the corresponding values computed from the data of Ivanova and P'ianov[4] on the angular and energy distributions of the cascade nucleons. These data were obtained in the calculation of the cascade for uranium at the proton energy of 660 Mev with the Monte Carlo method. In the calculation the creation of mesons was neglected.

The table shows qualitative agreement between the computed and experimental values. It should be kept in mind that the experimental values refer only to fission experiments.

From the experimental value for the parallel component of the momentum of the nucleus we can determine the excitation energy of the nucleus under the assumption, as in references 5 and 6, that the momenta of the cascade nucleons are transferred by way of one fast cascade particle in the direction of the incident proton beam. With our data, this gives $\overline{E}_f = 240$ Mev for uranium with $\overline{E}_p = 660$ Mev. This surpasses the value $\sim 160$ Mev of reference 6. Under the assumption that two fast cascade nucleons are emitted in the direction of the proton beam and perpendicular to it the measured values for the parallel and perpendicular components of the nucleus momentum yield for the excitation energy the value $\overline{E}_f =$ 145 Mev. It is seen that the presence of the perpendicular momentum component leads to a significantly lower value for $E_f$. It is obvious, however, that the second variant represents an extreme approximation just as the first variant does. It must also be pointed out that the perpendicular component of the momentum of the nucleus should be considered in the investigation of the angular distributions. The irregularity in the angular distribution of the fission fragments of bismuth in the 60 to 90° region in the laboratory system, found by Wolke and Gutman,[7] may possibly be explained by this fact, as these authors also noted.

In conclusion the author expresses his gratitude to Prof. N. A. Perfilov for a number of critical remarks, and to N. S. Ivanova and I. I. P'ianov for making available the computational data on the angular and energy distributions of the cascade nucleons of uranium.

[1] R. Serber, Phys. Rev. **72**, 1114 (1947).
[2] N. M. Steiner and J. A. Jungerman, Phys. Rev. **101**, 807 (1956).
[3] N. T. Porile and N. Sugarman, Phys. Rev. **107**, 1410 (1957).
[4] N. S. Ivanova and I. I. P'ianov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 416 (1956); Soviet Phys. JETP **4**, 367 (1957).
[5] V. I. Ostroumov, Dokl. Akad. Nauk SSSR **103**, 409 (1955).
[6] V. P. Shamov, **Физика деления атомных ядер. Приложение №1 к журн. Атомн. энергия за 1957 г.** (The Physics of Nuclear Fission. Supplement No. 1 to the "Atomic Energy," 1957), Atomizdat, M., 1957, p. 129.
[7] R. L. Wolke and J. R. Gutman, Phys. Rev. **107**, 850 (1957).

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## *ON THE MULTIPLE INTERACTION IN QUANTUM FIELD THEORY*

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THE problem of constructing the nucleon potential with the inclusion of multiple interactions by summing over the corresponding terms in the perturbation expansion was considered in reference 1 in the framework of pair mesodynamics.

In the present paper, an expression for the effective potential for the multiple interaction of two particles is found in closed form. The discussion is based on the usual methods of Feynman and Dyson under the assumption $||E| - m| \ll m$. In the expansion for the energy of the free particle (e.g., the electron)

$$|E_n| = m + p_n^2/2m + \dots \tag{1}$$

(here, as in the following, $\hbar = c = 1$) we can therefore restrict ourselves to the first term $|E_n| \approx m$. It is easily seen[1] that in this case the Green's function for the electron has the form

$$\begin{aligned} S^F(2,1) &\approx \frac{1+\beta}{2}\delta(r_1 - r_2)\, e^{-im(t_2 - t_1)}, \quad t_2 > t_1 \\ S^F(2,1) &\approx \frac{1-\beta}{2}\delta(r_1 - r_2)\, e^{im(t_2 - t_1)}, \quad t_2 < t_1. \end{aligned} \tag{2}$$

We note that the approximation (2) does not imply a transition to a theory with fixed sources ($m \to \infty$), since the Green's function $S^F$ would then depend only on the time.

Using (2) and carrying out all calculations in the coordinate representation, we obtain for the case of a process of order 2n, where the interacting particles interchange n virtual quanta,

$$S^{(2n)} = e^2\left(-\frac{e^2}{m^2}\right)^{n-1}\iint d^3r_1 d^3r_2\,|\varphi(r_1, r_2)|^2 \times \iint_{-\infty}^{\infty} dt_1 dt_2\,[D^F(x_1 - x_2)]^n. \quad (3)$$

Summing over n and introducing the effective potential,[2]

$$S_{i\to f} = \sum_{n=1}^{\infty} S^{(2n)} = -i\iint d^3r_1 d^3r_2 \varphi_f^*(r_1, r_2) \quad (4)$$

$$\times U(|r_1 - r_2|)\,\varphi_i(r_1, r_2)\lim_{t\to\infty}\int_{-\infty}^{t} e^{i\nu t_1}dt_1,$$

where $\nu = \epsilon_f - \epsilon_i = 0$. We find, finally,

$$U(r) = ie^2\int_{-\infty}^{\infty} D_F'(r, t)\,dt, \quad D_F' = \frac{D^F}{1 + r_0\lambda D^F}, \quad (5)$$

where $r_0$ is the classical electron radius, $\lambda$ is the Compton wavelength (in our notation $S^F$ and $D^F$ are equal to $\frac{1}{2}S^F$ and $\frac{1}{2}D^F$ in reference 2, respectively).

Such simple results were obtained only by excluding the poles of type $[m^2 - (\omega_2 + \ldots + \omega_k)^2]^{-1}$ for processes involving n virtual quanta, keeping in mind the character of the approximation, i.e., $|\omega_2 + \ldots + \omega_k| < m$. It can be shown that these restrictions have, in any case, no bearing on the finiteness of the potential at the origin, which follows from (5):

$$U(r) = e^2/\sqrt{r^2 + R^2}, \quad R = \sqrt{r_0\lambda/\pi} \quad (6)$$

The form of expression (6) agrees with the potential proposed in reference 3. We have

$$U(r)|_{r=0} = m\sqrt{\pi\alpha}, \quad \alpha = 1/137.$$

In the calculations for the pseudoscalar mesodynamics with pseudoscalar coupling the order of the time integrations has to be changed; but the final result agrees with Eqs. (3) to (5): the nucleon potential depends on the distance like $\int_{-\infty}^{\infty}\Delta_F'(r, t)\,dt$, where

$$\Delta_F' = \Delta^F/[1 + (G/M)^2\Delta^F],$$

M is the mass of the nucleon, and G is the coupling constant.

[1] A. Klein, Phys. Rev. 91, 740 (1953); 92, 1017 (1953).

[2] A. I. Akhiezer and V. B. Berestetskii, Квантовая электродинамика (Quantum Electrodynamics), M. Gostekhizdat, 1953. [Engl. Transl. Publ. by U. S. Dept. of Commerce].

[3] D. D. Ivanenko and V. Lebedev, J. Exptl. Theoret. Phys. (U.S.S.R.) 22, 639 (1952).

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## *LEVEL WIDTHS OF π-MESONIC ATOMS*

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If the interaction between slow negative pions and nuclei is considered as a perturbation to the Coulomb potential of a point source (see references 1 and 2), it is possible to estimate the potential of the meson-nucleus interaction. Denoting by v(r) the deviation of the interaction potential from its Coulomb value, we obtain, to first order of perturbation theory, the total shift $\Delta E_{nl}$ of the mesonic-atom level

$$\Delta E_{nl} = \int_0^{\infty} |\Psi_{nl}(r)|^2 v(r) r^2 dr, \quad (1)$$

and the phase $\tau_{kl}$ of scattering of a slow pion by a nucleus denoted by $I_l$ in reference 1), becomes

$$\tau_{kl} = -\frac{\pi\mu}{\hbar^2}k\int_0^{\infty} |\varphi_{kl}(r)|^2 v(r) r^2 dr. \quad (2)$$

Here k is the wave number of the meson, and v(r) differs from zero only when $r \le r_Z$ [$r_Z = (\hbar/\mu c)A^{1/3}$ is the nuclear radius] and depends little on the energy in the low-energy region (~1 Mev); $\Psi_{nl}(r)$ is the wave function of the bound state in the Coulomb field, which has the following form as $r/R_{nZ} \to 0$ $\left(2R_{nZ} = \frac{\hbar^2}{\mu e^2}\frac{n}{Z}\right)$

$$\Psi_{nl}(r) = \left[\frac{(n+l)!}{2n(n-l-1)!}\right]^{1/2}\frac{1}{(2l+1)!}\left(\frac{1}{R_{nZ}}\right)^{3/2}\left(\frac{r}{R_{nZ}}\right)^l \quad (3)$$

and $\varphi_{kl}(r)$ is the regular wave function of the continuous spectrum in the Coulomb attraction field, which becomes, as $(kr) \to 0$

$$\varphi_{kl}(r) = \sqrt{\frac{2}{\pi}}\,C_l\frac{(kr)^l}{(2l+1)!!}, \quad (4)$$

$$C_l^2 = \frac{2\pi|\alpha|}{1 - \exp\{-2\pi|\alpha|\}}\prod_{s=1}^{l}\left(1 + \frac{\alpha^2}{s^2}\right); \quad \alpha = -\frac{Ze^2}{\hbar c}\sqrt{\frac{\mu c^2}{2E_\pi}}.$$