ON THE Λ-HYPERON CREATION CROSS SECTION NEAR THE Σ-HYPERON CREATION THRESHOLD

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A singularity arises in the cross section of the $\pi^- + p \to K^0 + \Lambda^0$ reaction near the threshold of the $\pi^- + p \to K^0 + \Sigma^0$ reaction. An experimental study of this singularity affords a check on the validity of various models used to explain interactions between strange particles. The relative parity of the Λ and Σ hyperons can also be determined.

THE energy and angular dependences of elastic scattering of particles near the threshold of some inelastic process were investigated in many works (cf., e.g., references 1 and 2). It has been shown that a measurement of this dependence can yield information on the elastic-scattering phase shifts and on the spin and parity of particles created in the inelastic process. As will be shown below, analogous information can be obtained also by investigating the energy and angle dependences of one inelastic process near the threshold of another inelastic process. Such an analysis may prove particularly useful in the study of strange particles.

By way of an example, we examine the creation of strange particles through collisions between protons and π^- mesons with energies close to $E_0 = 890$ Mev. This energy is the threshold for the reaction*

$$\pi^- + p \to K^0 + \Sigma^0. \tag{1}$$

Along with reaction (1), there occur also: (a) elastic scattering of π mesons, (b) processes involving creation of 2, 3, 4, and 5π mesons, and (c) creation of Λ hyperons in a reaction

$$\pi^- + p \to K^0 + \Lambda^0, \tag{2}$$

whose threshold is 760 Mev. We shall investigate the cross section of reaction (2) near the threshold of reaction (1).

We shall assume henceforth that the K-meson spin is 0, and that the spins of the Λ and Σ hyperons is $\frac{1}{2}$. We examine both possible values of the relative parity of the Λ and Σ hyperons.

*The existence of a reaction $\pi^- + p \to K + \Sigma^-$ the threshold of which should be several Mev below that of reaction (1), will barely affect the formulas derived below, since Coulomb attraction exists between the particles created in this reaction (see references 1 and 3 on this subject).

It is quite clear that in the study of threshold effects it is enough to consider only those partial channels of reactions (1), (2a) and (2b) with identical total momenta, parities, and isotopic spins. Since reaction (1) is at the threshold, $\Sigma^0 + K^0$ are created in the S state, and the total momentum j is $\frac{1}{2}$. The orbital momentum of $\Lambda^0 + K^0$ will at the same time be either 0 or to 1 in the channel of interest to us, depending on the relative parity of the Λ and Σ hyperons. Since reaction (2) proceeds only via a state with isotopic spin $T = \frac{1}{2}$, only this state will interest us in cases (a) and (b) of reaction (1).

We write the total S matrix of the channel with $j = \frac{1}{2}$ and $T = \frac{1}{2}$ and a given parity in the following form

$$S = \begin{vmatrix} S_{11} & S_{12} \ldots & S_{1N} & m_1 k^{1/2} \\ S_{21} & S_{22} \ldots & S_{2N} & m_2 k^{1/2}. \\ \ldots & \ldots & \ldots & \ldots \\ S_{N1} & S_{N2} \ldots & S_{NN} & m_N k^{1/2} \\ m_1 k^{1/2} & m_2 k^{1/2} \ldots & m_N k^{1/2} & x \end{vmatrix}, \tag{3}$$

where the matrix element at the intersection of the i-th column and j-th line is the transition from the j-th channel into the i-th one. We shall number the channels as follows: i = 1 corresponds to the $\pi^- + p$ pair; i = 2 corresponds to the $\Lambda^0 + K^0$ pair, and i = 3, 4, . . . , N corresponds to different states with several π mesons. Finally, the last channel (N + 1) corresponds to the $\Sigma^0 + K^0$ pair. The quantities $m_i k^{1/2}$ in the last column represent the amplitudes of creation of $\Sigma_0 + K^0$ in various channels. As a consequence of the symmetry of the S matrix, they equal the inverse processes listed in the lowest row.

We know that near the creation threshold of two neutral particles (in our case $\Sigma^0 + K^0$) their creation amplitudes are proportional to $k^{1/2}$ (k is the wave vector of the created particles, k =

$\sqrt{2\mu(e-e_0)}$. We can therefore assume the m_i in (3) to be independent of the energy. The matrix element x describes elastic scattering of the K^0 meson by the Σ^0 hyperon; $x \to 1$ as $k \to 0$. The remaining matrix element can be rewritten

$$S_{ik} = s_{ik} + a_{ik}k + \ldots, \qquad (4)$$

where s_{ik} is the value of S_{ik} at the threshold and a_{ik} are certain coefficients which will now be determined. The matrix S is unitary ($SS^+ = 1$) and symmetrical ($S = \tilde{S}$). Above the threshold this leads to the following relations between the matrix elements:

$$\sum_{l=1}^{N}(s_{il}\overset{*}{a}_{jl} + \overset{*}{s}_{jl}a_{il}) = -m_i\overset{*}{m}_j, \qquad \sum_{l=1}^{N}\overset{*}{s}_{il}m_l = -\overset{*}{m}_i x. \qquad (5)$$

Below the threshold the channel $\Sigma^0 + K^0$ is closed, and the matrix S is reduced in rank from (N + 1) to N. In addition, the wave vector in (4) now becomes imaginary. Taking this into account we can readily obtain

$$\sum_{l=1}^{N}(-s_{il}\overset{*}{a}_{jl} + \overset{*}{s}_{jl}a_{il}) = 0. \qquad (6)$$

Using (5), (6), and the condition $\sum_{l=1}^{N} s_{il}s^*_{kl} = \delta_{ik}$, which follows from the unitarity of the S matrix at the threshold, we obtain

$$a_{ik} = {}^1/_2\, m_i m_k, \qquad (7)$$

where we took it into account that $x \to 1$ when $k \to 0$.

Having determined the energy dependence of the S matrix, we can now ascertain how reaction (2) behaves near E_0, which is the threshold of reaction (1).

The cross section for the creation of $\Lambda^0 + K^0$ is (in the c.m. system)

$$\sigma_\Lambda(\theta, E) = |g(\theta, E)|^2 + |h(\theta, E)|^2,$$

and the polarization of the Λ^0 hyperons is given by

$$P_\Lambda(\theta, E) = 2\,\mathrm{Im}\, h(\theta, E)\, g^*(\theta, E)/\sigma_\Lambda(\theta, E),$$

where the amplitudes g and h are determined, as usual, by

$$g(\theta, E) = \frac{1}{2ik_1}\sum_l [(l+1)M_l^{l+1/2} + lM_l^{l-1/2}]P_l(\theta),$$
$$h(\theta, E) = \frac{1}{2ik_1}\sum_l [M_l^{l+1/2} - M_l^{l-1/2}]P_l^{(1)}(\theta). \qquad (8)$$

Here k_1 is the wave vector of the colliding p + π^-, m_l^j the matrix element that describes the creation of $\Lambda^0 + K^0$ in a state with a total momentum j and an orbital momentum l, P_l and $P_l^{(1)}$ the ordinary and adjoint Legendre polynomials, θ the angle of emission of the Λ^0 hyperon, and E the total energy of the system.

Near the threshold of the $\Sigma^0 + K^0$ creation, as follows from formulas (5) and (7), all the elements M_l^j are independent of the energy, with the exception of the one characterized by the same quantum numbers as the state of $\Sigma_0 + K^0$. This last element is of the form

$$M_l^{1/2}(E) = M_l^{1/2}(E_0) + {}^1/_2\, m_1 m_2 k. \qquad (9)$$

We can thus calculate the energy dependence of the cross section and of the polarization of reaction (2) near E_0, assuming that the entire dependence is contained in the matrix element (9).

If the Σ hyperon has the same parity as the Λ hyperon, then $l = 0$ in formula (9) and we obtain from (8)

$$g(\theta, E) = g(\theta) + \alpha k/2ik_1,\quad h(\theta, E) = h(\theta),\quad \alpha = m_1 m_2/2.$$

Hereinafter $\varphi(\theta) \equiv (\theta, E = E_0)$, where φ is any quantity.

The cross section of reaction (2) and the polarization of the Λ hyperons are:

$$\sigma(\theta, E) = \sigma(\theta) + \frac{|k|}{k_1}\begin{cases}-\mathrm{Im}\, g(\theta)\alpha^* & E > E_0\\ \mathrm{Re}\, g(\theta)\alpha^* & E < E_0\end{cases},$$
$$P(\theta, E) = P(\theta) + \frac{|k|}{k_1}\begin{cases}\mathrm{Re}\, f(\theta)\alpha^* & E > E_0\\ \mathrm{Im}\, f(\theta)\alpha^* & E < E_0\end{cases}, \qquad (10)$$

where $f(\theta) = (h(\theta) - iP(\theta)g(\theta))/\sigma(\theta)$.

If the parity of the Σ hyperon opposes that of the Λ hyperon, we have $l = 1$ in formula (9) and

$$g(\theta, E) = g(\theta) + \frac{\alpha\cos\theta}{2ik_1}k,\quad h(\theta, E) = h(\theta) + \frac{\alpha\sin\theta}{2ik_1}k,$$
$$\sigma(\theta, E) = \sigma(\theta) + \frac{|k|}{k_1}\begin{cases}-\mathrm{Im}\, l(\theta)\alpha^* & E > E_0\\ \mathrm{Re}\, l(\theta)\alpha^* & E < E_0\end{cases}, \qquad (11)$$
$$P(\theta, E) = P(\theta) + \frac{|k|}{k_1}\begin{cases}\mathrm{Re}\, f_1(\theta)\alpha^* & E > E_0\\ \mathrm{Im}\, f_1(\theta)\alpha^* & E < E_0\end{cases}.$$

We have used here the following notation:

$$f_1(\theta) = (m(\theta) - iP(\theta)\,l(\theta))/\sigma(\theta),$$
$$m(\theta) = h(\theta)\cos\theta - g(\theta)\sin\theta,\quad l(\theta) = g(\theta)\cos\theta + h(\theta)\sin\theta.$$

It is seen from (10) and (11) that near E_0 the cross section and the polarization are linear functions of $|k| \sim |E - E_0|^{1/2}$, and are singular near the threshold as functions of energy. The singularity may be a peak, a dip, or a step (for more details see references 1 to 3). On the basis of rather reasonable estimates we can expect the energy width of such a singularity to be on the order of $|k| \gtrsim \mu_\pi$. This singularity is proportional to $\alpha = m_1 m_2/2$, where m_2 is by definition the amplitude of the following reaction

$$K + \Lambda \rightarrow K + \Sigma \qquad (12)$$

divided by $\sqrt{k}$. Consequently the greater the probability of reaction (12), the larger the singularity in the reaction (2) near E_0.

What can be predicted, on the basis of existing model representations of the interactions of strange particles, as regards the cross section of reaction (12) and, consequently, as regards the size of the singularity in reaction (2)^2?

In the model proposed by Gell-Mann,[4] π mesons interact strongly with baryons and moderately with K mesons. The amplitude of reaction (12), $m_2 k^{1/2}$, will be less in this model than the amplitude of reaction (1), $m_1 k^{1/2}$, since reaction (1) contains only one moderate interaction, while reaction (12) contains two, absorption and K-meson emission. Reaction (2), like reaction (1) contains one moderate interaction and the amplitudes of these reactions are apparently of the same order of magnitude. Using the language of perturbation theory, we can say that the quantity $\sigma(\theta)$ in formulas (10) and (11) is proportional to g^2, while the terms describing the singularity are proportional to g^4, where g is the constant of the moderate interaction. One can therefore expect on the basis of Gell-Mann's theory that the singularity of reaction (2) will be small in the vicinity of E_0.

An entirely different result is obtained by using the model considered by one of the authors.[5] According to this second model the creation of strange particles by collision between ordinary particles is due to the moderate interaction, while transitions of type (12) are due to the strong interaction. Consequently, within the framework of the second model, $m_2 > m_1$, the terms describing the singularity, like the cross section itself, are proportional to g^2, and the singularity in the cross section of reaction (2) would therefore be expected to be comparable with the value of the cross section of this reaction itself.

Thus, if the singularities predicted by formulas (10) or (11) are observed in the cross section for the creation of Λ hyperons and in their polarization, this would serve as an argument in favor of the second model.

It follows from formulas (10) and (11) that, by investigating reaction (2) near the threshold of reaction (1), it is possible to determine the relative parity of the Λ and Σ hyperons.* We shall demonstrate this by assuming, for the sake of being definite, that only S and P waves participate in reaction (2). This means that the amplitudes g and h do not contain higher powers of $\cos\theta$ and $\sin\theta$ than the first. On the other hand, it is seen from (10) and (11) that the angular distribution of the singularity in the cross section is determined by the value of $g(\theta)$ if the Λ and Σ hyperons have the same parity, and by the quantity $g(\theta)\cos\theta + h(\theta)\sin\theta$ if the parities are opposite. The first quantity does not contain $\cos^2\theta$ but the second does contain it. Therefore, by measuring experimentally the angular distribution of the singularity in reaction (2) and then expanding it in powers of $\cos\theta$, it is possible to determine the relative parity of the Λ and Σ hyperons. Similar reasoning can be applied if D waves also participate in the reaction along with the S and P waves. The actual number of partial waves in reaction (2) can be determined from the angular distribution of this reaction.

It also follows from formulas (10) and (11) that with sufficiently accurate measurements it is possible to find (with accuracy to within a common phase factor) the amplitudes $g(\theta)$ and $h(\theta)$. The possibility of total determination of the amplitudes $g(\theta)$ and $h(\theta)$ becomes evident from a count of the number of equations obtained for these amplitudes from experiment, comparing it with the number of unknown parameters in $g(\theta)$ and $h(\theta)$ (see reference 2). It must be noted that at present there is no other method whatever for the determination of the amplitudes $g(\theta)$ and $h(\theta)$, since it is essential to know, for an ordinary phase-shift analysis of reaction (2), not only the cross section and polarization in this reaction, but also in all other reactions occurring during $\pi^- + p$ collisions at this energy.

The above argument can be extended to include other cases of strange-particle creation. In particular, it would be interesting to investigate the reactions $\bar{K} + p \rightarrow \Lambda(\Sigma) + \pi$ near the threshold of creation of a cascade hyperon $\bar{K} + p \rightarrow \Xi + K$.

*Professors Flowers and Chinovskii have communicated to the authors that the possibility of determining the relative parity of Λ and Σ hyperons by investigating reaction (2) near the threshold of reaction (1) has been indicated by Adair.

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