DEPOLARIZATION OF μ^--MESONS IN HYDROGEN, DEUTERIUM, AND TRITIUM

V. B. BELIAEV and B. N. ZAKHAR' EV

Joint Institute for Nuclear Research

Submitted to JETP editor May 19, 1958; resubmitted June 9, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 996-1000 (October, 1958).

A number of effects that depolarize μ^--mesons in hydrogen, deuterium and tritium, are considered. Complete depolarization in hydrogen and tritium occurs before the mesic atom is slowed down to thermal energies. Depolarization probabilities are given for the case of capture of the mesic atom by the K-orbit and in the case of formation of mesic molecules. A correction $\sim \mu/M$ was included in the calculations to allow for the motion of the nuclei. An approximate allowance is made for the difference in the hyperfine structure levels.

THE capture of polarized μ^- mesons is of interest in the investigation of the nature of the interaction between μ^- mesons and protons. However, the situation is made complicated by the fact that the mesons may be depolarized by the medium. In this paper this question is investigated for the cases of hydrogen, deuterium, and tritium.*

Partial depolarization of the meson occurs when it is captured by the K-orbit of the hydrogen mesic atom. A polarized meson with $s_z = 1/2$ can form together with a proton a system having a total spin either $F = 1$, or $F = 0$. The component F_z takes on the values 1 and 0 with equal probability. (In an unpolarized medium a polarized meson has equal probability of encountering a proton with its spin parallel or antiparallel to the z axis.) In the case $F_z = 0$ the meson "forgets" the initial direction of its spin (it becomes depolarized). However, the state $F_z = 1$ "remembers" the direction of the μ^- spin (depolarization does not occur). Thus, the probability of depolarization during the formation of mesic hydrogen is equal to $1/2$.

Similar arguments in the case of tritium and deuterium lead respectively to the values $1/2$ and $2/3$.

The main effect that depolarizes μ^--mesons which enter liquid hydrogen is the so-called "charge exchange" of the mesic atoms — the transfer of μ^- from the orbit of one atom to the orbit of another atom (this was pointed out to us by Zel'dovich). The probability of such processes

$$(p_1)_\mu + p_2 \to p_1 + (p_2)_\mu; \quad (d_1)_\mu + d_2 \to d_1 + (d_2)_\mu; \quad (t_1)_\mu + t_2 \to (t_2)_\mu + t_1 \tag{1}$$

can be computed by the method proposed by Zel'dovich and Sakharov.[2]

A system consisting of two identical nuclei and a meson can exist either in a symmetric Σ_g or an antisymmetric Σ_u state* with respect to a reversal of sign of the meson coordinates (inversion). By utilizing the fact that there are no transitions between the states Σ_u and Σ_g, we regard any state of our system as a superposition of these two noncoherent waves (Σ_u and Σ_g form a complete set of eigenfunctions of the interaction Hamiltonian).

Consider a mesic atom $(p_1)_\mu$ incident on a proton p_2. If we denote the state of the three particles in the case when μ^- is in the orbit of the first (second) proton by Φ_{p_1} (Φ_{p_2}), then

$$\Sigma_u = (\Phi_{p1} - \Phi_{p2}) / \sqrt{2}, \quad \Sigma_g = (\Phi_{p1} + \Phi_{p2}) / \sqrt{2}. \tag{2}$$

For large distances between $(p_1)_\mu$ and p_2 we can take the wave function Φ_{p_1} in the form of a converging wave:

$$\Psi_{\text{init}} = \Phi_{p1} = (e^{-ipr}/r)(\Sigma_u + \Sigma_g)/\sqrt{2}. \tag{3}$$

After scattering, the waves Σ_u and Σ_g will be shifted in phase by α_u and α_g respectively:

$$\Psi_{\text{final}} = (e^{ipr+i\alpha_u}/r)\Sigma_u/\sqrt{2} + (e^{ipr+i\alpha_g}/r)\Sigma_g/\sqrt{2} \tag{4}$$

or alternatively:

$$\Psi_{\text{final}} = (e^{ipr}/2r)[\Phi_{p1}(e^{i\alpha_u} + e^{i\alpha_g}) + \Phi_{p2}(e^{i\alpha_g} - e^{i\alpha_u})]. \tag{5}$$

The square of the absolute value of the coefficient of Φ_{p_2} determines the probability of charge exchange:

$$W = \tfrac{1}{2}[1 - \cos(\alpha_u - \alpha_g)]. \tag{6}$$

We now determine α_u and α_g. They are equal

*An estimate of the charge-exchange cross section at thermal energies of the mesic atom has been made by Gershtein[1] for the case $(p_1)_\mu + p_2 \to p_1 + (p_2)_\mu$.

*We have in mind the ground states of the system.

TABLE I. Phases for the scattering of mesic atoms by nuclei

Energy E (ev), c.m.s.	In state Σ_u			In state Σ_g		
	Medium					
	Hydrogen	Deuterium	Tritium	Hydrogen	Deuterium	Tritium
100	−1.890	−2.936	−3.740	2.389	3.182	3.662
10	−0.646	−1.042	−1.360	3.173	5.104	5.749
1	−0.211	−0.343	−0.450	3.192	5.888	6.207
0.5	−0.145	−0.238	−0.313	3.186	6.011	6.248
0.1	−0.065	−0.106	−0.140	3.160	6.160	6.267
0.05	−0.047	−0.077	−0.101	3.157	6.195	6.276
0.01	−0.021	−0.034	−0.045	3.147	6.243	6.278

TABLE II. Cross sections for the scattering of mesic atoms by nuclei in 10^{-19} cm^2

Energy E(ev), c.m.s.	In state Σ_u			In state Σ_g		
	Medium					
	Hydrogen	Deuterium	Tritium	Hydrogen	Deuterium	Tritium
100	0.466		0.0545	0.242	0.421	
10	1.874	1.925	1.645		2.208	0,447
1	2.268	2.874	3.26	0.131	3.831	0.0998
0.5	2.130	2.830	3.225	0.1834	3.686	0.0421
0.1	2.178	2.890	3.35	0.156	3.900	0.0423
0.05	2.250	3.020	3.46	0.242	3.960	0.0176
0.01	2.280	2.990	3.48	0.151	4.160	0.0463

to twice the scattering phases δ_u and δ_g. Indeed, the asymptotic expression for the s component of the plane incident plus the spherical diverging waves $e^{ikz} + f(\theta)e^{ikr}/r$ will be given by:[3]

$$\frac{e^{i\delta}\sin(kr+\delta)}{kr} = \frac{1}{2ikr}\left[e^{2i\delta+ikr} - e^{-ikr}\right].$$

To find the scattering phases it is more convenient to solve, instead of the Schrödinger equation, the equivalent nonlinear first-order equation, obtained from the Schrödinger equation

$$u'' + (k^2 - V)u = 0 \qquad (7)$$

by the substitution $u = \exp\left(\int \varphi\, dr\right)$, where[6] $\varphi = k \cot[kr + y(r)]$.

We thus have

$$y'(r) = -\alpha V(r)\sin^2(\beta r + y(r)) \qquad (8)$$

together with the boundary condition $y(0) = 0$, where $\alpha = 2me^2/k\hbar^2$, $\beta = ka_0$, m is the reduced mass of the two heavy particles, a_0 is the Bohr radius of the mesic atom, $y(r)$ is a function that approaches the scattering phase $\delta(k)$ as $r \to \infty$, $V(r)$ is the effective interaction potential between the nuclei. We shall denote the latter by $V_1(r)$ when the three particles are in the Σ_u state, and by $V_2(r)$ when they are in the Σ_g state. Within the region of attraction, the potential $V_2(r)$ was chosen in the form of Morse's function with parameters taken[4,5] in agreement with spectroscopic data for the molecular ion H_2^+.

$V_1(r)$ is a repulsive potential and $V_2(r)$ is an attractive potential. Both potentials satisfy the conditions

$$\lim_{r\to 0} r^2 V(r) = 0; \quad \lim_{r\to\infty} rV(r) = 0,$$

imposed on $V(r)$ in Drukarev's paper.[6] Since V_1 and V_2 become infinite at $r = 0$ the boundary condition was taken in somewhat altered form: $y(\epsilon) = 0$, where ϵ is a very small quantity. The phases are stable with respect to small changes in ϵ.

Since the ratio μ/M of the meson mass to the nuclear mass is not negligible, appropriate corrections must be introduced into the potentials given above. The values of such corrections for the molecular ion H_2^+ are given by Dalgarno.[7] By means of a simple recalculation we obtain them for the case $(H_2)_\mu^+$ of interest to us. The results of numerical calculations carried out by S. Lomnev are given in Table I.

The values of the cross sections for processes (1) are given in Table II.

The scattering cross section in the case of deuterium in the state Σ_g is considerably larger than the cross section in hydrogen and in tritium. This is explained by the fact that in the mesic molecular ion $(d_2^+)_\mu$ there is a level close to 0, which produces a resonance. At the same time in the case

TABLE III

Energy E(ev), c.m.s.	Probability of charge exchange			Charge exchange cross sections, 10^{-19} cm²		
	Medium					
	Hydrogen	Deuterium	Tritium	Hydrogen	Deuterium	Tritium
100	0.824	0.027	0.809	0.426	0.007	0.139
10	0.393	0.019	0.541	2.032	0.049	0.932
1	0.067	0.003	0.133	3.464	0.0776	2.292
0.5	0.035	0.0012	0.075	3.570	0.061	2.550
0.1	0.007	0.0003	0.015	3.619	0.067	2.585
0.05	0.004	0.00013	0.0088	3.978	0.066	2.992
0.01	0.0005	0.00003	0,0016	2.585	0.0776	2.757

of the repulsive potential we obtain a smooth dependence of the cross section on the nuclear masses, as expected in view of the absence of bound states in the case of Σ_u.

The probabilities and the cross sections for charge exchange are given in Table III. The probabilities of charge exchange W are given in Table III in dimensionless units; the charge exchange cross sections are $\sigma = 4\pi k^{-2} W$.

In the case of deuterium the probability of losing the polarization as a result of one charge exchange is equal to $^2/_3$, just as in the case of capture into the K orbit of the mesic atom.

It should be noted that during the collision time $\tau < 10^{-16}$ sec at energies $E > 0.01$ ev the interaction between the spins does not have time to flip them over, and therefore the "forgetting" occurs in the interval between transfers. The energy of interaction of the μ^--meson spin with the nuclear spin is given by $E \sim 0.2$ ev. The characteristic time of the spin-spin interaction (the time for depolarization in states with $F_z = 0$) is of order $\hbar/E \sim 10^{-15}$ sec. The time which the μ^- spends in the orbit of a mesic atom between successive charge exchanges is greater than 10^{-11} sec, as may be seen from Table III. From this it follows that in the state $F = 1$, $F_z = 0$ the μ^- has time to become completely depolarized.

The phenomenon of charge exchange is possible only if the nuclear charge is $Z = 1$, otherwise the mesic atom will not be neutral and because of the Coulomb barrier the nuclei will be unable to approach each other to a distance $\sim 10^{-11}$ cm at which a transfer of the μ^- meson can occur. For this reason one of the effects mentioned by Day and Morrison[8] will not occur.

So far we have not taken into account the difference between the hyperfine structure levels of the mesic atom. At energies > 0.2 ev its effect on the scattering process can be neglected. At lower energies one should introduce into the charge exchange cross section a correction factor v_f/v_i where v_f is the velocity after the collision, while v_i is the velocity before the collision.[2,3] An exact calculation to take into account the energy of the hyperfine structure has been carried out by Gershtein.[1]

When the energy of the mesic atom is comparable with the binding energy of the hydrogen molecule, we must take into account the fact that the mesic atom is scattered not by free protons but by protons bound in the H_2 molecule. Owing to the very small size of the mesic atom in comparison with the hydrogen molecule, this is carried out in a manner similar to that in which binding is taken into account in the scattering of slow neutrons in hydrogen.[9,10] For the case of mesic atoms of thermal energy such a calculation has been made by Gershtein.[11]

After slowing down, the μ^- are captured into the orbit of one of the nuclei. The binding energy released goes into electron conversion (Auger effect) and into the breaking up of the hydrogen molecule. The mesic atom formed in this process receives a recoil energy on the order of several tens of electron volts.

In each collision of the mesic atom with nuclei it will lose on the average half its energy until it slows down to thermal velocities. By making use of such an outline of the various processes it is not difficult to follow the process of depolarization of the μ^- meson. From Table III it may be seen that in hydrogen and in tritium the complete depolarization of the μ^- meson due to charge exchange occurs before the mesic atom is slowed down to energies of several electron volts. In deuterium depolarization due to charge exchange practically does not occur at energies above thermal. One may hope that because of the level close to zero in the mesic molecular ion $(d_2^+)_\mu$ the latter will be formed prior to the complete depolarization of the muon. After the $(d_2^+)_\mu$ ion has been formed the process of "forgetting" the spin orientation will cease.

If the deuterium contains a small number of tritium nuclei the meson will soon transfer to one of them, and since such a process is irreversible further charge exchange processes will not occur. Such an effect can also prevent the complete loss of polarization.* It would be very interesting to check this experimentally. It should also be noted that if hydrogen nuclei could be polarized, the depolarization of the μ^- due to charge exchange and to the formation of molecules could be avoided. However, at the present time this cannot be accomplished.

Shmushkevich has called attention to the depolarization effect due to the spin-orbit coupling during the successive transitions of the meson between the levels of the mesic atom. Results of the experimental work carried out by Ignatenko et al.[12] indicate that the depolarization in such a case does not exceed 80%.

We take pleasure in expressing our gratitude to Academician Ia. B. Zel'dovich for directing this work, to S. S. Gershtein for very useful discussions, and also to Prof. Ia. A. Smorodinskii and A. E. Ignatenko for their interest in this work.

The authors are indebted to S. Lomnev for carrying out the numerical calculations on an electronic computer.

*A similar effect in hydrogen in the presence of a small admixture of deuterium does not affect the depolarization, since $W_{depol} > W_{charge\ exch}$ (pd).

[1] S. S. Gershtein, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 463 (1958), Soviet Phys. JETP **7**, 318 (1958).

[2] Ia. B. Zel'dovich and A. D. Sakharov, J. Exptl. Theoret. Phys. (U.S.S.R.), **32**, 947 (1957), Soviet Phys. JETP **5**, 775 (1957).

[3] L. D. Landau and E. M. Lifshitz, Квантовая механика (Quantum Mechanics) Moscow, 1948 [Engl. Transl. Pergamon, 1958].

[4] P. Gombas, (The Many-Particle Problem in Quantum Mechanics) [Russian Transl., IIL, 1953].

[5] P. M. Morse, Phys. Rev. **34**, 57 (1929).

[6] G. F. Drukarev, J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 247 (1949).

[7] A. Dalgarno, Proc. Roy. Soc. **A237**, 383 (1956).

[8] T. B. Day and P. Morrison, Phys. Rev. **107**, 912 (1957).

[9] H. Bethe, Nuclear Physics (Russ. Transl.) part 2, p. 112, 1948.

[10] M. Hammermesh and J. Schwinger, Phys. Rev. **69**, 145 (1946).

[11] S. S. Gershtein, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 993 (1958), Soviet Phys. JETP **7**, 685 (1958).

[12] Ignatenko, Egorov, Khalupa, and Chultem, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 894 (1958), Soviet Phys. JETP **8**, this issue, p. 621.

Translated by G. Volkoff
208