acting particles interchange n virtual quanta,

$$S^{(2n)} = e^2\left(-\frac{e^2}{m^2}\right)^{n-1}\iint d^3r_1 d^3r_2\,|\varphi(r_1, r_2)|^2 \times \iint_{-\infty}^{\infty} dt_1 dt_2\,[D^F(x_1 - x_2)]^n. \tag{3}$$

Summing over n and introducing the effective potential,[2]

$$S_{i\to f} = \sum_{n=1}^{\infty} S^{(2n)} = -i\iint d^3r_1 d^3r_2 \varphi_f^*(r_1, r_2) \times U(|r_1 - r_2|)\,\varphi_i(r_1, r_2)\lim_{t\to\infty}\int_{-\infty}^{t} e^{i\nu t_1}dt_1, \tag{4}$$

where $\nu = \epsilon_f - \epsilon_i = 0$. We find, finally,

$$U(r) = ie^2\int_{-\infty}^{\infty} D'_F(r, t)\,dt, \quad D'_F = \frac{D^F}{1 + r_0\lambda D^F}, \tag{5}$$

where r_0 is the classical electron radius, λ is the Compton wavelength (in our notation S^F and D^F are equal to $\frac{1}{2}S^F$ and $\frac{1}{2}D^F$ in reference 2, respectively).

Such simple results were obtained only by excluding the poles of type $[m^2 - (\omega_2 + \ldots + \omega_k)^2]^{-1}$ for processes involving n virtual quanta, keeping in mind the character of the approximation, i.e., $|\omega_2 + \ldots + \omega_k| < m$. It can be shown that these restrictions have, in any case, no bearing on the finiteness of the potential at the origin, which follows from (5):

$$U(r) = e^2/\sqrt{r^2 + R^2}, \quad R = \sqrt{r_0\lambda/\pi} \tag{6}$$

The form of expression (6) agrees with the potential proposed in reference 3. We have

$$U(r)|_{r=0} = m\sqrt{\pi\alpha}, \quad \alpha = 1/137.$$

In the calculations for the pseudoscalar mesodynamics with pseudoscalar coupling the order of the time integrations has to be changed; but the final result agrees with Eqs. (3) to (5): the nucleon potential depends on the distance like $\int_{-\infty}^{\infty}\Delta'_F(r, t)\,dt$, where

$$\Delta'_F = \Delta^F/[1 + (G/M)^2\Delta^F],$$

M is the mass of the nucleon, and G is the coupling constant.

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LEVEL WIDTHS OF π-MESONIC ATOMS

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If the interaction between slow negative pions and nuclei is considered as a perturbation to the Coulomb potential of a point source (see references 1 and 2), it is possible to estimate the potential of the meson-nucleus interaction. Denoting by v(r) the deviation of the interaction potential from its Coulomb value, we obtain, to first order of perturbation theory, the total shift ΔE_{nl} of the mesonic-atom level

$$\Delta E_{nl} = \int_0^{\infty} |\Psi_{nl}(r)|^2 v(r)\,r^2dr, \tag{1}$$

and the phase τ_{kl} of scattering of a slow pion by a nucleus denoted by I_l in reference 1), becomes

$$\tau_{kl} = -\frac{\pi\mu}{\hbar^2}k\int_0^{\infty}|\varphi_{kl}(r)|^2 v(r)\,r^2dr. \tag{2}$$

Here k is the wave number of the meson, and v(r) differs from zero only when $r \le r_Z$ [$r_Z = (\hbar/\mu c)A^{1/3}$ is the nuclear radius] and depends little on the energy in the low-energy region (~1 Mev); $\Psi_{nl}(r)$ is the wave function of the bound state in the Coulomb field, which has the following form as $r/R_{nZ} \to 0$ $\left(2R_{nZ} = \frac{\hbar^2}{\mu e^2}\frac{n}{Z}\right)$

$$\Psi_{nl}(r) = \left[\frac{(n+l)!}{2n(n-l-1)!}\right]^{1/2}\frac{1}{(2l+1)!}\left(\frac{1}{R_{nZ}}\right)^{3/2}\left(\frac{r}{R_{nZ}}\right)^l \tag{3}$$

and $\varphi_{kl}(r)$ is the regular wave function of the continuous spectrum in the Coulomb attraction field, which becomes, as $(kr) \to 0$

$$\varphi_{kl}(r) = \sqrt{\frac{2}{\pi}}C_l\frac{(kr)^l}{(2l+1)!!}, \tag{4}$$

$$C_l^2 = \frac{2\pi|\alpha|}{1 - \exp\{-2\pi|\alpha|\}}\prod_{s=1}^{l}\left(1 + \frac{\alpha^2}{s^2}\right); \quad \alpha = -\frac{Ze^2}{\hbar c}\sqrt{\frac{\mu c^2}{2E_\pi}}.$$

Considering in mesonic atoms only levels with values of n such that $r_Z/R_{nZ} \ll 1$, but to which perturbation theory is still applicable (n bounded from above), considering in the continuous spectrum such slow mesons for which $kr_Z \ll 1$, and using furthermore Eqs. (3) and (4), we obtain in lieu of (1) and (2)

$$\Delta E_{nl} = \frac{(n+l)!}{2n(n-l-1)!\,[(2l+1)!]^2}\left(\frac{1}{R_{nZ}}\right)^{2l+3}\int_0^{r_Z} r^{2(l+1)} v(r)\,dr, \quad (5)$$

$$\tau_{hl} = -\frac{2\mu}{\hbar^2}\frac{C_l^2}{[(2l+1)!!]^2}k^{2l+1}\int_0^{r_Z} r^{2(l+1)} v(r)\,dr. \quad (6)$$

Comparison of formulas (5) and (6) shows a simple connection between the level shift in the mesonic atom and the phase of scattering of a low-energy meson by a nucleus. This connection is a particular case of the more general relation obtained by Byers.[3]

In what follows we shall treat the imaginary part of the interaction potential Im v, which leads to a broadening of the levels (particularly by the 1S level) in mesonic atoms and to the absorption of pions when scattered by nuclei. We shall assume the imaginary part of the potential to be independent of r (the results of the calculations depend little on the shape of the potential and depend on r_Z, the average radius of the nuclear matter). Then, using (5) and using for Im ΔE_{1S} the value (0.45 ± 0.07) kev* measured experimentally by West and Bradley[4] for beryllium-9, we obtain for the S state Im v ≈ 1.5 Mev. An analogous estimate can be made using the two-nucleon model of absorption of pions by nuclei.[5] In this case (see, for example, reference 3), we obtain for the S state

$$\operatorname{Im}\tau = \Gamma\frac{Z}{6\pi}\left(\frac{\mu c}{\hbar}\right)10^{-27}C_0^2\,k\ \mathrm{cm}^2, \quad (7)$$

where the parameter Γ characterizes the probability of absorption of a pion by a pair of nucleons in the nucleus, referred to the absorption in deuterium. Comparing this expression with formula (6) at $l = 0$ and taking $\Gamma = \sim 5$ (see reference 6), we get Im v = 0.28Γ ≈ 1.4 Mev, which is in approximate agreement with the quantity obtained above. Thus the complex portion of the potential, responsible for the absorption of slow mesons, is small.

Knowing the width of the 1S level, we can estimate the lifetime of the negative pion at this level for nuclear capture, and compare it with the lifetime for decay. This is of interest because Fry and White[7] reported observed cases of decay of negative pions stopped in emulsion and gave an approximate value of 10^3 for the ratio of the decay probability to the nuclear-absorption probability (for the light elements C, N, and O). Taken together with an approximate pion lifetime of 10^{-8} seconds, this gives approximately 10^{-11} seconds for the lifetime for nuclear capture. This contradicts, as noted by Gol'danskii and Podgoretskii,[8] the strong interaction between the pion and the nuclear matter. Since the time that the meson hits the 1S orbit of the mesonic atom amounts to approximately 10^{-13} seconds, the observed decay should occur obviously on this orbit. Yet the lifetime for nuclear capture, obtained by West and Bradley from a measurement of the level width ($\Delta t\ \mathrm{Im}\ \Delta E \approx \hbar$) is approximately 10^{-18} seconds, which contradicts the results of Fry and White.

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*Reference 4 gives the total level width $\gamma = (8/3)$ Im $\Delta E_{1S} = (1.2 \pm 0.2)$ kev.

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EXCHANGE EFFECTS IN FERROMAGNETIC RESONANCE

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In the present paper we find a single dispersion law for transverse electromagnetic waves and for