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DOUBLE ELASTIC SCATTERING OF DEUTERONS IN A MAGNETIC FIELD

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A treatment is given of the double elastic scattering of a beam of deuterons in a magnetic field. An expression is obtained for the angular distribution of double elastic scattering. The special cases of longitudinal and transverse fields are considered.

1. Mendlowitz and Case[1] have given a theoretical treatment of the double elastic scattering of an electron beam for the case in which the beam is acted on between the scatterers by a magnetic field which is constant in time and homogeneous. The presence of an anomalous magnetic moment of the electron brings about an additional scattering, and measurements of this provide a possibility of experimental determination of the anomalous moment.

The present paper is devoted to the theoretical study of the double elastic scattering of a beam of particles with unit spin, in a magnetic field. It turns out that in this case (unlike that of particles with spin one-half) measurement of the double scattering in a magnetic field provides additional information about the scattering amplitude.

2. The amplitude for scattering of a deuteron by a nucleus with spin zero has the form[2]

$$F(\vartheta, \varphi) = A(\vartheta) + B(\vartheta)(\mathbf{S}\cdot\mathbf{n}) + C(\vartheta)(\mathbf{S}\cdot\mathbf{n})^2 + \frac{1}{2}D(\vartheta)\{(\mathbf{S}\cdot\mathbf{k}_0)(\mathbf{S}\cdot\mathbf{k}) + (\mathbf{S}\cdot\mathbf{k})(\mathbf{S}\cdot\mathbf{k}_0)\}, \quad (1)$$

where $\mathbf{k}_0$ and $\mathbf{k}$ are unit vectors along the momenta of the deuteron before and after the scattering; $\mathbf{n} = \mathbf{k}_0 \times \mathbf{k} / \sin\vartheta$ is the unit vector perpendicular to the plane of the scattering; $\mathbf{S}$ is the spin operator of the deuteron; and A, B, C, D are functions of the scattering angle and the energy of the deuteron.

The cross section for the scattering of a polarized beam is calculated by the formula

$$I(\vartheta, \varphi) = \mathrm{Sp}\{F(\vartheta, \varphi)\rho F^+(\vartheta, \varphi)\}, \quad (2)$$

where ρ is the spin density matrix of the deuteron beam,

$$\rho = \frac{1}{3}\sum_{jM}\langle T_{jM}\rangle T_{jM}^+, \quad (3)$$

in writing which we have introduced the following operators

$$T_{1\pm1} = \mp\frac{\sqrt{3}}{2}(S_x \pm iS_y), \qquad T_{10} = \sqrt{\frac{3}{2}}S_z,$$
$$T_{2\pm2} = \frac{\sqrt{3}}{2}(S_x \pm iS_y)^2,$$
$$T_{2\pm1} = \mp\frac{\sqrt{3}}{2}\{(S_x \pm iS_y)S_z + S_z(S_x \pm iS_y)\},$$
$$T_{20} = (3S_z^2 - 2)/\sqrt{2}, \quad (4)$$

and $\langle T_{jM}\rangle$ means the average value of T_{jM}.

If instead of $\langle T_{jM}\rangle$ we insert the polarization obtained after the first scattering,

$$\langle T_{jM}\rangle I_0(\vartheta) = {}^1/_3\,\mathrm{Sp}\{F(\vartheta, \varphi)F^+(\vartheta, \varphi)T_{jM}\}, \quad (5)$$

and if we use a coordinate system in which the plane of the first scattering is the xz plane and that of the second scattering contains the z axis (in other words, the z direction coincides with the direction of the deuteron momentum after the first scattering), then we get for the differential cross section of the double scattering:

$$I(\vartheta_1, \vartheta, \varphi) = I_0(\vartheta)\{1 + \langle T_{20}(\vartheta_1)\rangle\langle T_{20}(\vartheta)\rangle + 2[\langle T_{11}(\vartheta_1)\rangle\langle T_{11}(\vartheta)\rangle^* + \langle T_{21}(\vartheta_1)\rangle\langle T_{21}(\vartheta)\rangle]\cos\varphi + 2\langle T_{22}(\vartheta_1)\rangle\langle T_{22}(\vartheta)\rangle\cos 2\varphi\}, \quad (6)$$

where ϑ_1 and ϑ are the angles of the first and second scatterings, φ is the angle between the planes of the two scatterings, and $I_0(\vartheta)$ is the differential cross section for the scattering of an unpolarized beam,

$$I_0(\vartheta) = |A|^2 + \frac{2}{3}(|B|^2 + |C|^2) + \frac{1}{2}\left(\cos^2\vartheta + \frac{1}{3}\right)|D|^2 + \frac{4}{3}\mathrm{Re}[A^*(C + \cos\vartheta D)] + \frac{2}{3}\mathrm{Re}(C^*D)\cos\vartheta. \quad (7)$$

The average values used in Eq. (5) are given by the formulas

$$\langle T_{11}\rangle I_0(\vartheta) = -i\frac{2}{\sqrt{3}}\,\mathrm{Re}\left[\left(A + C + \frac{1}{2}\cos\vartheta\, D\right)B^*\right],$$

$$\langle T_{20}\rangle I_0(\vartheta) = -\frac{\sqrt{2}}{6}(|B|^2 + |C|^2) + \frac{\sqrt{2}}{8}\left(3\cos^2\vartheta - \frac{1}{3}\right)|D|^2$$
$$-\frac{\sqrt{2}}{3}\,\mathrm{Re}\,[(A - \cos\vartheta D)C^*] + \frac{2\sqrt{2}}{3}\cos\vartheta\,\mathrm{Re}\,(A^*D)$$
$$+\frac{\sqrt{2}}{2}\sin\vartheta\,\mathrm{Im}\,(B^*D),$$

$$\langle T_{21}\rangle I_0(\vartheta) = -\frac{\sqrt{3}}{6}\sin\vartheta\cos\vartheta\,|D|^2$$
$$-\frac{\sqrt{3}}{3}\sin\vartheta\,\mathrm{Re}\,[(A + C)D^*] + \frac{\sqrt{3}}{3}\cos\vartheta\,\mathrm{Im}\,(B^*D),$$

$$\langle T_{22}\rangle I_0(\vartheta) = -\frac{\sqrt{3}}{6}\Big\{|B|^2 + |C|^2 + \frac{\sin^2\vartheta}{4}|D|^2$$
$$+ 2\,\mathrm{Re}\,[(A + \cos\vartheta D)C^*] + \sin\vartheta\,\mathrm{Im}\,(B^*D)\Big\}.$$

We note that we have the following relations:

$$\langle T_{1-M}\rangle = (-1)^{M+1}\langle T_{1M}\rangle;\quad \langle T_{2-M}\rangle = (-1)^M\langle T_{2M}\rangle. \tag{8}$$

3. It is well known that owing to the existence of an anomalous magnetic moment of the electron the precession of the electron spin in a magnetic field is at a somewhat more rapid rate than that of the orbital angular momentum; therefore, generally speaking, when a polarized electron beam passes through a magnetic field there will be a change of the polarization (we deal here with the direction of the polarization relative to the direction of the beam).[1,3] We note that for particles with spin one-half there will be a change of the original polarization only in cases in which the particle has an anomalous magnetic moment (i.e., the Lande g-factor is not equal to two). In the case of a particle with spin unity, on the other hand, there will be a change of the original polarization in the magnetic field even in the case in which there is no anomalous magnetic moment (i.e., even if the Lande g-factor is equal to unity).

According to reference 1 the change of the density matrix under the action of the magnetic field is given by

$$\rho(t) = \exp[i(\omega_L\mathbf{L} + \omega_S\mathbf{S})\cdot\mathbf{h}t]\,\rho(0)\exp[-i(\omega_L\mathbf{L} + \omega_S\mathbf{S})\cdot\mathbf{h}t],$$

where $\rho(0)$ is the density matrix before the application of the magnetic field, $\rho(t)$ is the density matrix at the time t after the application of the field, $\mathbf{h}$ is the unit vector along the field, and

$$\omega_L = eH/mc,\quad \omega_S = geH/2mc. \tag{9}$$

Introducing the notations

$$\xi = (\omega_S - \omega_L)t,\quad \eta = \omega_L t, \tag{10}$$

we get

$$\rho(t) = \exp[i\mathbf{J}\cdot\mathbf{h}\eta]\exp[i\mathbf{S}\cdot\mathbf{h}\xi]\,\rho(0)\exp[-i\mathbf{S}\cdot\mathbf{h}\,\xi]\exp[-i\mathbf{J}\cdot\mathbf{h}\,\eta]$$

(where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is the total angular momentum).

It is easy to see that the differential cross section for double elastic scattering reduces to the form

$$I(\vartheta, \varphi) = Q(t)\,\mathrm{Sp}\,\{F(\vartheta, \varphi)\rho'(t)F^+(\vartheta, \varphi)\}\,Q^{-1}(t), \tag{11}$$

where

$$\rho'(t) = \exp[i\mathbf{S}\cdot\mathbf{h}\xi]\,\rho(0)\exp[-i\,\mathbf{S}\cdot\mathbf{h}\,\xi], \tag{12}$$

$$Q(t) = \exp[i\mathbf{L}\cdot\mathbf{h}\,\eta]. \tag{13}$$

Calculation gives (see Appendix)

$$\rho'(t) = \frac{1}{3} + \frac{1}{2}P_i(t)S_i + T_{ik}(t)S_iS_k, \tag{14}$$

where

$$P_i(t) = \cos\xi\,P_i + (1 - \cos\xi)h_ih_kP_k + \sin\xi\varepsilon_{ikn}h_nP_k, \tag{15}$$

$$T_{ik}(t) = \cos 2\xi T_{ik} + (\cos\xi - \cos 2\xi)h_l(h_iT_{kl} + h_kT_{il})$$
$$-\sin\xi(\cos\xi - 1)h_nh_m(h_i\varepsilon_{kln} + h_k\varepsilon_{iln})T_{lm}$$
$$+\sin\xi\cos\xi\,h_n(\varepsilon_{iln}T_{kl} + \varepsilon_{kln}T_{il}) + (\cos\xi - 1)^2h_ih_kh_lh_mT_{lm}$$
$$-\sin^2\xi\cdot\delta_{ik}h_lh_mT_{lm}, \tag{16}$$

$$P_i = \mathrm{Sp}\,[S_i\rho(0)] = P_i(0),$$
$$T_{ik} = \mathrm{Sp}\left\{\left[\frac{1}{2}(S_iS_k + S_kS_i) - \frac{2}{3}\delta_{ik}\right]\rho(0)\right\} = T_{ik}(0).$$

It is easy to verify that

$$T_{ik}(t) = T_{ki}(t),\qquad T_{ii}(t) = 0.$$

For the further calculations it is convenient to use the irreducible spin tensors of Racah;[4] the expressions for the average values of these quantities are as follows:

$$\langle T_{1\pm1}(t)\rangle = \mp(\sqrt{3}/2)\{P_x(t) \pm iP_y(t)\};$$
$$\langle T_{10}(t)\rangle = \sqrt{3/2}\,P_z(t);$$
$$\langle T_{2\pm2}(t)\rangle = (\sqrt{3}/2)\{T_{xx}(t) - T_{yy}(t) \pm 2iT_{xy}(t)\}; \tag{17}$$
$$\langle T_{2\pm1}(t)\rangle = \mp\sqrt{3}\{T_{xz}(t) \pm iT_{yz}(t)\};$$
$$\langle T_{20}(t)\rangle = (3/\sqrt{2})\,T_{zz}(t).$$

Thus by using Eqs. (15), (16), and (17) we can express $\langle T_{jM}(t)\rangle$ in terms of $\langle T_{jM}(0)\rangle$.

For $\rho'(t)$ it is easy to obtain the following formula:

$$\rho'(t) = \frac{1}{3}\sum_{jM}\langle T_{jM}(t)\rangle T^+_{jM}. \tag{18}$$

Let us consider the cases of longitudinal and transverse fields. In the case of a longitudinal field (i.e., $\mathbf{h}$ parallel to z), Eqs. (15), (16), (17), and (8) give

$$\langle T_{1\pm1}(t)\rangle = e^{\mp i\xi}\langle T_{1\pm1}\rangle,\ \langle T_{10}(t)\rangle = 0,$$
$$\langle T_{2\pm2}(t)\rangle = e^{\mp 2i\xi}\langle T_{2\pm2}\rangle;$$
$$\langle T_{2\pm1}(t)\rangle = e^{\mp i\xi}\langle T_{2\pm1}\rangle;\ \langle T_{20}(t)\rangle = \langle T_{20}\rangle.$$

For the transverse field, on the other hand (h parallel to y), we get

$$\langle T_{1\pm1}(t)\rangle = \langle T_{1\pm1}\rangle, \quad \langle T_{10}(t)\rangle = 0,$$

$$\langle T_{2\pm2}(t)\rangle = \frac{1}{2}(1+\cos^2\xi)\langle T_{2\pm2}\rangle$$

$$\pm\frac{1}{2}\sin 2\xi\langle T_{2\pm1}\rangle + \frac{1}{2}\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{20}\rangle,$$

$$\langle T_{2\pm1}(t)\rangle = \cos 2\xi\langle T_{2\pm1}\rangle$$

$$\mp\frac{1}{2}\sin 2\xi\langle T_{2\pm2}\rangle \pm \frac{1}{2}\sqrt{\frac{3}{2}}\sin 2\xi\langle T_{20}\rangle,$$

$$\langle T_{20}(t)\rangle = \frac{1}{2}(\cos 2\xi + \cos^2\xi)\langle T_{20}\rangle$$

$$+\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{22}\rangle - \sqrt{\frac{3}{2}}\sin 2\xi\langle T_{21}\rangle.$$

In the case of the longitudinal field the operator Q(t) takes the form

$$Q(t) = \exp[\eta\partial/\partial\varphi],$$

and for the differential cross section for double elastic scattering we get

$$\begin{aligned} I_\parallel(\vartheta_1, \vartheta, \varphi) = I_0(\vartheta)\{1 &+ \langle T_{20}(\vartheta_1)\rangle\langle T_{20}(\vartheta)\rangle \\ &+ 2[\langle T_{11}(\vartheta_1)\rangle\langle T_{11}(\vartheta)\rangle^* \\ &+ \langle T_{21}(\vartheta_1)\rangle\langle T_{21}^*(\vartheta)\rangle]\cos(\varphi+\eta+\xi) \\ &+ 2\langle T_{22}(\vartheta_1)\rangle\langle T_{22}(\vartheta)\rangle\cos 2(\varphi+\eta+\xi)\}. \end{aligned} \qquad (19)$$

In the case of the transverse magnetic field

$$Q(t) = \exp[\eta\cos\varphi\partial/\partial\vartheta - \eta\sin\varphi\cot\vartheta\partial/\partial\varphi],$$

and the differential cross section for double elastic scattering takes the form

$$\begin{aligned} I_\perp(\vartheta_1, \vartheta, \varphi) = I_0(\vartheta)\{1 &+ a(\vartheta_1+\eta\cos\varphi, \vartheta) \\ &+ b(\vartheta_1+\eta\cos\varphi, \vartheta)\cos(\varphi-\eta\sin\varphi\cot\vartheta_1) \\ &+ c(\vartheta_1+\eta\cos\varphi, \vartheta)\cos 2(\varphi-\eta\sin\varphi\cot\vartheta_1)\}, \end{aligned} \qquad (20)$$

where

$$a(\vartheta_1, \vartheta) = \langle T_{20}(\vartheta)\rangle\Big\{\frac{1}{2}(\cos 2\xi+\cos^2\xi)\langle T_{20}(\vartheta_1)\rangle$$

$$+\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{22}(\vartheta_1)\rangle - \sqrt{\frac{3}{2}}\sin 2\xi\langle T_{21}(\vartheta_1)\rangle\Big\},$$

$$b(\vartheta_1, \vartheta) = 2\Big[\langle T_{21}(\vartheta)\rangle\Big\{\cos 2\xi\langle T_{21}(\vartheta_1)\rangle - \frac{1}{2}\sin 2\xi\langle T_{22}(\vartheta_1)\rangle$$

$$+\frac{1}{2}\sqrt{\frac{3}{2}}\sin 2\xi\langle T_{20}(\vartheta_1)\rangle\Big\} + \langle T_{11}(\vartheta)\rangle^*\langle T_{11}(\vartheta_1)\rangle\Big],$$

$$c(\vartheta_1, \vartheta) = 2\langle T_{22}(\vartheta)\rangle\Big\{\frac{1}{2}(1+\cos^2\xi)\langle T_{22}(\vartheta_1)\rangle$$

$$+\frac{1}{2}\sin 2\xi\langle T_{21}(\vartheta_1)\rangle + \frac{1}{2}\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{20}(\vartheta_1)\rangle\Big\}. \qquad (21)$$

4. Repeated scatterings are used for the determination of the scattering amplitude from experimental data. To determine the coefficients in the scattering amplitude (1) one needs seven independent relations (for the four absolute values and three phase differences). By measurement of the intensity of the scattering of an unpolarized beam of deuterons, we can obtain one relation between the coefficients A, B, C, and D.

Let us consider experiments on simple double scattering (i.e., in the absence of a magnetic field), confining ourselves to the case in which the two scatterings occur under almost identical conditions (i.e., $\vartheta \approx \theta_1$, and also the energies are approximately equal). It can be seen from (6) that experiments on simple double scattering give three relations for the coefficients of the scattering amplitude. In fact,

$$\Big(\frac{1}{2\pi}\int_0^{2\pi} I d\varphi - I_0\Big)\Big/ I_0 = \langle T_{20}(\vartheta)\rangle^2, \qquad (22)$$

$$(I(\varphi=0) - I(\varphi=\pi))/2I_0 = 2[|\langle T_{11}(\vartheta)\rangle|^2 + \langle T_{21}(\vartheta)\rangle^2],$$

$$\Big[\frac{1}{2}I(\varphi=0) + \frac{1}{2}I(\varphi=\pi) - I\Big(\varphi=\frac{\pi}{2}\Big)\Big]\Big/2I_0 = 2\langle T_{22}(\vartheta)\rangle^2.$$

Thus experiments on simple double scattering do not allow us to determine $\langle T_{11}\rangle$, that is, the degree of polarization received by the beam in the first scattering* (this situation is different from that in the case of spin one-half).

Let us consider, finally, experiments on double scattering in a transverse magnetic field (the case of the longitudinal field gives nothing new as compared with simple double scattering). From the expression (20) one can obtain

$$[I_\perp(\varphi=0) - I_\perp(\varphi=\pi)]/2I_0$$

$$= {}^1/_2\{a(\vartheta+\eta, \vartheta) - a(\vartheta-\eta, \vartheta)$$

$$+ b(\vartheta+\eta, \vartheta) + b(\vartheta-\eta, \vartheta) + c(\vartheta+\eta, \vartheta) - c(\vartheta-\eta, \vartheta)\}.$$

We shall choose the value of the magnetic field strength and the time during which the deuteron is in the field (i.e., the time interval between the scatterings) in such a way that (with n an integer)

$$\eta = 2\pi n, \text{ i.e., } Ht = 2\pi(mc/e)n. \qquad (23)$$

Then

$$[I_\perp(\varphi=0) - I_\perp(\varphi=\pi)]/2I_0$$

$$= 2\Big[|\langle T_{11}(\vartheta)\rangle|^2 + \langle T_{21}(\vartheta)\rangle\Big\{\cos 2\xi\langle T_{21}(\vartheta)\rangle$$

$$-\frac{1}{2}\sin 2\xi\langle T_{22}(\vartheta)\rangle + \frac{1}{2}\sqrt{\frac{3}{2}}\sin 2\xi\langle T_{20}(\vartheta)\rangle\Big\}\Big],$$

$$[I_\perp(\varphi=0) + I_\perp(\varphi=\pi)]/2I_0$$

$$= 1 + \langle T_{20}(\vartheta)\rangle\Big\{\frac{1}{2}(\cos 2\xi + \cos^2\xi)\langle T_{20}(\vartheta)\rangle$$

*We note, however, that if the energy of the incident deuterons is such that only elastic scattering is possible, then one can apply the unitarity relations,[4] in virtue of which the number of quantities to be measured is reduced to four.

$$+\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{22}(\vartheta)\rangle - \sqrt{\frac{3}{2}}\sin 2\xi\langle T_{21}(\vartheta)\rangle\Big\}$$
$$+ 2\langle T_{22}(\vartheta)\rangle\Big\{\frac{1}{2}(1+\cos^2\xi)\langle T_{22}(\vartheta)\rangle + \frac{1}{2}\sin 2\xi\langle T_{21}(\vartheta)\rangle$$
$$+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{20}(\vartheta)\rangle\Big\}, \qquad (24)$$

$$\Big[\frac{1}{2}I_\perp(\varphi=0) + \frac{1}{2}I_\perp(\varphi=\pi) - I_\perp\Big(\varphi=\frac{\pi}{2}\Big)\Big]\Big/ I_0$$
$$= -(2I_0)^{-1}[I_\perp(\varphi=0) - I_\perp(\varphi=\pi)]\sin(2\pi n\cot\vartheta)$$
$$+ 2[1+\cos(4\pi n\cot\vartheta)]\langle T_{22}(\vartheta)\rangle\Big\{\frac{1}{2}(1+\cos^2\xi)\langle T_{22}(\vartheta)\rangle$$
$$+\frac{1}{2}\sin 2\xi\langle T_{21}(\vartheta)\rangle + \frac{1}{2}\sqrt{\frac{3}{2}}\sin^2\xi\langle T_{20}(\vartheta)\rangle\Big\}.$$

We note that with the choice of Ht indicated above we have

$$\xi = 2\pi n(g/2 - 1). \qquad (25)$$

Thus we have obtained six relations — Eqs. (22) and (24). Of course, not all these relations are independent of each other, but they are sufficient for the determination of $\langle T_{11}\rangle$, $\langle T_{20}\rangle$, $\langle T_{21}\rangle$, and $\langle T_{22}\rangle$. Having determined these quantities, and knowing also the expression $I_0(\vartheta)$, we shall have five equations for the seven unknown quantities in the scattering amplitude.

We have examined in detail the case of a field directed along the y axis. If the field is directed along the x axis, i.e., parallel to the plane of the first scattering and perpendicular to the momentum of the deuteron after the first scattering, then no new information is obtained about the scattering amplitude (beyond that obtained in the case of the field parallel to the y axis). The same is true for the case of an arbitrary direction of the field.

Thus, although measurement of the double elastic scattering in a magnetic field does give additional information about the scattering amplitude, this information is still not sufficient to determine it completely.

Accordingly we see that double elastic scattering in a magnetic field gives less information than simple triple scattering (the latter gives complete information about the scattering amplitude). This is due to the fact that the magnetic field leaves unchanged the absolute value of the polarization vector, and also the sum of the squares of the components of the tensor $\langle T_{2M}\rangle$.

APPENDIX

According to Eq. (12)

$$\rho'(t) = \exp[i\mathbf{S}\cdot\mathbf{h}\,\xi]\,\rho(0)\exp[-i\mathbf{S}\cdot\mathbf{h}\,\xi]. \qquad (A.1)$$

Expanding $\exp[\pm i\mathbf{S}\cdot\mathbf{h}\xi]$ in a power series in ξ and using the formula for reduction of tensors of third rank in S_i to tensors of lower ranks,

$$S_k S_m S_n = i\varepsilon_{kml} S_l S_n + i\varepsilon_{mnl} S_k S_l + \delta_{kn} S_m - i\varepsilon_{kmn}, \qquad (A.2)$$

we get

$$\exp[\pm i\mathbf{S}\cdot\mathbf{h}\xi] = 1 \pm i(\mathbf{S}\cdot\mathbf{h})\sin\xi + (\mathbf{S}\cdot\mathbf{h})^2(\cos\xi - 1). \qquad (A.3)$$

Substituting Eq. (A.3) into (A.1), applying Eq. (A.2), and recalling that[5]

$$\rho(0) = {}^1/_3 + {}^1/_2 P_i S_i + T_{ik} S_i S_k,$$

one readily obtains Eqs. (14) to (16) of the main text.

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