DEPOLARIZATION OF μ^+ MESONS IN METALS

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The problem of the depolarization of μ^+ mesons in metals is treated on the assumption that mesonium is formed. It is shown that the exchange interaction of the electron in mesonium with the electron fluid of the metal decidedly diminishes the depolarization of μ^+ mesons. On reasonable assumptions about the dimensions of mesonium in a metal, estimates are obtained that are in good agreement with the experimental data.

THE problem of the depolarization of μ^+ mesons in condensed media has taken on importance in connection with the discovery of the nonconservation of parity in weak interactions.[1-3]

As a consequence of the nonconservation of parity in the $\pi^+ \to \mu^+$ decay, the μ^+ mesons are produced polarized along the direction of their motion, and in the two-component theory of the neutrino[2-4] this polarization is complete. To determine the polarization of the μ^+ mesons one can use the nonconservation of parity in the subsequent $\mu^+ \to e^+$ decay.

When integrated over the energy, the angular distribution of the decay positrons has the form[2] $1 + a\cos\theta$. Here θ is the angle between the momentum of the μ^+ meson and the direction of emission of the positron, and $a = Pa_0$, where P is the polarization of the μ^+ meson and a_0 depends on the type of interaction chosen in the theory.

It is possible, however, that during the time elapsing from the production of the μ^+ meson until its decay there can be appreciable depolarization owing to the interaction of the μ^+ meson with the medium.

The experimental data show that the angular anisotropy of the positrons depends strongly on the substance in which the μ^+ mesons are stopped.[5] In particular, in metals the anisotropy is larger than in many other substances. The lifetime of the μ^+ meson ($\tau_0^+ \approx 2\times 10^{-6}$ sec) is evidently not long enough for any appreciable depolarization to occur in metals.

One of the possible mechanisms for the depolarization of μ^+ mesons is the formation of mesonium — the bound system of a μ^+ meson and an electron.

It is readily verified that owing to the hyperfine structure of the ground state of mesonium the formation of this system ($\mu^+ + e^-$) leads, in the absence of a surrounding medium, to depolarization of the μ^+ meson by an average factor of two. This value of the depolarization is obtained on the assumption that both directions of the spin of the captured electron are equally probable.

In the case of formation of mesonium in condensed media one must also take into account irreversible processes of interaction with the medium. This means that the behavior of the density matrix ρ of the system of the spins of the μ^+ meson and electron is described by the Wangsness-Bloch equation:[6,7]

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[\hat{H}, \rho] - \frac{\pi}{\hbar}\,\mathrm{Sp}_b\,[\hat{\hat{V}}, [\hat{\hat{V}}, \rho\rho^{(b)}]], \tag{1}$$

where the index (b) refers to the medium and $\rho^{(b)}$ is the equilibrium density matrix of the medium. Here we have: $\hat{H} = (\epsilon/2)\sigma_k^{(\mu)}\sigma_k^{(e)}$, with $\epsilon = 16\mu_e\mu_\mu/3r_0^3$ in vacuum, where $r_0 = \hbar^2/m_e e^2$ is the Bohr radius; $\sigma_k^{(\mu)}$, $\sigma_k^{(e)}$ are the Pauli spin matrices of the μ^+ meson and the electron, respectively; and $\hat{\hat{V}}$ is the part of the operator $\hat{V}$ for the interaction of mesonium with the medium which is diagonal in the energy. ($\overline{V}_{nk} = V_{nk}[\delta(E_n^{(b)} - E_k^{(b)})]^{1/2}$; $E_n^{(b)}$ and $E_k^{(b)}$ are energy levels of the medium.)

The density matrix ρ can be represented as a linear combination of operators:

$$\rho = \sum_{\varkappa k}\rho_{\varkappa k}U_\varkappa^{(\mu)}U_k^{(e)}, \tag{2}$$

$$U_0^{(\mu)} = \chi^{(\mu)}/\sqrt{2};\; U_0^{(e)} = \chi^{(e)}/\sqrt{2};$$

$$U_j^{(\mu)} = \sigma_j^{(\mu)}/\sqrt{2};\; U_i^{(e)} = \sigma_i^{(e)}/\sqrt{2},$$

normalized by the condition

$$\mathrm{Sp}\,(U_i U_k) = \delta_{ik}\,,$$

where $\chi^{(\mu)}$, $\chi^{(e)}$ are unit operators. The coefficients $\rho_{\kappa k}$ are the average values of the corresponding operators:

$$\rho_{\varkappa k} = \langle U_{\varkappa}^{(\mu)} U_k^{(e)} \rangle. \tag{3}$$

As the medium to be considered we take a metal in which all the valence electrons take part in the conductivity. In such a metal the centers of the crystal lattice do not produce any magnetic fields with which the spin of the μ^+ meson could interact, and we neglect the direct interaction of the μ^+ meson with the conduction electrons. Therefore in our case $\hat{V}$ is the operator for the interaction of the spin of the captured electron with the conduction electrons.

Assuming that the relaxation times of the electron spin in the three independent directions are the same, we find without difficulty:

$$\begin{aligned} \frac{\partial \rho_{0i}}{\partial t} &= \frac{\varepsilon}{\hbar} \sum_{\varkappa k} e_{ki\varkappa}\, \rho_{\varkappa k} - \frac{W}{\hbar} \rho_{0i}\,, \\ \frac{\partial \rho_{j0}}{\partial t} &= \frac{\varepsilon}{\hbar} \sum_{\varkappa k} e_{\varkappa jk}\, \rho_{\varkappa k}\,, \\ \frac{\partial \rho_{ji}}{\partial t} &= \frac{\varepsilon}{\hbar} \sum_{\varkappa k} (e_{kij}\, \rho_{0k} + e_{\varkappa ji}\, \rho_{\varkappa 0}) - \frac{W}{\hbar} \rho_{ji}\,; \end{aligned} \tag{4}$$

$\hbar/W$ is the relaxation time of the spin of the captured electron in the metal; $e_{i\kappa k}$ is the antisymmetric unit tensor.

The general solution of the system (4) has the form:

$$\begin{aligned} \rho_{0i} &= a_{0i} \exp\{-\gamma_0 t\} + b_{0i} \exp\{-(\gamma_r - i\gamma_i)\, t\} \\ &\quad + b_{0i}^* \exp\{-(\gamma_r + i\gamma_i)\, t\}, \\ \rho_{j0} &= a_{j0} \exp\{-\gamma_0 t\} + b_{j0} \exp\{-(\gamma_r - i\gamma_i)\, t\} \\ &\quad + b_{j0}^* \exp\{-(\gamma_r + i\gamma_i)\, t\}, \\ \rho_{ji} &= a_{ji} \exp\{-\gamma_0 t\} + b_{ji} \exp\{(\gamma_r - i\gamma_i)\, t\} \\ &\quad + b_{ji}^* \exp\{-(\gamma_r + i\gamma_i)\, t\}. \end{aligned} \tag{5}$$

Using Eqs. (4) and (5) we get the following system of algebraic equations:

$$\begin{aligned} &-\gamma_0 a_{0i} = (\varepsilon/\hbar) e_{ki\varkappa}\, a_{\varkappa k} - (W/\hbar)\, a_{0i}\,, \\ &-(\gamma_r - i\gamma_i) b_{0i} = (\varepsilon/\hbar)\, e_{\varkappa ki}\, b_{\varkappa k} - (W/\hbar)\, b_{0i}\,, \\ &-\gamma_0 a_{j0} = (\varepsilon/\hbar)\, e_{\varkappa jk}\, a_{\varkappa k}\,, \\ &-(\gamma_r - i\gamma_i)\, b_{j0} = (\varepsilon/\hbar)\, e_{\varkappa jk}\, b_{\varkappa k}\,, \\ &-\gamma_0 a_{ji} = (\varepsilon/\hbar)\, (e_{kij}\, a_{0k} + e_{kji}\, a_{k0}) - (W/\hbar)\, a_{ji}\,, \\ &-(\gamma_r - i\gamma_i)\, b_{ij} = (\varepsilon/\hbar)\, (e_{kji}\, b_{k0} + e_{kij}\, b_{0k}) - (W/\hbar)\, b_{ji}\,. \end{aligned} \tag{6}$$

On the assumption that at the instant of its capture the electron is unpolarized and that there is no polarization correlation ρ_{ji}, we have for the initial conditions:

$$\begin{aligned} &a_{0i} + b_{0i} + b_{0i}^* = 0, \\ &a_{j0} + b_{j0} + b_{j0}^* = \rho_{j0}^{(0)}, \quad a_{ji} + b_{ji} + b_{ji}^* = 0, \end{aligned} \tag{7}$$

where $\rho_{j0}^{(0)}$ is the value of the polarization vector ρ_{j0} of the μ^+ meson at the initial time.

As will become evident later, in metals $W \gg \epsilon$. Keeping the first few terms of the expansion in the small parameter ϵ/W, we readily obtain the relations

$$\begin{aligned} \rho_{j0} &= \rho_{j0}^{(0)} \left\{ \left[1 + 2\left(\frac{\varepsilon}{W}\right)^2\right] e^{-2\varepsilon^2 t/\hbar W} \right. \\ &\qquad \left. - 2\left(\frac{\varepsilon}{W}\right)^2 \cos\left(\sqrt{2}\,\frac{\varepsilon}{\hbar}\, t\right) e^{-Wt/\hbar} \right\}, \\ \rho_{0i} &= \rho_{i0}^{(0)} \left\{ 2\left(\frac{\varepsilon}{W}\right)^2 e^{-2\varepsilon^2 t/\hbar W} - \sqrt{2}\left(\frac{\varepsilon}{W}\right) \sin\left(\sqrt{2}\,\frac{\varepsilon}{\hbar}\, t\right) e^{-Wt/\hbar} \right\}, \\ \rho_{ji} &= \rho_{k0}^{(0)} \left\{ \left(\frac{\varepsilon}{W}\right) e^{-2\varepsilon^2 t/\hbar W} - \left(\frac{\varepsilon}{W}\right) \cos\left(\sqrt{2}\,\frac{\varepsilon}{\hbar}\, t\right) e^{-Wt/\hbar} \right\} e_{jik}. \end{aligned}$$

Thus the relaxation time of the μ^+ meson spin turns out to be given by $\hbar W/\epsilon^2 \gg \hbar/\epsilon$, and thus during times of the order of $\hbar/\epsilon$, which are characteristic of the hyperfine splitting of mesonium, there is no halving of the polarization. Physically this means that the strong interaction with the conduction electrons breaks the coupling of the captured electron to the spin of the μ^+ meson, so that the depolarization of the μ^+ meson is slowed down.*

To obtain an estimate of the relaxation time of the electron spin in the metal we note the fact that the interaction of the electron in the mesonium atom with the conduction electrons is mainly an exchange interaction. When the mesonium atom collides with an electron with its spin opposite to that of the electron in the mesonium atom, the electrons can change places. Since such exchange collisions are due to Coulomb forces they are predominant over the magnetic interactions, which are relativistic effects.

For the number of exchange collisions in unit time we get:

$$\frac{W}{\hbar} = \frac{m}{\pi^2 \hbar^3} \int_0^\infty \bar{n}(E)\, \sigma(E)\, (1 - \bar{n}(E))\, E\, dE, \tag{8}$$

where $\sigma(E)$ is the cross section of the exchange interaction, which depends on the energy E of the conduction electron, and

$$\bar{n}(E) = (e^{(E-\mu)/kT} + 1)^{-1}$$

is the Fermi distribution. Computing the expression (8) under the conditions $kT \ll \mu$, $\mu = E$. (μ is the chemical potential, and E_0 is the Fermi limit energy) and with the approximation $\sigma(E_0) \sim r_0^2$, we readily find:

*Similar considerations have been given earlier in a qualitative form by Ia. B. Zel'dovich.

$$W = \frac{mE_0\,kT}{\pi^2\,\hbar^2}\,\sigma(E_0) \sim \frac{mE_0\,r_0^2}{\pi^2\,\hbar^2}\,kT;$$

here $\hbar^2/mr_0^2 = me^4/\hbar^2$ is the atomic unit of energy, and E_0 is also of the order of an atomic energy. Thus

$$W \sim kT \approx 300°.$$

Since $\epsilon = 16\mu_e\mu_\mu/3r_0^3 \approx 0.1°$, $W \gg \epsilon$, as was assumed above.

For the relaxation time of the μ^+ meson spin we get: $\tau_{rel} = \hbar W/2\epsilon^2 \sim 10^{-7}$ sec, i.e., $\tau_{rel} < \tau_0^+$. With such a ratio of the times some depolarization should occur. It must be noted, however, that in the calculations the radius of mesonium was taken to be the Bohr radius. In actual fact, in condensed media it can be somewhat larger. If we note further that the relaxation time of the μ^+ meson spin is very sensitive to the value assumed for the radius r_0 ($\tau_{rel} \sim r_0^6$, since $\tau_{rel} \sim \epsilon^{-2}$ and $\epsilon \sim r_0^{-3}$), when with reasonable assumptions regarding the radius of mesonium in metals we can get satisfactory agreement with experiment. Verification of the assumed depolarization mechanism could be obtained by experiments to determine the angular anisotropy of the positrons from $\pi^+ \to \mu^+ \to e$ decay at low temperatures.

The depolarization caused by the formation of mesonium in metals should increase with decrease of the temperature (since $W \sim kT$), whereas with other depolarization mechanisms the behavior will evidently be the opposite of this.

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