ON THE MECHANISM OF STRIPPING IN REACTIONS INVOLVING THE CAPTURE OF TWO NUCLEONS

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Reactions of the (n, t) type are examined qualitatively. Two competing processes are discussed: (a) the process of "successive stripping" (n-d-t), and (b) the process of simultaneous capture of two nucleons.

As is known, a characteristic peculiarity of the angular distributions in the stripping reactions (d, p) and (d, n) and in the pick-up reactions (p, d) and (n, d) is the presence of a maximum in the region of small angles whose position permits the determination of the orbital angular momentum l of the captured nucleon.[1] The experimental data (e.g., references 2 to 4) show that a pick-up process may also occur in the reactions (n, t), (d, t), (d, α), and similar ones. In this connection attempts[5-7] were made to give a theoretical foundation for the stripping mechanism in a more general form, as an extension of the direct interaction process.

In the present paper we give a qualitative discussion of the reactions of the type (n, t), (p, t), (n, He^3), and (p, He^3).*

We consider the expression for the amplitude of the reverse reaction (t, n). In analogy to the usual stripping theory,[8] the amplitude is determined by the integral equation

$$\psi = \psi_0 - G\,(V_{01} + V_{02})\,\psi. \tag{1}$$

Here 0 is the index referring to the emitted neutron, 1, 2 are the indices of the nucleons captured by the nucleus, ψ_0 is the wave function of the initial system (nucleus A + triton), G is the Green's function for the Schrödinger equation of the final system (nucleus (A + 2) + neutron), and V_{ij} is the interaction potential of the two nucleons. In Born approximation the ψ on the right hand side is changed to ψ_0. In calculations using the method of distorted waves the Schrödinger equation contains the interaction between the neutron with the nucleus (A + 2), for example, in the form of the optical potential.

*The reaction (p, t) for Li^7 was considered by A. I. Baz' (Paper delivered by A. A. Ogloblin at the Conference on Nuclear Reactions in Moscow, 1957).

Generally speaking, two processes contribute to the amplitude of the reaction: (a) a transition of type (n-d-t) with the formation of a deuteron in the intermediary stage, and (b) a direct transition (n-t), made possible by the fact that the nucleons in the triton, as well as (on account of pair interactions) the nucleons in the nucleus, do not have a definite energy, so that the initial and final states of the nucleons are not orthogonal.

In general the angular distributions of the particles emitted in processes (a) and (b) may be different. We consider these distributions on the basis of the shell model of the nucleus. In this case the reaction cross section is determined, in the Born approximation, by the square of the matrix element

$$\begin{aligned} I = 2\left(\frac{n(n-1)}{2}\right)^{1/2} \langle l^{n-2}\,\alpha_2 L_2 S_2 J_2 T_2;\\ \mathbf{k}_t S_t\,|\,V_{01}\,|\,l^n\,\alpha_1 L_1 S_1 J_1 T_1;\ \mathbf{k}_n S_n\rangle, \end{aligned} \tag{2}$$

where we assume LS coupling, having in view the application of this formula to the lightest nuclei with a p shell.*

After summing the square of the matrix element over the magnetic quantum numbers of the initial and final states by the method of Levinson,[9] we obtain the following expression:

$$\begin{aligned} I^2 &= 2n\,(n-1)\,(2L_1+1)\,(2S_1+1)\\ &\times (2J_1+1)\,(2J_2+1)\,(2S_t+1)\\ &\times \sum (2S+1)^{-1}\,(-1)^{\rho}\,(l^n\,\alpha_1\,L_1\,S_1\,T_1\,|\,l^{n-2}\,\alpha_2\,L_2\,S_2\,T_2;\ l^2\,LST)^2\\ &\times C^2_{T_2T}\,(T_1\,M_{T_1};\ M_{T_2}\,0)\,C^2_{T^1/_2}\,(^1/_2 - {}^1/_2;\ 0 - {}^1/_2)\,D^2_{\mathbf{k}_n,\,\mathbf{k}_t}\,(l^2,\,L)\\ &\times \sum_{\lambda} (2\lambda+1)\,(-1)^{\mu}\,W(S_1\,S_1\,L_1\,L_1,\ \lambda J_1)\,W(S_2\,S_2\,S_1\,S_1;\ \lambda S_1)\\ &\times W(L_1\,L_1\,L_2\,L_2;\ \lambda L)\,W(L_2\,L_2\,S_2\,S_2;\ \lambda J_2);\\ \rho &= J_2 - J_1 + S + 2S_2; \end{aligned}$$

*The qualitative conclusions are the same for jj coupling.

$$\mu = J_1 + J_2 + L + 2(S_1 + S_2 + L_1 + L_2);$$

$$D^2_{k_n,\, k_t}(l^2, L) = (2L+1)^{-1} \sum_{M_L} |\langle k_t(0, 1, 2)| V_{01} | l^2 LM_L(1, 2); k_n(0)\rangle|^2. \quad (3)$$

C and W denote, respectively, the Clebsch-Gordan and Racah coefficients; $(l^n\alpha_1 | l^{n-2}\alpha_2; l^2)$ is the parentage coefficient.[10,11] The wave functions of the free particles are normalized to unit amplitude. L is the total orbital angular momentum of the two nucleons escaping from the nucleus. A non-zero contribution to expression (3) comes only from those values of L which satisfy the condition

$$L = 0,\ 2,\ \ldots,\ 2l; \quad \mathbf{S}_t + \mathbf{J}_2 = \mathbf{L} + \mathbf{J}_1 + \mathbf{S}_n.$$

The expression for the differential cross section has the usual form:

$$\frac{d\sigma}{d\Omega} = \frac{M_n M_t}{4\pi^2\hbar^4}\,\frac{k_t}{k_n}\,\frac{1}{(2S_n+1)(2J_1+1)}\, I^2. \quad (4)$$

In either process (a) or (b), the angular distributions are very similar in form to the somewhat smeared curves characteristic for the usual stripping process.[1] The difference between the processes (a) and (b) is that the value of the orbital angular momentum of the captured nucleon in the usual stripping theory is replaced by l in case (a), and by L in case (b). The interference term has, of course, a more complicated form. As in the usual stripping theory (for neutrons of energy ~15 to 20 Mev), the cross section for the transition (b) with L = 2 will, other conditions being equal, be one order of magnitude smaller than the cross section for the transition (b) with L = 0.

Figure 1 shows the angular distribution for the process (a) with $l = 1$ (nucleus with a p shell) obtained by an approximate treatment of the "successive stripping" (p-d-t). It was assumed that the energy of the system is conserved in the intermediate stage. The calculation was made for the reaction $Li^7(p, t)Li^6$ with $E_p = 12$ Mev, since experimental data are available for this case.[2]

Figure 2 shows the angular distributions for the process (b) with L = 0 at $E_p = 12$ and 35 Mev. Figure 3 shows the case L = 2 at the same energies. The angular distributions for process (b) are also calculated approximately. In the computation, oscillator wave functions were used for the nucleons bound in the nucleus. This permits the use of a representation that separates the center-of-mass motion of the two nucleons from their relative motion;[12] in the result, the integration in the matrix element becomes much simpler, as it is extended over the whole range of the argument. (This procedure affects the angular distribution very little[13]). Furthermore, the interaction between the nucleons was assumed to have the form of a delta function. The internal wave function of the triton was taken from the paper of Newns.[15] In the general case, when several values L contribute to expression (3), and when the intermediary deuterons in process (a) have sufficient energy, the angular distribution will be the sum of several contributions, and its analysis will be more difficult. An example for this case is the reaction $Li^7(p, t)Li^5$, where the sum over L in formula (3) comprises two values, L = 0 and L = 2, with approximately equal parentage coefficients.[10,11] We do not know the ratio of the amplitudes for processes (a) and (b). The experimental angular distributions[2] indicate the presence of two peaks: one with a maximum at 0°, and another, with half the amplitude, at ~55°. It may be assumed that the second peak is caused by

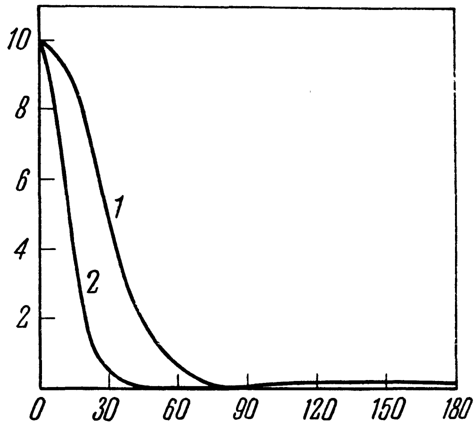

FIG. 2. Angular distribution for process (b); reaction $Li^7(p, t)Li^5$, L = 0. Curve 1) $E_p = 12$ Mev, curve 2) $E_p = 35$ Mev.

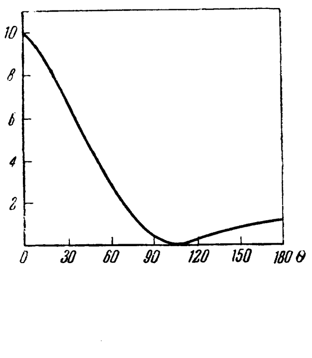

FIG. 1. Angular distribution for process (a); reaction $Li^7(p, t)Li^5$, $E_p = 12$ Mev, $l = 1$. Abscissa: angles in the center-of-mass system; ordinate: differential cross sections in arbitrary units.

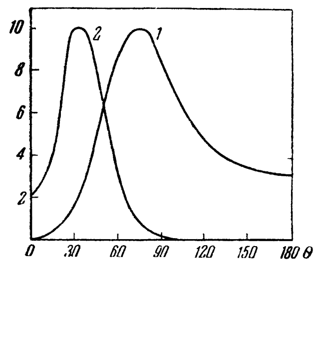

FIG. 3. Angular distribution for process (b); reaction $Li^7(p, t)Li^5$, L=2. Curve 1) Ep= 12 Mev, curve 2) Ep= 35 Mev.

either the interference of processes (a) and (b) for $L = 2$, or by process (b) with $L = 2$.

A whole series of cases exists where the situation is less complicated, and where the analysis is less ambiguous. In particular, we have such a case when the sum over L in expression (3) contains only one term, as, for example, in the reaction $Li^6(n, t)He^4$, where $L = 0$. The experimental angular distribution[4] is in qualitative agreement with the theory (Figs. 1 and 2). However, in this case the angular distributions for processes (a) and (b) are similar, which makes their separation impossible. A separation may be possible if the expression (3) contains only the term $L = 2$, for example (Fig. 3). This may be the case if the initial value of J is significantly different from the final value, as, for example, in the reactions $B^{10}(n, t)Be^8$ and $C^{12}(n, t)B^{10}$ with formation of the final nuclei in the ground state. However, experimental data for these reactions are missing up to now.

Processes (a) and (b) may also be separated in the reactions (n, t), (p, t), etc. for which the threshold for deuteron formation E_d^0 is above the threshold for triton formation E_t^0, where $E_d^0 - E_t^0$ is sufficiently large. It is of interest here to compare the angular distributions of the tritons for incident particles of energies below the threshold E_d^0, when only process (b) is possible, and for those of energies above the threshold E_d^0, when both processes are possible. If the probability of process (a) is comparable with the probability of process (b), some changes can be expected in the behavior of the excitation curve in the region of incident energies corresponding to the threshold for process (a). Thus, for the reaction $Be^9(n, t)Li^7$ the thresholds E_t^0 and E_d^0 are respectively equal to 11.5 and 16.2 Mev, while the corresponding values for the reaction $C^{12}(p, t)C^{10}$ are 5.8 and 17.9 Mev. At present, experimental data have only been published for the reaction $Be^9(n, t)Li^7$ at incident energies between E_d^0 and E_t^0.[3] The measured angular distribution has one maximum at 40°. The interpretation of this result is not clear, since expression (3) should get the largest contribution from $L = 0$.

We can also hope for a separation of the processes (a) and (b) if $l > L$. Examples for this may be found among the reactions with nuclei with the shell $1f_{7/2}$. For example, we can expect that in the reaction $Ca^{42}(p, t)Ca^{40}$ ($l = 3$, $L = 0, 1$) the angular distribution for the process (b) has a peak around 0°, while the maximum of the differential cross section for the process (a) will supposedly lie in the region of large angles.

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