ON THE HYDRODYNAMIC THEORY OF MULTIPLE PRODUCTION OF PARTICLES

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Submitted to JETP editor May 9, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 978-981 (October, 1958)

The problem of symmetries in the angular and energy distributions of secondary particles produced in nucleon-nucleus collisions or in collisions of two nuclei is considered on the basis of the hydrodynamic theory of multiple production of particles. It is shown that such symmetry appears in a certain special system of coordinates which is close to the center-of-mass system.

A hydrodynamic theory of multiple production of particles in the collision of two identical particles has been developed in a paper by Landau.[1] In this case it is obvious that the angular and energy distribution of the particles produced by the collision will be symmetrical in the center-of-mass system with respect to the plane perpendicular to the direction of motion of the colliding particles. In the present paper we shall show that in the collision of a nucleon with a nucleus the angular and energy distributions will also be approximately symmetrical in a certain system of coordinates which is close to the center-of-mass system.

It is convenient to conduct the calculations in the reference system in which the colliding particles have equal and opposite velocities. Because of the strong Lorentz contraction of the nucleon and nucleus the disturbance produced in the nucleus by the nucleon cannot spread far in the transverse direction during the time of the collision; we can therefore assume that the nucleon interacts only with a tube of nuclear matter lying directly in front of it.[2] Also because of the Lorentz contraction the motion of the system in the first moments after the collision will be in only one dimension. In proving the symmetry we need consider only the one-dimensional stage of the motion of the nuclear matter.

As has been shown by Khalatnikov,[3] an arbitrary one-dimensional motion of a medium in the extreme relativistic case is described by a potential χ which satisfies the equation

$$3\frac{\partial^2\chi}{\partial\eta^2}-\frac{\partial^2\chi}{\partial y^2}-2\frac{\partial\chi}{\partial y}=0, \qquad (1)$$

where $y = \ln(T/T_0)$, $\eta = \tanh^{-1} v$, T and v are the temperature and speed of the medium, and T_0 is the initial temperature.* The coordinate x and the time t are expressed in terms of the potential χ in the following way:

$$x=e^{-y}\left(\frac{\partial\chi}{\partial y}\sinh\eta-\frac{\partial\chi}{\partial\eta}\coth\eta\right), \qquad (2)$$

$$t=e^{-y}\left(\frac{\partial\chi}{\partial y}\coth\eta-\frac{\partial\chi}{\partial\eta}\sinh\eta\right). \qquad (3)$$

Thus if the function $\chi(\eta, y)$ has been found, Eqs. (2) and (3) can be used to find the quantities $\eta(x, t)$ and $y(x, t)$, provided that

$$\partial(\eta, y)/\partial(x, t)\neq 0.$$

In the present paper we confine ourselves to the case in which the ratio n of the length of the tube to the dimensions of a nucleon, which is approximately the same as the number of nucleons in the tube, does not exceed 3.7. In this case (see reference 4), after passage of the shock waves beyond the boundaries of the system, a flow of the matter into empty space begins, in the form of running waves of rarefaction. This motion is described by the formulas:[3]

$$\eta=\pm\sqrt{3}\,y, \qquad (4)$$

$$(x-x_1)/(t-t_1)=(v\pm c)/(1\pm vc). \qquad (5)$$

Here $c = 3^{-1/2}$ is the speed of sound; the sign + refers to the wave propagated in the positive x direction, and the sign − refers to the wave propagated in the opposite direction. The instant t_1 is that of the start of the efflux of the matter, and x_1 is the coordinate of the edge of the system at this instant. We choose the origin of our system in such a way that for the wave travelling to the right we have $x_1 = t_1 = 0$. We denote the corresponding quantities for the wave going to the left by l and t_0. As can easily be shown, $l = \frac{1}{4}(n+1)d$, $t_0 = \frac{3}{4}(n-1)d$, where d is the thickness of the relativistically contracted nucleon.

*The speed of light is set equal to unity.

According to Eq. (4) $\partial(\eta, y)/\partial(x, t) = 0$ in the region of the running wave, so that the running wave is just that particular solution that is not contained in the general solution of Eq. (1).

After the encounter of the two running waves a region comes into existence in which the variables η and y are independent, and which is accordingly described by Eq. (1). At the boundaries of this region and the running waves the conditions (4) and (5) must be fulfilled; using Eqs. (2) and (3), we get from these the following boundary conditions for χ:

$$\chi = 0 \text{ for } \eta = \sqrt{3}y, \tag{6}$$

$$\chi = -le^y \sinh \eta + t_0(e^y \cosh \eta - 1) \quad \text{for } \eta = -\sqrt{3}y. \tag{7}$$

The solution of (1) with the boundary conditions (6) and (7) can be obtained in the most convenient form by the following approach. We shall seek the solution in the regions $\eta > 0$ and $\eta < 0$ separately, and require that the two solutions agree for $\eta = 0$. Let us first find the solution for $\eta < 0$. To do this we introduce the new variables

$$\alpha = -\eta, \quad \beta = \eta - \sqrt{3}y, \quad u = \chi e^y.$$

After this Eq. (1) and the boundary condition (6) take the forms:

$$u_{\alpha\alpha} - 2u_{\alpha\beta} + u/3 = 0, \tag{8}$$

$$u = 0 \quad \text{for} \quad \beta = 0. \tag{9}$$

We carry out a Laplace transformation on the variable β and seek the solution in the form $U = a(p)e^{k\alpha}$. The quantity k is found from the conditions (8) and (9):

$$k = p - \sqrt{p^2 - 1/3},$$

where we must take the minus sign for the square root. Making the inverse Laplace transformation, we get the solution in the form

$$u = \frac{1}{2\pi i}\int \exp\{-\sqrt{3}yp + \eta\sqrt{p^2 - 1/3}\}\, a(p)\, dp. \tag{10}$$

Let us now consider the region $\eta > 0$. After the replacements

$$\eta' = -\eta,$$

$$u' = u + l \sinh \eta e^{2y} - t_0 e^y(\cosh \eta e^y - 1),$$

the problem reduces to the previous one, so that for $\eta > 0$

$$u_1 = \frac{1}{2\pi i}\int \exp\{-\sqrt{3}yp - \eta\sqrt{p^2 - 1/3}\}\, a_1(p)\, dp - l \sinh \eta e^{2y} + t_0 e^y(\cosh \eta e^y - 1). \tag{11}$$

The unknown functions $a(p)$ and $a_1(p)$ are found from the conditions that $u = u_1$ and $\partial u/\partial\eta = \partial u_1/\partial\eta$ for $\eta = 0$. Simple calulations give:

$$a(p) = -\frac{l}{2}\frac{1}{(p + 2/\sqrt{3})\sqrt{p^2 - 1/3}} + \frac{t_0}{2}\left(\frac{1}{p + 2/\sqrt{3}} - \frac{1}{p + 1/\sqrt{3}}\right), \tag{12}$$

$$a_1(p) = -\frac{l}{2}\frac{1}{(p + 2/\sqrt{3})\sqrt{p^2 - 1/3}} - \frac{t_0}{2}\left(\frac{1}{p + 2/\sqrt{3}} - \frac{1}{p + 1/\sqrt{3}}\right). \tag{13}$$

After this, by means of Eqs. (10) and (12) or (11) and (13) we get the solution:*

$$\chi = \frac{\sqrt{3}}{2}\left(l - t_0\frac{\partial}{\partial\eta}\right)e^y \int_{\eta/\sqrt{3}}^{y} e^{-2y'} I_0\left(\sqrt{y'^2 - \eta^2/3}\right) dy' + \frac{\sqrt{3}}{2} t_0 \frac{\partial}{\partial\eta} \int_{\eta/\sqrt{3}}^{y} e^{-y'} I_0\left(\sqrt{y'^2 - \eta^2/3}\right) dy'. \tag{14}$$

For the collision of identical particles it is obvious that $t_0 = 0$, and the solution found here is the same as that obtained by Khalatnikov.

By means of Eqs. (1) to (3) we can easily obtain the expressions for the derivatives of t and x with respect to y and η:

$$\frac{\partial t}{\partial y} = e^{-y}\left(\frac{\partial\psi}{\partial y}\cosh\eta - \frac{\partial\psi}{\partial\eta}\sinh\eta\right), \quad \frac{\partial t}{\partial\eta} = e^{-y}\left(\frac{\partial\psi}{\partial\eta}\cosh\eta - \frac{1}{3}\frac{\partial\psi}{\partial y}\sinh\eta\right),$$

$$\frac{\partial x}{\partial y} = e^{-y}\left(\frac{\partial\psi}{\partial y}\sinh\eta - \frac{\partial\psi}{\partial\eta}\cosh\eta\right), \quad \frac{\partial x}{\partial\eta} = e^{-y}\left(\frac{\partial\psi}{\partial\eta}\sinh\eta - \frac{1}{3}\frac{\partial\psi}{\partial y}\cosh\eta\right). \tag{15}$$

Here $\psi = \partial\chi/\partial y - \chi$. The calculation of $\partial\psi/\partial y$ and $\partial\psi/\partial\eta$ gives:

$$\frac{\partial\psi}{\partial y} = \frac{\sqrt{3}}{2}e^{-y}\left(l\frac{\partial}{\partial y} - t_0\frac{\partial}{\partial\eta} - l\right)I_0(z), \tag{16}$$

$$\frac{\partial\psi}{\partial\eta} = \frac{\sqrt{3}}{2}e^{-y}\left(l\frac{\partial}{\partial\eta} - \frac{1}{3}t_0\frac{\partial}{\partial y} - \frac{1}{3}t_0\right)I_0(z); \tag{17}$$

$$z = \sqrt{y^2 - \eta^2/3}.$$

For an overwhelming majority of the particles z

*The solution of the one-dimensional problem for $n \leq 3.7$ has also been obtained in somewhat different form in reference 5. It is not difficult to reduce the solution there obtained to the form (14), which is convenient for our purposes.

is large at the end of the one-dimensional state. It we use the first two terms of the asymptotic expansion of the Bessel function, Eqs. (16) and (17) can be written

$$\frac{\partial\psi}{\partial y} = \frac{\sqrt{3}}{2} l e^{-y}\left(\frac{\partial}{\partial y} - 1\right) I_0(z') - \frac{t_0^2}{8\sqrt{3}l} e^{-y} \frac{I_1(z')}{z'}, \quad (18)$$

$$\frac{\partial\psi}{\partial\eta} = \frac{\sqrt{3}}{2} l e^{-y} \frac{\partial}{\partial\eta'} I_0(z'), \quad (19)$$

where we have introduced the new variables

$$\eta' = \eta + t_0/2l, \quad z' = \sqrt{y^2 - \eta'^2/3}.$$

For our proof we must expand the right members of Eqs. (18) and (19) in Taylor's series in the differences $\eta' - \eta = \eta_0$.

The choice of the variable η' means that we go over into a new coordinate system moving relative to the original system with the speed $V = \tanh\eta_0$. In fact,

$$v' = \tanh\eta' = (v + V)/(1 + vV).$$

To sufficient accuracy for practical purposes this new coordinate system coincides with the center-of-mass system (the relative velocity of these systems is given by

$$\tanh\left[\frac{3}{2}\frac{n-1}{n+1} - \tanh^{-1}\frac{n-1}{n+1}\right] \leqslant 0.2,$$

since we are considering tubes that are not very long, $n \leq 3.7$).

It can be seen from Eqs. (18) and (19) that $\partial\psi/\partial y$ is an even function of η' and $\partial\psi/\partial\eta'$ is an odd function. From this and Eq. (15) one easily obtains the relations $x'(-\eta', y) = -x'(\eta', y)$, $t'(-\eta', y) = t'(\eta', y)$, from which our assertion about the symmetry of the emission of the particles in this system follows. The deviation from this symmetry in the region described by the solution (14) is of the same order asymptotically as the third term in the asymptotic expansion of the Bessel function, i.e., extremely small at the end of the one-dimensional stage. The running waves are practically entirely responsible for the disturbance of the symmetry. This disturbance is also small, since in the region of the running waves there are very few particles (of the order of one[5,6]). It can be seen from Eqs. (18) and (19) that if we confine ourselves to the first term of the asymptotic expansion of the Bessel function the one-dimensional stage will be described by the same formulas as in the case of the collision of identical particles (i.e., when $t_0 = 0$). Therefore to this accuracy the angular and energy distributions of the particles will be the same as in the collision of identical particles.

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Translated by W. H. Furry
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