DIFFRACTION SCATTERING OF 6.15-Bev PROTONS BY PROTONS

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A phase shift analysis is carried out for the diffraction scattering of 6.15-Bev protons by protons. The spatial distribution of the absorbing field is deduced from results of an analysis using the semiclassical approximation. The value 0.79×10^{-13} cm is obtained for the mean-square proton-proton nuclear interaction radius.

1. SCATTERING OF IDENTICAL PARTICLES AT HIGH ENERGIES

We consider the interaction of identical particles, which is taken to be independent of their spins (s). The differential cross section for elastic scattering is described by the well-known formula* (see, for example, reference 1):

$$\frac{d\sigma}{d\omega} = \Big\{ |f(\vartheta)|^2 + |f(\pi-\vartheta)|^2 \pm \frac{1}{2s+1}[f(\vartheta)f(\pi-\vartheta)^* + f(\vartheta)^* f(\pi-\vartheta)]\Big\}. \tag{1}$$

Here, $f(\vartheta)$ is the scattering amplitude and ϑ is the scattering angle. The upper sign refers to integral s, the lower, to half-integral s. The experimental data[2] indicate that elastic p-p scattering at high energies goes mainly through small angles. In this case, if $f(\vartheta)$ has significant values only for small ϑ, then in this region of angles it can be seen from Eq. (1) that

$$d\sigma/d\omega = |f(\vartheta)|^2. \tag{2}$$

Here

$$f(\vartheta) = \frac{i\lambda}{2}\sum_{l=0}^{\infty}(2l+1)(1-\beta_l)P_l(\cos\vartheta). \tag{3}$$

In Eq. (3) the notation is as usual; in particular, $\beta_l = \exp\{2i\eta_l\}$, where $\eta_l = \alpha_l + i\xi_l$ and η_l is the scattering phase.

We assume that all of the elastic scattering at high energies is diffraction (by "diffraction" we mean here that approximation in which all $\alpha_l = 0$). Then the scattering amplitude will be imaginary, which substantially simplifies the phase shift analysis. In fact, because of the orthogonality of the Legendre polynomials in Eq. (3) it is easy to obtain

$$i\lambda(1-\beta_l) = \int_{-1}^{+1} P_l(x) f(x)\,dx. \tag{4}$$

($x = \cos\vartheta$). From Eq. (2) we find

$$f(x) = e^{i\gamma}\sqrt{d\sigma/d\omega}. \tag{5}$$

Substituting Eq. (5) into Eq. (4) we obtain

$$i\lambda(1-\beta_l) = \int_{-1}^{+1} P_l(x)\, e^{i\gamma}\sqrt{\frac{d\sigma}{d\omega}}\,dx. \tag{6}$$

Since the scattering amplitude is imaginary in the case considered, γ can take on only the values $\pi/2$ and $3\pi/2$. From Eq. (3) it can be seen that $\gamma = \pi/2$ at zero angle. The differential scattering cross section changes monotonically with ϑ at high energies and is never zero in the measured range of angles. Taking this latter fact into account, and also the fact that $f(\vartheta)$ is a continuous function of ϑ, we make the simplest assumption, that $\gamma = \pi/2$ at all angles. Then Eq. (6) has the form

$$\lambda(1-\beta_l) = \int_{-1}^{+1} P_l(x)\sqrt{\frac{d\sigma}{d\omega}}\,dx. \tag{7}$$

Some theoretical basis for the assumptions made has been given by Belenkii.[3] The phenomenological analysis of elastic π-p and p-p scattering showed that for $E_\pi \gtrsim 1$ Bev and $E_p \gtrsim 5$ Bev all of the elastic scattering can be considered diffractional.[4,5] We give below a phase-shift analysis of elastic p-p scattering at 6.15 Bev within the framework of these assumptions.

2. ANALYSIS OF p-p SCATTERING AT 6.15 Bev

The experimental data are described by empirical formulae of the form

$$d\sigma/d\omega = a^2/(b-\cos\vartheta)^2, \tag{I}$$

$$d\sigma/d\omega = c\exp(-k\vartheta^2). \tag{II}$$

*All considerations in this article are in the center-of-mass system of the colliding particles.

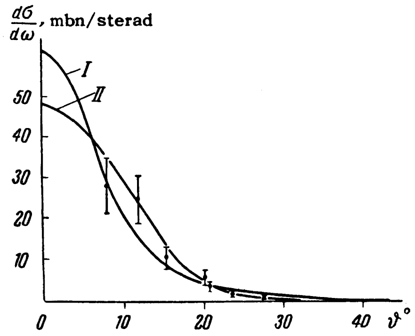

FIG. 1. Differential cross section for the elastic p – p interaction at $E_p = 6.15$ Bev. The experimental values are taken from Ref. 2, with account of a systematic error of ± 15%. The solid curves give the approximating functions $d\sigma, d\omega = 0.0244/(1.02\text{-}\cos\vartheta)^2$ mbn/sterad (I) and $d\sigma/d\omega = 48.0 \exp\{-19.2\,\vartheta^2\}$ mbn/sterad (II).

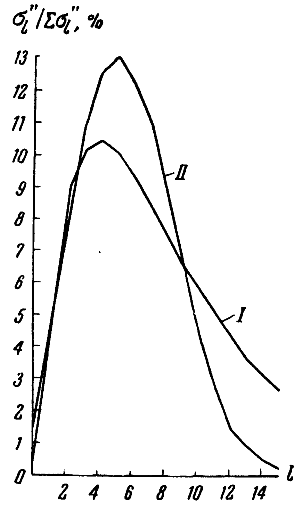

FIG. 3. Values of the partial contributions to the total inelastic cross section in percentages for curves (I) and (II).

Curves of type (I), with suitable choice of a and b, go up sharply at small angles and give values for the cross section in excess of the experimental ones at large angles. Curves of type (II) go up more slowly towards zero angle, within the limits of experimental error, and drop sharply at larger angles (see Fig. 1).

Calculations were carried out for various choices of the constants a, b, c, and k, satisfying the experimental data. In the future, only the results of a phase shift analysis for the empirical curves in Fig. 1 will be given. Various other possible choices of the constants give analogous results.

For curve (I), β_l was calculated from Eq. (7) using the Legendre function of the second type.[4] For the Gaussian curve (II) it was assumed that[6]

$$1 - \beta_l = \alpha_0 \exp\{-0.026\, l^2\}.$$

The value of α_0 was determined from β_0, calculated from Eq. (7).

Ambiguity in the choice of approximation at large angles leads to a large indeterminacy in values of β_0 and β_1 for curve (I); we therefore omit them from our consideration.

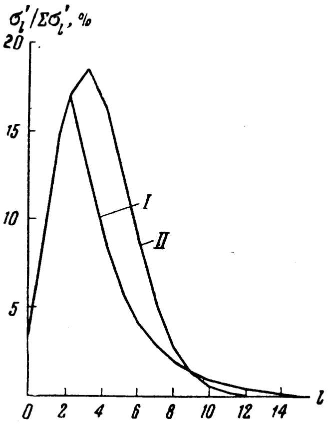

FIG. 2. Values of the partial contributions to the total cross section for elastic scattering in percentages for curves (I) and (II).

According to general scattering theory, the partial contributions to the cross section for elastic and inelastic scattering are respectively equal to

$$\sigma_l' = \pi\lambda^2 (2l + 1)(1 - \beta_l)^2, \qquad (8)$$

$$\sigma_l'' = \pi\lambda^2 (2l + 1)(1 - \beta_l^2). \qquad (9)$$

Using the values of β_l obtained earlier, σ_l' and σ_l'' were calculated. In Figs. 2 and 3 the values of partial contributions σ_l' and σ_l'' are given in percentages of the corresponding (elastic or inelastic) cross sections as a function of l.

It can be seen from Fig. 2 that the main contributions to the elastic scattering come from the partial waves with $l = 0$ to 8. The main contributions to the inelastic scattering come from waves with $l = 1$ to 12 (Fig. 3).

In order to check the possibility of neglecting contributions from waves with $l > 15$ for the approximations chosen, the total interaction cross section σ_t was calculated from the well-known formula

$$4\pi\lambda\, \mathrm{Im}\, f(0°) = \sigma_t. \qquad (10)$$

Substituting f(0°) from our empirical formulae (I) and (II) we find σ_t equal to 36.0 mbn and 32.2 mbn respectively. On the other hand, $\sigma_t^{15} = \sum_{l=0}^{15} (\sigma_l' + \sigma_l'')$ is equal to 32.3 mbn and 31.7 mbn, respectively, which agree well with values of σ_t calculated from Eq. (10). The values $\sigma' = \sum_{l=0}^{15} \sigma_l' = 7.6$ to 7.8 mbn agree well with the experimental value[2] of 8 mbn for the cross section for elastic scattering.

We consider the results obtained using the quasiclassical approximation. The assumption that all elastic scattering is diffractional means that the scattering system is purely absorptive.

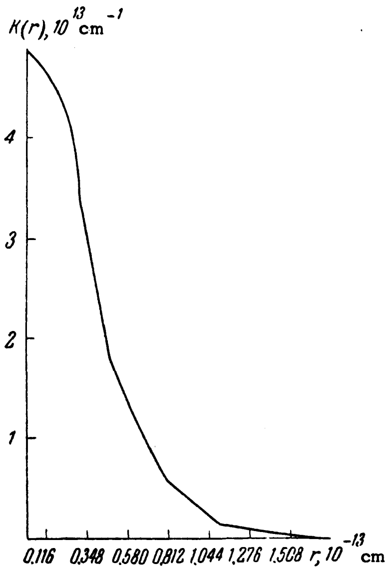

FIG. 4. Absorption coefficient for the nucleon-nucleon system of a function of distance between the centers of the nucleons.

In order to find the absorption coefficient K(r), results of the phase shift analysis for the curve (II) were used; these describe the experimental data well even if the systematic error of ±15% is not taken into account. K(r) was calculated by the method considered in reference 7. In our case r can be considered as the distance between "centers" of the nucleons. The results of calculation are given in Fig. 4.

From the formula

$$\overline{\rho^2} = \int_0^R r^4 K(r)\, dr \Big/ \int_0^R r^2 K(r)\, dr \qquad (11)$$

the value of the mean square radius of nuclear proton-proton interaction was found to be equal to 0.79×10^{-13} cm. This value agrees well with the electromagnetic radius of the proton, equal to 0.78×10^{-13} cm.[8]

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