MAGNETIC TRANSITIONS BETWEEN COLLECTIVE EXCITED STATES OF EVEN-EVEN NUCLEI

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The reduced probability for an M1 transition between two 2^+ states and the ratio of this probability to that for an E2 transition between the same states is calculated. The small value of this ratio is in agreement with the experimental data.

INTRODUCTION

In a previous paper,[1] assuming that the shape of the nucleus is not axially symmetric, the authors showed that various energy levels of nonaxial nuclei are satisfactorily explained if one assumes that they are assigned to be rotational states. The possibility of giving explicit expressions for wave functions of nuclear rotational states in terms of the parameters β and γ enables one to compute relative intensities of different transitions from a given excited state to other rotational states.

The intensity rules for transitions between rotational states of axial nuclei were investigated in a paper of Alaga, Alder, Bohr, and Mottelson.[2] In our paper,[1] we determined the probabilities of electric quadrupole transitions between rotational states of nonaxial even-even nuclei.

In the present paper we shall compute the probabilities of magnetic dipole transitions between rotational states having spins 2^+, 2^+. Such levels are observed in Se^{76}, Te^{122}, Os^{188}, Os^{186}, Pt^{192}, and other nuclides. As we have shown,[1] the knowledge of the ratio of the energy of the second 2^+ level to that of the first enables us to determine the parameter γ and the ratio of the reduced probabilities for the electric quadrupole transitions $B(E2; 22\rightarrow 21)/B(E2; 22\rightarrow 0)$. However, according to the general selection rules, an M1 transition is possible between two states of spin 2 and the same parity, in addition to the E2 transition. As the experiments[3] show, the intensity ratio $T(M1)/T(E2)$ is a few hundredths for the nuclei listed above. Such values are a definite indication that the lowest energy levels of spin 2 cannot be attributed to single-particle excitations, since in that case the ratio of the intensities of the competing magnetic dipole and electric quadrupole transitions can be given by the formula (cf. reference 4, Par. 31):

$$T(MJ)/T(E, J+1) \sim \{25(2J+1)/A^{2/3}(\hbar\omega)_{\mathrm{Mev}}\}^2;$$

from which one finds that this ratio is 10^4 for $A \sim 30$ and $\hbar\omega \sim 100$ kev, while it is ~ 10 for heavy nuclei and $\hbar\omega \sim 1$ Mev. Later we shall show that our picture of the rotational nature of the 2^+, 2^+ excited levels of even-even nuclei enables one to explain the experimentally observed value of $T(M1)/T(E2)$.

1. THE MAGNETIC DIPOLE MOMENT OPERATOR IN THE UNIFORM NUCLEAR MODEL

The collective excitations of the nucleus are described in the uniform model of A. Bohr and Mottelson by wave functions depending on five coordinates α_μ, which determine (relative to a fixed coordinate system) the deviation of the nuclear shape from spherical:

$$R(\vartheta\varphi) = R_0\left(1 + \sum_{\mu=-2}^{2} \alpha_\mu Y_{2\mu}\right), \tag{1.1}$$

They can also be described by the two parameters β and γ, which determine the shape of the nucleus in the coordinate system fixed in the principal axes of the ellipsoid of inertia of the nucleus, and the three Euler angles which fix the orientation of these axes relative to a fixed system of coordinates.

If the equilibrium values of β and γ are different from zero, the lowest collective excited states of the nucleus correspond to energy levels of an asymmetric top (cf. reference 1). The wave functions of these states are eigenfunctions of the square of the total angular momentum. In classical theory the components of this angular momentum are expressed in terms of the coordinates α_μ and velocities $\dot{\alpha}_\mu$ by the formula

$$J_\nu = i\sqrt{6}(-1)^\nu B \sum_\mu (21, \nu+\mu, -\nu \mid 2\mu)\, \dot{\alpha}_\mu \alpha^*_{\mu+\nu},$$
$$\nu = 0, 1, -1, \tag{1.2}$$

where B is the mass parameter of the theory; $(21, \nu+\mu, -\nu \mid 2\mu)$ are vector addition coefficients. In quantum theory the components of the angular momentum of the asymmetric top are operators acting on generalized spherical functions D^J_{MK} which depend on the Euler angles,

$$J_\nu D^J_{MK} = (-1)^\nu \sqrt{J(J+1)}\,(J1, M+\nu, -\nu \mid JM)\, D^J_{M+\nu, K}. \tag{1.3}$$

We now proceed to compute the magnetic dipole operator corresponding to collective motions in even-even nuclei. According to Bohr and Mottelson, this operator is given by the formula

$$\mathfrak{M}(1\mu) = \mu_0 g_R \int \mathbf{R}(\mathbf{r})\, \nabla (rY_{1\mu})\, d\tau, \tag{1.4}$$

where $\mu_0 = e\hbar/2Mc$ is the nuclear magneton; g_R is the gyromagnetic ratio corresponding to collective motion of the nucleons in the nucleus;

$$\mathbf{R}(\mathbf{r}) = BR_0^{-5} \sum_\lambda \dot{\alpha}_\lambda [\mathbf{r} \times \nabla (r^2 Y_{2\lambda})]$$

is the angular momentum density.

We write the operator ∇ as a sum of two terms:

$$\nabla = \frac{\mathbf{r}}{r}\frac{\partial}{\partial r} - \frac{i}{r^2}[\mathbf{r} \times \mathbf{L}],$$

where $\mathbf{L} \equiv -i[\mathbf{r} \times \nabla]$; we can then write

$$[\mathbf{r} \times \nabla (r^2 Y_{2\lambda})] = ir^2 \mathbf{L} Y_{2\lambda}.$$

Also noting that

$$\nabla (rY_{1\mu}) = (-1)^\mu \sqrt{3/4\pi}\, \mathbf{e}_\mu,$$

where the $\mathbf{e}_\mu$ are unit vectors which are given in terms of the unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ of a Cartesian coordinate system by the relations $\mathbf{e}_0 = \mathbf{e}_z$, $\mathbf{e}_{1,-1} = \mp(\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$, and

$$(\mathbf{e}_\mu \cdot \mathbf{L})\, Y_{2\lambda} = (-1)^\mu \sqrt{6}\,(21\lambda+\mu, -\mu \mid 2\lambda)\, Y_{2,\lambda+\mu},$$

we can write (1.4) in the form

$$\mathfrak{M}(1\mu) = \mu_0 g_R \frac{3B}{\sqrt{2\pi}} R_0^{-5} \times \sum_\lambda (21, \lambda+\mu, -\mu \mid 2\lambda)\, \dot{\alpha}_\lambda \int d\Omega \int_0^{R(\vartheta\varphi)} dr\, r^4 Y_{2,\lambda+\mu}, \tag{1.5}$$

where $R(\vartheta\varphi)$ is defined in (1.1)

Carrying out the integration and using (1.2), we can write (1.5) as a series

$$\mathfrak{M}(1\mu) = \mathfrak{M}_0(1\mu) + \mathfrak{M}_1(1\mu) + \cdots, \tag{1.6}$$

where

$$\mathfrak{M}_0(1\mu) = \mu_0 g_R \frac{1}{2}\sqrt{\frac{3}{\pi}}\, J_\mu, \tag{1.7}$$

$$\mathfrak{M}_1(1\mu) = \mu_0 g_R \frac{5\sqrt{6}}{7\pi} \sum_\nu (21, \mu-\nu, \nu \mid 1\mu)\, \alpha^*_{\mu-\lambda} J_\nu. \tag{1.8}$$

The matrix elements of the operator (1.7) between different rotational states are equal to zero. To obtain nonvanishing matrix elements, which determine the intensity of the magnetic dipole radiation between rotational states, one must include the operator (1.8) of the next approximation, which contains the collective coordinates α_μ and the projections J_ν of the total angular momentum.

2. PROBABILITY OF MAGNETIC DIPOLE TRANSITIONS BETWEEN ROTATIONAL STATES

We calculate the reduced probability for magnetic dipole transition between two rotational states having spin 2 and the same parity. According to reference 1, the wave functions of these states can be written as

$$\psi_{21m} = (5/8\pi^2)^{1/2}\,[a_1 D^2_{m0} + b_1 (D^2_{m2} + D^2_{m,-2})/\sqrt{2}],$$
$$\psi_{22m} = (5/8\pi^2)^{1/2}\,[a_2 D^2_{m0} + b_2 (D^2_{m2} + D^2_{m,-2})/\sqrt{2}], \tag{2.1}$$

where

$$a_1 N_1 = -[\sin\gamma \sin 3\gamma + 3\cos\gamma\cos 3\gamma + \sqrt{9 - 8\sin^2 3\gamma}],$$
$$b_1 N_1 = 3\sin\gamma\cos 3\gamma - \cos\gamma \sin 3\gamma,$$

$$N_1^2 = 2\sqrt{9 - 8\sin^2 3\gamma} \times [\sqrt{9 - 8\sin^2 3\gamma} + \sin\gamma\sin 3\gamma + 3\cos\gamma\cos 3\gamma],$$

$$a_2 N_2 = \sqrt{9 - 8\sin^2 3\gamma} - \sin\gamma\sin 3\gamma - 3\cos\gamma\cos 3\gamma,$$
$$b_2 N_2 = 3\sin\gamma\cos 3\gamma - \cos\gamma\sin 3\gamma,$$

$$N_2^2 = 2\sqrt{9 - 8\sin^2 3\gamma} \times [\sqrt{9 - 8\sin^2 3\gamma} - \sin\gamma\sin 3\gamma - 3\cos\gamma\cos 3\gamma].$$

The reduced probability for magnetic dipole radiation in the transition from the 22 state to the 21 state is given by

$$B(M1;\ 22 \to 21) = \frac{1}{5} \sum_{m'\mu m} |(21m' \mid \mathfrak{M}(1\mu) \mid 22m)|^2. \tag{2.2}$$

Substituting the values (1.8) and (2.1) in (2.2), we find

$$B(M1;\ 22 \to 21) = \frac{90}{49\pi^2}\mu_0^2 g_R^2 \beta^2 \frac{\sin^2 3\gamma}{9 - 8\sin^2 3\gamma}. \tag{2.3}$$

The reduced probability for magnetic dipole transition between the 22 and 21 states is equal to zero for an axial nucleus $(\gamma = 0)$, and increases

Nucleus	$E_{22}-E_{21}$ (kev)	$T(M1)/T(E2)$	Percent of E2 transition (exptl)
Se^{76}	643	$9.8\cdot 10^{-2}$	98 ± 1
Te^{122}	693	$1.9\cdot 10^{-2}$	92 ± 4
Os^{186}	627	$6.5\cdot 10^{-3}$	99 ± 1
Os^{188}	480	$1.04\cdot 10^{-2}$	99.6

with increasing γ, reaching its maximum value for $\gamma = 30°$.

As we have previously shown,[1] the value of γ can be determined from the ratio of the energies E_{22}/E_{21} of the two 2^+ levels. Then the measurement of the reduced probability for magnetic dipole transition enables us, by means of (2.4), to compute the gyromagnetic ratio g_R for collective motion of the nucleons in the nucleus.

Using the formula for the reduced probability of electric quadrupole transition between these same levels[1]

$$B(E2;\ 22 \rightarrow 21) = (10e^2Q_0^2/7\pi)\sin^2 3\gamma/(9-8\sin^2 3\gamma),$$

we get the ratio of the reduced probabilities for magnetic dipole and electric quadrupole transitions:

$$\frac{B(M1;\ 22\rightarrow 21)}{B(E2;\ 22\rightarrow 21)} = \frac{144}{7\pi}\left(\frac{\mu_0 g_R \beta}{eQ_0}\right)^2 = \frac{80}{7}\left(\frac{\mu_0 g_R}{eZR_0^2}\right)^2, \quad (2.4)$$

where R_0 is the nuclear radius. It is interesting to note that this ratio is independent of γ and β.

The ratio of the γ-ray intensities emitted in the two types of radiation is determined from the ratio (2.4) of the reduced probabilities by the formula

$$T(M1)/T(E2) = (0.03k^2)^{-1}\frac{B(M1;\ 22\rightarrow 21)}{B(E2;\ 22\rightarrow 21)}, \quad (2.5)$$

where $k = (E_{22} - E_{21})/\hbar c$.

The table contains values of $T(M1)/T(E2)$ for $22 \rightarrow 21$ transitions in a few nuclei, as obtained from (2.5) using the values $\mu_0 = 5.05 \times 10^{-24}$ erg/Gauss, $g_R = 0.4$; $R_0 = 1.2\,A^{1/3} \times 10^{-13}$ cm. The last column gives the intensities of the $22 \rightarrow 21$ transition from the data of Lindquist and Marklund.[3]

[1] A. S. Davydov and G. F. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 440 (1958), Soviet Phys. JETP **8**, 303 (1959).

[2] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab Mat. fys. Medd. **29**, No. 9 (1955).

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[4] M. E. Rose, Multipole Fields, Wiley, 1955 (Russ. Transl. IIL, 1957).

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