to obtain the probability for the emission of a nucleon in the direction $\mathbf{n}_N \equiv \mathbf{p}_N/|\mathbf{p}_N|$ and with the polarization ζ_N, we must integrate the expression for the decay probability over the electron and neutrino variables, subject to conservation laws. Some of the details of the calculation are given in Appendix A. The result is

$$dW(\zeta_Y;\ E_N, \mathbf{n}_N, \zeta_N) = \frac{\sqrt{E_N^2 - m_N^2}\, dE_N}{2(2\pi)^3}\frac{d\Omega_N}{4\pi}\frac{\sqrt{[R_N - (m_e + m_\nu)^2][R_N - (m_e - m_\nu)^2]}}{R_N^3} T^N, \tag{11}$$

where

$$R_N = m_Y^2 + m_N^2 - 2m_Y E_N, \tag{12}$$

$$T^N = T_1^N + T_2^N(\zeta_Y\cdot\mathbf{n}_N) + T_3^N(\zeta_N\cdot\mathbf{n}_N) + T_4^N(\zeta_Y\cdot\mathbf{n}_N)(\zeta_N\cdot\mathbf{n}_N) + T_5^N[(\zeta_Y\cdot\zeta_N) - (\zeta_Y\cdot\mathbf{n}_N)(\zeta_N\cdot\mathbf{n}_N)]. \tag{13}$$

In (13) T_1^N describes the nucleon-energy distribution, T_2^N the emission asymmetry, T_3^N and T_4^N the longitudinal polarization, and T_5^N the transverse polarization. The T_j^N are given by

$$\begin{aligned}
T_1^N &= (\xi^2 + \eta^2)\{(m_Y - E_N)(m_Y E_N - m_N^2)\Gamma_1^N - {}^1/_3 m_Y(E_N^2 - m_N^2)\Gamma_2^N\} - \xi\eta m_N R_N[\Gamma_1^N + \Gamma_2^N];\\
T_2^N &= (\xi^2 - \eta^2)\sqrt{E_N^2 - m_N^2}\{(m_Y E_N - m_N^2)\Gamma_1^N - {}^1/_3 m_Y(m_Y - E_N)\Gamma_2^N\};\\
T_3^N &= (\xi^2 - \eta^2)\sqrt{E_N^2 - m_N^2}\{m_Y(m_Y - E_N)\Gamma_1^N - {}^1/_3(m_Y E_N - m_N^2)\Gamma_2^N\};\\
T_4^N &= (\xi^2 + \eta^2)\{m_Y(E_N^2 - m_N^2)\Gamma_1^N - {}^1/_3(m_Y - E_N)(m_Y E_N - m_N^2)\Gamma_2^N\} + \xi\eta m_N R_N[\Gamma_1^N - \Gamma_2^N];\\
T_5^N &= -{}^1/_3(\xi^2 + \eta^2)m_N R_N \Gamma_2^N + \xi\eta[E_N(m_Y^2 + m_N^2) - 2m_Y m_N^2][\Gamma_1^N - \Gamma_2^N];\\
\Gamma_1^N &\equiv [R_N + m_e^2 - m_\nu^2][R_N - m_e^2 + m_\nu^2];\quad \Gamma_2^N \equiv [R_N - (m_e + m_\nu)^2][R_N - (m_e + m_\nu)^2].
\end{aligned} \tag{14}$$

If we set $m_e = m_\nu = 0$ in Eqs. (11) to (14), we obtain the formulas of the V-A theory for the polarization and emission asymmetry of the electrons in the decay of a μ meson (the index Y would refer to the μ meson and N to the electron). Such formulas have been given before.[3,4]

In analogy with Eq. (11), we have for the electron

$$dW(\zeta_Y; E_e, \mathbf{n}_e, \zeta_e) = \frac{\sqrt{E_e^2 - m_e^2}\, d\Omega_e}{2(2\pi)^3 \quad 4\pi} \frac{\sqrt{[R_e - (m_N + m_\nu)^2][R_e - (m_N - m_\nu)^2]}}{R_e^3} T^e, \tag{15}$$

where

$$R_e = m_Y^2 + m_e^2 - 2m_Y E_e; \tag{16}$$

$$T^e = T_1^e + T_2^e(\zeta_Y \cdot \mathbf{n}_e) + T_3^e(\zeta_e \cdot \mathbf{n}_e) + T_4^e(\zeta_Y \cdot \mathbf{n}_e)(\zeta_e \cdot \mathbf{n}_e) + T_5^e[(\zeta_Y \cdot \zeta_e) - (\zeta_Y \cdot \mathbf{n}_e)(\zeta_e \cdot \mathbf{n}_e)]; \tag{17}$$

$$\begin{gathered}
T_1^e = \eta^2\{(m_Y E_e - m_e^2)(m_Y - E_e)\Gamma_1^e - {}^1\!/_3\, m_Y(E_e^2 - m_e^2)\Gamma_2^e\} + \xi^2 E_e R_e[\Gamma_1^e + \Gamma_2^e] - 2\xi\eta m_N(m_Y E_e - m_e^2)R_e[R_e - m_N^2 + m_\nu^2];\\
T_2^e = -\sqrt{E_e^2 - m_e^2}\,\{\eta^2[(m_Y E_e - m_e^2)\Gamma_1^e - {}^1\!/_3\, m_Y(m_Y - E_e)\Gamma_2^e] + \xi^2 R_e[\Gamma_1^e + \Gamma_2^e] + 2\xi\eta m_Y m_N R_e[R_e - m_N^2 + m_\nu^2]\};\\
T_3^e = -\sqrt{E_e^2 - m_e^2}\,\{\eta^2[m_Y(m_Y - E_e)\Gamma_1^e - {}^1\!/_3(m_Y E_e - m_e^2)\Gamma_2^e] + \xi^2 R_e[\Gamma_1^e + \Gamma_2^e] - 2\xi\eta m_Y m_N R_e[R_e - m_N^2 + m_\nu^2]\};\\
T_4^e = \eta^2\{m_Y(E_e^2 - m_e^2)\Gamma_1^e - {}^1\!/_3(m_Y - E_e)(m_Y E_e - m_e^2)\Gamma_2^e\} + \xi^2 m_e R_e[\Gamma_1^e + \Gamma_2^e] + 2\xi\eta m_Y m_N(m_Y - E_e)R_e[R_e - m_N^2 + m_\nu^2];\\
T_5^e = -{}^1\!/_3\,\eta^2 m_e R_e \Gamma_2^e + \xi^2 E_e R_e[\Gamma_1^e + \Gamma_2^e] + 2\xi\eta m_N(m_Y E_e - m_e^2)R_e[R_e - m_N^2 + m_\nu^2];\\
\Gamma_1^e \equiv [R_e + m_N^2 - m_\nu^2][R_e - m_N^2 + m_\nu^2]; \quad \Gamma_2^e \equiv [R_e - (m_N + m_\nu)^2][R_e - (m_N - m_\nu)^2].
\end{gathered} \tag{18}$$

The formulas for polarization, emission asymmetry, and energy distribution of the antineutrino can be obtained from Eqs. (15) to (18) by interchanging the e and ν indices and applying the substitution (10).

4. DISCUSSION OF RESULTS

The total decay probability and integrated emission asymmetry are obtained from Eq. (15) by summing over the directions of ζ_e and integrating over E_e between the limits m_e and $[m_Y^2 + m_e^2 - (m_N + m_\nu)^2]/2m_Y$.

An analogous procedure should be applied to Eq. (11). The results may be written as

$$dW(\zeta_Y, \mathbf{n}_i) = W_0[1 - \alpha_0^i(\zeta_Y \cdot \mathbf{n}_i)]\, d\Omega_i / 4\pi, \quad i = N, e. \tag{19}$$

The energy integral entering into the expression for W_0 must be evaluated numerically if none of the masses vanish. For $m_\nu = 0$ the integral can be performed analytically and we find

$$W_0 = \frac{m_Y^5}{2(2\pi)^3}[(\xi^2 + \eta^2)A_1 + \xi\eta A_2], \tag{20}$$

where

$$\begin{gathered}
A_1 = \left(\frac{m_N}{m_Y}\right)^4\left[1 - \left(\frac{m_e}{m_Y}\right)^4\right]\ln y_1 + \left(\frac{m_e}{m_Y}\right)^4\left[1 - \left(\frac{m_N}{m_Y}\right)^4\right]\ln y_2 + \frac{z}{24}\left\{1 + \left(\frac{m_N}{m_Y}\right)^6 + \left(\frac{m_e}{m_Y}\right)^6 + 12\left(\frac{m_N}{m_Y}\right)^2\left(\frac{m_e}{m_Y}\right)^2\right.\\
\left. - 7\left[\left(\frac{m_N}{m_Y}\right)^2 + \left(\frac{m_e}{m_Y}\right)^2 + \left(\frac{m_N}{m_Y}\right)^4 + \left(\frac{m_e}{m_Y}\right)^4 + \left(\frac{m_N}{m_Y}\right)^2\left(\frac{m_e}{m_Y}\right)^4 + \left(\frac{m_N}{m_Y}\right)^4\left(\frac{m_e}{m_Y}\right)^2\right]\right\};\\
A_2 = \frac{m_N}{m_Y}\left\{\left[\left[1 - \left(\frac{m_e}{m_Y}\right)^2\right]^2 + \left(\frac{m_N}{m_Y}\right)^2\right]\ln y_1 - \left(\frac{m_e}{m_Y}\right)^4\left[1 - \left(\frac{m_N}{m_Y}\right)^2\right]\ln y_2 - \frac{z}{24}\left[1 + \left(\frac{m_N}{m_Y}\right)^4 - 2\left(\frac{m_e}{m_Y}\right)^4 + 10\left(\frac{m_N}{m_Y}\right)^2\right.\right.\\
\left.\left. - 5\left(\frac{m_e}{m_Y}\right)^2 - 5\left(\frac{m_N}{m_Y}\right)^2\left(\frac{m_e}{m_Y}\right)^2\right]\right\};\\
z = \left\{\left(1 + \frac{m_N}{m_Y} + \frac{m_e}{m_Y}\right)\left(1 + \frac{m_N}{m_Y} - \frac{m_e}{m_Y}\right)\left(1 - \frac{m_N}{m_Y} - \frac{m_e}{m_Y}\right)\left(1 - \frac{m_N}{m_Y} + \frac{m_e}{m_Y}\right)\right\}^{1/2};\\
y_1 = \frac{m_Y}{2m_N}\left[1 + z + \left(\frac{m_N}{m_Y}\right)^2 - \left(\frac{m_e}{m_Y}\right)^2\right], \quad y_2 = \frac{m_Y}{2m_e}\left[1 + z - \left(\frac{m_N}{m_Y}\right)^2 + \left(\frac{m_e}{m_Y}\right)^2\right]
\end{gathered} \tag{21}$$

The coefficients α_0^i of Eq. (19) can also be given in analytic form, but the resultant expressions are unwieldy. It is simpler to calculate them for each specific case by a numerical integration of T_2^N and T_2^e over the energy. We give above a table of values of W_0 and α_0 for the decays enumerated in (1). Since at this time only the order of magnitude of these quantities is of interest, we make use of the simplifying assumptions $\xi = 0$, $\eta = \sqrt{2}\,G = 2 \times 10^{-49}$ erg-cm (i.e., $C_V = -C_A = G/\sqrt{2}$, where G is the constant of Feynman and Gell-Mann[1]). This means that no renormalization is taken into account for either the vector or axial-vector coupling con-

Probabilities for Leptonic Decays of Hyperons and Emission Asymmetries of the Produced Particles

Decay mode	W_0 (sec^{-1})	$\tau \times 10^{10}$ (sec)	$W_0\tau$ (%)	α_0^N or $\alpha_0^{Y'}$	α_0^e or α_0^μ
$\Lambda^0 \to p + e^- + \tilde{\nu}$	$7.4 \cdot 10^7$	2.8	2.1	0.6	0.05
$\Lambda^0 \to p + \mu^- + \tilde{\nu}$	$1.2 \cdot 10^7$	2.8	0.33	0.4	0.06
$\Sigma^- \to n + e^- + \tilde{\nu}$	$5.4 \cdot 10^8$	1.8	9.6	0.55	0.175
$\Sigma^- \to n + \mu^- + \tilde{\nu}$	$2.4 \cdot 10^8$	1.8	4.3	0.5	0.09
$\Sigma^- \to \Lambda^0 + e^- + \tilde{\nu}$	$2.1 \cdot 10^6$	1.8	0.037	0.65	
$\Sigma^+ \to \Lambda^0 + e^+ + \nu$	$1.4 \cdot 10^6$	0.9	0.013	0.5	
$\Xi^- \to \Lambda^0 + e^- + \tilde{\nu}$	$1.6 \cdot 10^8$	1	1.6	0.6	
$\Xi^- \to \Lambda^0 + \mu^- + \tilde{\nu}$	$4.3 \cdot 10^7$	1	0.4	0.5	

stants. The table also lists values of τ, the experimental hyperon lifetime, and of $W_0\tau$ which is the ratio of hyperon β decays to total number of decays.

It follows from the above that approximately $1/7$ of the total number of decays of the Σ^- hyperon and $1/20$ of the decays of the Λ^0 hyperon should proceed through the leptonic modes.* Therefore a contradiction with the universal V-A interaction would arise should the absence of these decay modes be confirmed. On the other hand, taking into account renormalization and the energy form factor of the interaction may decrease the value of W_0.

As can also be seen from the table the integrated emission asymmetry of the electrons and μ mesons is small. This smallness is a consequence of the assumption $C_A = -C_V$. It follows that a measurement of lepton-emission asymmetry would provide a sensitive test of this assumption.

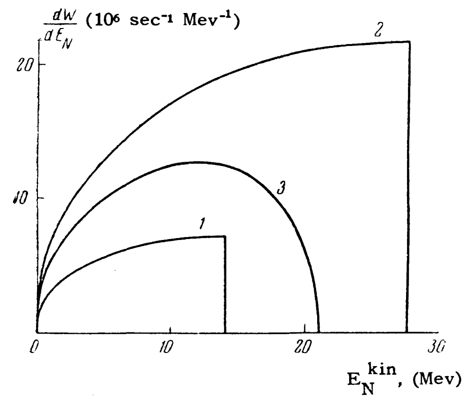

FIG. 1. Dependence of the hyperon decay probability on the energy of the produced nucleon.

*The values for the decay probabilities for $\Lambda^0 \to p + e^- + \tilde{\nu}$ and $\Sigma^- \to n + e^- + \tilde{\nu}$ quoted by Feynman and Gell-Mann are somewhat smaller than those given in the table. This discrepancy disappears if it is assumed that the quantity C(x) introduced in reference 1 has been set approximately equal to unity by the authors in their calculations.

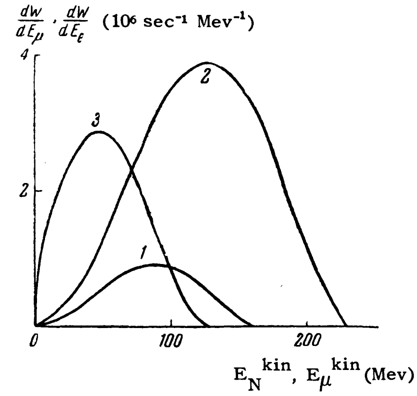

FIG. 2. Dependence of the hyperon decay probability on the electron or on the μ-meson energy.

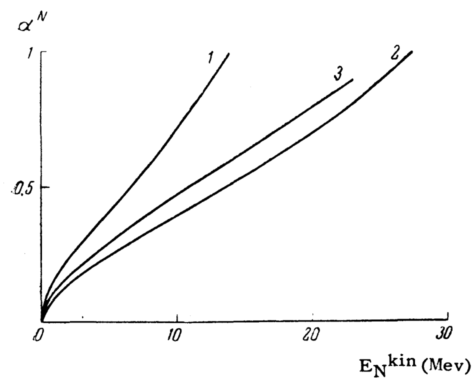

FIG. 3. Energy dependence of the nucleon emission asymmetry.

Figure 1 gives the dependence of the hyperon decay probability on the kinetic energy of the nucleon $E_N^{kin} \equiv E_N - m_N$. Figure 2 gives the dependence on the energy of the electron, E_e^{kin}, or of the μ meson E_μ^{kin}. The energy dependences of the asymmetry coefficients of the nucleon $[\alpha^N(E_N) \equiv -T_2^N/T_1^N]$ and of the electron and μ meson $[\alpha^e(E_e) \equiv -T_2^e/T_1^e$ and $\alpha^\mu(E_\mu) \equiv -T_2^\mu/T_1^\mu]$ are shown in Figs. 3 and 4 respectively.

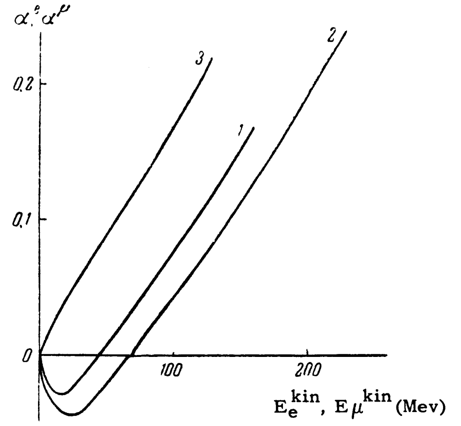

FIG. 4. Energy dependence of the electron or μ-meson emission asymmetry.

In all figures the curves labelled 1 refers to the decay $\Lambda^0 \to p + e^- + \tilde{\nu}$, those labelled 2 to $\Sigma^- \to n + e^- + \tilde{\nu}$, and those labelled 3 to $\Sigma^- \to n + \mu^- + \tilde{\nu}$. The remaining decays in (1) are characterized by analogous curves.

APPENDIX A

Calculation of Decay Probability

If the interaction Hamiltonian is of the form (3), the decay probability of type (2) is given by

$$dW = 2\pi \frac{d^3p_N}{(2\pi)^3} \frac{d^3p_e}{(2\pi)^3} d^3p_\nu \delta(p_Y - p_N - p_e - p_\nu) |M|^2, \quad \text{(A.1)}$$

where

$$|M|^2 = 4\mathrm{Sp}\{\eta^2 R_1^N \gamma_\mu R_1^Y \gamma_\nu + \xi^2 R_2^N \gamma_\mu R_2^Y \gamma_\nu + \eta\xi [R_3^N \gamma_\mu R_4^Y \gamma_\nu + R_4^N \gamma_\mu R_3^Y \gamma_\nu]\} \mathrm{Sp}\{R_1^e \gamma_\mu R_1^\nu \gamma_\nu\}. \quad \text{(A.2)}$$

Here

$$\left.\begin{aligned} R_1^i &= a\rho^i \bar{a}, \quad R_2^i = \bar{a}\rho^i a, \\ R_3^i &= a\rho^i a, \quad R_4^i = \bar{a}\rho^i \bar{a} \end{aligned}\right\} i = Y, N, e, \nu, \quad \text{(A.3)}$$

$$a \equiv (1 + \gamma_5)/2, \quad \bar{a} \equiv (1 - \gamma_5)/2, \quad \text{(A.4)}$$

and ρ^i is the usual density matrix for the i-th particle.[5]

It is easy to show that $(\hat{A} \equiv \gamma_\mu A_\mu)$:

$$\begin{aligned} R_1^i &= -ia\hat{t}_i / 4E_i, \quad R_2^i = -i\bar{a}\hat{t}'_i / 4E_i, \\ R_3^i &= a(m_i + \hat{\zeta}'_i \hat{p}_i) / 4E_i, \quad R_4^i = \bar{a}(m_i - \hat{\zeta}'_i \hat{p}_i) / 4E_i, \end{aligned} \quad \text{(A.5)}$$

where

$$t_i \equiv p_i - m_i \zeta'_i, \quad t'_i \equiv p_i + m_i \zeta'_i, \quad \text{(A.6)}$$

where

$$\zeta'_i = \zeta_i + (\zeta_i \cdot \mathbf{p}_i)\mathbf{p}_i / m_i(E_i + m_i), \quad \zeta'_{i0} = (\zeta_i \cdot \mathbf{p}_i) / m_i. \quad \text{(A.7)}$$

ζ'_i differs from ζ_i by having the component along the $\mathbf{p}_i$ direction increased E_i/m_i times, and ζ'_{i0} is defined by the equation

$$(\zeta'_i p_i) \equiv \zeta_{i\mu} p_{i\mu} = 0. \quad \text{(A.8)}$$

When Eq. (A.5) is introduced into Eq. (A.2) the result is

$$\begin{aligned} |M|^2 = (1/4 E_Y E_N E_e E_\nu) \{ & \eta^2 (t_Y t_\nu)(t_N t_e) + \xi^2 (t'_Y t_e)(t'_N t_\nu) \\ & + \eta\xi [m_Y m_N (t_e t_\nu) - m_Y \{(\zeta'_N t_e)(p_N t_\nu) - (\zeta'_N t_\nu)(p_N t_e)\} \\ & + m_N \{(\zeta'_Y t_e)(p_Y t_\nu) - (\zeta'_Y t_\nu)(p_Y t_e)\} \\ & - \frac{1}{8} \mathrm{Sp}(\hat{\zeta}'_N \hat{p}_N \gamma_\mu \hat{\zeta}'_Y \hat{p}_Y \gamma_\nu) \\ & \times (-(t_e)_\mu (t_\nu)_\nu - (t_e)_\nu (t_\nu)_\mu + (t_e)_\alpha (t_\nu)_\beta \varepsilon_{\alpha\beta\mu\nu})]\} \end{aligned} \quad \text{(A.9)}$$

($\epsilon_{\alpha\beta\mu\nu}$ is the totally antisymmetric tensor with components $0, \pm 1$).

The last trace in Eq. (A.9) is easy to calculate; subsequent substitution into Eq. (A.1) yields all possible distributions and correlations in the decay of a hyperon. However, for our purposes, it is not necessary to write out the general expression since $\zeta_N = 0$ and the last trace vanishes in all cases, except Eqs. (11) to (14). In the case when $\zeta_N \neq 0$ the calculation is carried out in the coordinate system in which $\mathbf{p}_Y = \mathbf{p}_N$, i.e., $\mathbf{p}_e = -\mathbf{p}_\nu$. In that case the last term of Eq. (A.9) after integration over the directions of $\mathbf{p}_e$ reduces to

$${}^1\!/_4 \sum_\mu \mathrm{Sp}(\hat{\zeta}'_N \hat{p}_N \gamma_\mu \hat{\zeta}'_Y \hat{p}_Y \gamma_\mu)\,(p_e)_\mu (p_\nu)_\mu. \quad \text{(A.10)}$$

It is easy to see that (no summation over μ)

$$(p_e)_\mu (p_\nu)_\mu = -{}^1\!/_3\,\mathbf{p}_e^2 - \delta_{\mu 4}(E_e E_\nu - {}^1\!/_3\,\mathbf{p}_e^2). \quad \text{(A.11)}$$

Introducing Eqs. (A.11) and (A.8) into Eq. (A.10) gives

$$(E_e E_\nu - {}^1\!/_3\,\mathbf{p}_e^2)[(\zeta'_N \zeta'^*_Y)(p_N p^*_Y) - (\zeta'_N p^*_Y)(p_N \zeta'^*_Y)], \quad \text{(A.12)}$$

where * indicates that the purely imaginary fourth component of the given vector should be taken with the opposite sign.

The averaging over the directions of $\mathbf{p}_e$ in the remaining terms of Eq. (A.9), as well as the analogous integration over the directions of $\mathbf{p}_N$ in the case when electron polarization is studied, is elementary; after this Eq. (A.9) is expressed in terms of $\mathbf{p}_i$, E_i and the unchanged vectors ζ_i. The resultant expression is substituted into Eq. (A.1) and the expressions (11) to (18) are obtained after a Lorentz transformation to the frame $\mathbf{p}_Y = 0$ is performed.

We note that rule (10) for the interchange of electron and neutrino follows directly from Eqs. (A.9) and (A.6). In the most general case, when $\zeta_N \neq 0$, rule (10) should be amplified to include the condition $\zeta_N \to -\zeta_N$.

APPENDIX B

Universal Four-Fermion Interactions with Parity Nonconservation

As is well known,[6] five types of parity conserving interactions exist, universal in a narrower sense than that used in reference 1. The universality is defined by the condition that in going from the ordering

$$H_{\text{int}} = \sum_j g_j (\bar{\psi}_2 O_j \psi_1)(\bar{\psi}_3 O_j \psi_4) \qquad \text{(A.13)}$$

to the ordering

$$H_{\text{int}} = \sum_j f_j (\bar{\psi}_2 O_j \psi_4)(\bar{\psi}_3 O_j \psi_1) \qquad \text{(A.14)}$$

one should have $f_j = pg_j$ for all j (with the same $p = \pm 1$). The indices 1, 2, 3, 4 refer to the particles participating in the reaction, which need not be the decay of the particle labeled 1. One has $p = +1$ for the covariants V-A, S + P − T and 2(S − P) − (A + V), whereas $p = -1$ for 2(S − P) + (A + V) and 3(S + P) + T. It is easy to show, by the method of reference 7, that these interactions are also universal for the covariants with the primed constants, however then 2(S − P) − (A + V) has $p = -1$ and 2(S − P) + (A + V) has $p = +1$. Consequently, when both C_j and C'_j are nonzero, the last two combinations of covariants are no longer universal. Of the remaining three, 3(S + P) + T corresponds to $p = -1$ and, apparently, contradicts experimental data. For example, the Michel parameter determining the energy spectrum in μ-decay comes out equal to 0.25 whereas the experiment gives 0.68 ± 0.02.

The V-A and S + P − T covariants both have $p = 1$. They give similar results in the calculation of various processes and one may give a simple rule for going over from one to the other.

It is shown in reference 1 that the interaction corresponding in the notation of Lee and Yang[8] to

$$\begin{gathered} C_V = C'_V = -C_A = -C'_A \equiv G/\sqrt{2}, \\ C_S = C'_S = C_P = C'_P = C_T = C'_T = 0, \end{gathered} \qquad \text{(A.15)}$$

may be brought into the form

$$H_{\text{int}} = 4\sqrt{8}\,G\,[(\varphi_2^* \varphi_1)(\varphi_3^* \varphi_4) - (\varphi_2^* \sigma \varphi_1)(\varphi_3^* \sigma \varphi_4)], \qquad \text{(A.16)}$$

where

$$\varphi_i = {}^1\!/_2 (v_i + w_i), \quad \psi_i = \begin{pmatrix} v_i \\ w_i \end{pmatrix}. \qquad \text{(A.17)}$$

In turn, the interaction

$$\begin{gathered} C_S = -C'_S = C_P = -C'_P = -C_T = C'_T \equiv G/\sqrt{2}, \\ C_V = C'_V = C_A = C'_A = 0 \end{gathered} \qquad \text{(A.18)}$$

(the relative sign of C and C′ is here different than in the V-A covariant to give electron polarization correctly in normal β decay) leads to

$$H_{\text{int}} = 4\sqrt{8}\,G\,[(\varphi_2^* \varphi'_1)(\varphi_3^* \varphi'_4) - (\varphi_2^* \sigma \varphi'_1)(\varphi_3^* \sigma \varphi'_4)], \qquad \text{(A.19)}$$

where

$$\varphi'_1 = {}^1\!/_2 (v_i - w_i). \qquad \text{(A.20)}$$

Consequently the transition from V − A to S + P − T requires the replacement of φ_1 and φ_4 by φ'_1 and φ'_4 i.e. a change in sign of w_1 and w_4. Since $w_i = \sigma \cdot \mathbf{p}_i v_i / (E_i + m_i)$ this implies changing of the sign of $\mathbf{p}_1$ and $\mathbf{p}_4$ in the matrix element. On the other hand, the sign of the momentum does not change in the conservation laws for free particles or in the $\exp\{i(\mathbf{p}_e + \mathbf{p}_\nu)\cdot\mathbf{r}\}$ in forbidden transitions for the β decay. However, as can be seen from Eq. (A.9) for free particles, when $\xi = 0$ and $\eta = \sqrt{2}\,G$, the square of the matrix element contains the variables of the first and fourth particle only in the combination $(t_1 t_4)$, for which the sign change of $\mathbf{p}_1$ and $\mathbf{p}_4$ is equivalent to a sign change of $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_4$ with unchanged $\mathbf{p}_1$ and $\mathbf{p}_4$. Consequently, for free particles, the transition from Eq. (A.15) to (A.18) is accomplished by

$$\zeta_1 \to -\zeta_1, \quad \zeta_4 \to -\zeta_4. \qquad \text{(A.21)}$$

Precisely such a sign change for $\boldsymbol{\zeta}_1$ was noted in reference 4 in the discussion of μ-meson decay.

In the case of β decay, particles 1 and 4 are the neutron (n) and antineutrino (ν). The change in sign of $\mathbf{p}_1$ is equivalent to the introduction of the additional matrix $\beta = \gamma_4$ into the nuclear matrix element. For the neutrino we have $w_\nu = \sigma \cdot \mathbf{n}_\nu v_\nu$ $(\mathbf{n}_\nu = \mathbf{p}_\nu / |\mathbf{p}_\nu|)$, and strictly speaking it is $\mathbf{n}_\nu$ and not $\mathbf{p}_\nu$ that changes sign. The quantity that enters into $\exp\{i(\mathbf{p}_e + \mathbf{p}_\nu)\cdot\mathbf{r}\}$ is $\mathbf{p}_\nu = |\mathbf{p}_\nu|\mathbf{n}_\nu$. In order that $\mathbf{p}_\nu$ remain unchanged we must require that $|\mathbf{p}_\nu|$ change sign along with $\mathbf{n}_\nu$. Consequently the transition from Eq. (A.15) to (A.18) is accomplished, in the case of β decay, by

$$\int O \to \int O\beta, \quad \mathbf{n}_\nu \to -\mathbf{n}_\nu, \quad |\mathbf{p}_\nu| \to -|\mathbf{p}_\nu|. \qquad \text{(A.22)}$$

This leads to a different sign in the β-ν correlation in allowed transitions and to a change in the energy spectrum in forbidden transitions. Both these phenomena are well known (see, e.g., reference 9).

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SPLITTING OF ATOMIC TERMS WITH INTEGER TOTAL ANGULAR MOMENTUM IN A MAGNETIC CRYSTAL

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A method is proposed for finding the degeneracy caused by magnetic symmetry. The method is used to examine the nature of the splitting of atomic terms in a magnetic crystal for all cases of magnetic symmetry, under the assumption that the total angular momentum of the atom is integral. The results are compared with the splitting of atomic terms in a nonmagnetic crystal. It is found that the magnetic interaction in the crystal does not always remove the degeneracy of the atomic energy levels completely. It is shown that the results obtained are applicable to finding the splitting of terms of an atom in a nonmagnetic crystal which is placed in a magnetic field.

1. The field due to the other atoms of the crystal acts on an atom located in a crystal. This field has a definite symmetry which depends on both the crystal symmetry and on the position of the atom under consideration. The symmetry of the crystal field is a subgroup of the symmetry class of the crystal. The crystalline field can be treated as a perturbation which splits the energy levels of the unperturbed atom. This splitting is completely dependent on the symmetry of the crystal field at the point where the atom is located.

The question of splitting of atomic terms under the action of the crystalline field was treated by Bethe.[1] If we assume that the perturbation due to the crystal field is so small that the spin-orbit coupling in the atom is not broken down, we can start from a state of the free atom which is given by its total angular momentum J and its parity. Under the influence of the crystal field, the symmetry of the atom is reduced, which leads to partial or complete lifting of the degeneracy of the atomic state. To find this splitting we must expand the irreducible representation of the symmetry group of the free atom in irreducible representations of the symmetry group of the crystal field. The irreducible representation of the symmetry group of the free atom (spherical symmetry) gives the term of the unperturbed atom, while the irreducible representations of the symmetry group of the crystal field which are found in the expansion give the components in the splitting of the term.

We must however make one important reservation. The free atom is symmetric under time inversion. If the crystal field is purely electric, it is also symmetric under the time inversion R. Then the perturbed atom must also possess this symmetry. One might think that the symmetry with respect to R could be taken into account in the usual scheme of expansion in irreducible representations by assuming that the symmetry group of the crystal field also includes the time inver-