Letters to the Editor

ON A POSSIBLE STATISTICAL DESCRIPTION OF SYSTEMS OF PARTICLES INTERACTING WITH THE FIELD

Iu. L. KLIMONTOVICH

Moscow State University

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To investigate the emission of electromagnetic waves by charged particles in dielectrics and decelerating media, we consider the kinetic equations for systems of electrons and oscillators of the transverse electromagnetic field, It is assumed that the charge of the electrons is compensated by a uniformly distributed background of positive charge.

A state of the system in question is determined by the coordinates and momenta of the electrons and the coordinates $Q_\mathbf{k}$ and momenta $P_\mathbf{k}$ of the field oscillators with the various wave numbers $\mathbf{k}$. We introduce a distribution function $f(\mathbf{q}_1, \ldots, \mathbf{q}_N, \mathbf{p}_1, \ldots, \mathbf{p}_N, Q_\mathbf{k}, \ldots, P_\mathbf{k}, \ldots, t)$ which gives the probabilities of different states of the system.[1]

To obtain the kinetic equations for the first electron distribution function $f_1(\mathbf{q}, \mathbf{p}; t)$ and the first field-oscillator distribution function $F_1(Q_\mathbf{k}, P_\mathbf{k}; t)$ we construct a suitable chain of equations connecting distribution functions of different orders. The approximation of the higher distribution functions in terms of the lower is carried out in the same way as in the paper of Bogoliubov and Gurov.[2]

If the initial distribution of the field oscillators is the equilibrium distribution and the electrons are in near-equilibrium state, we get for the distribution function f_1 an equation of the Fokker-Planck type in the phase space:

$$\frac{\partial f_1}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f_1}{\partial \mathbf{q}} - \frac{e^2 n}{c^2}\frac{\partial}{\partial t}\int \frac{1}{|\mathbf{q}-\mathbf{q}'|}\left\{\mathbf{p} f_1\left(\mathbf{q}', \mathbf{p}; t - \frac{|\mathbf{q}-\mathbf{q}'|}{c/\sqrt{\varepsilon}}\right) + \frac{1}{4\pi} \times \int \frac{\mathrm{grad}_{\mathbf{q}''}\,\mathrm{div}_{\mathbf{q}''}\,[\mathbf{p} f_1(\mathbf{q}'', \mathbf{p}; t - |\mathbf{q}-\mathbf{q}'|\sqrt{\varepsilon}/c)]}{|\mathbf{q}'-\mathbf{q}''|}\,d\mathbf{q}''\right\} d\mathbf{q}'\,d\mathbf{p}\,\frac{\partial f_1}{\partial \mathbf{p}} = \frac{\partial}{\partial p_\alpha} B_{\alpha\beta}\frac{\partial f_1}{\partial p_\beta} + \frac{\partial}{\partial \mathbf{p}}(\mathbf{A} f_1), \tag{1}$$

where the diffusion coefficients $B_{\alpha\beta}$ and the systematic friction coefficient $\mathbf{A}$ are given by the following expressions:

$$B_{\alpha\beta} = \frac{e^2 \varkappa T}{2\pi}\int \delta\left(\omega_k - \frac{\mathbf{k}\cdot\mathbf{p}}{m}\right) a_{k\alpha} a_{k\beta}\, d\mathbf{k},$$

$$\mathbf{A} = \frac{e^2}{2\pi m}\int \delta\left(\omega_k - \frac{\mathbf{k}\cdot\mathbf{p}}{m}\right)\mathbf{a}_k(\mathbf{a}_k\cdot\mathbf{p})\,d\mathbf{k}, \quad \omega_k = \frac{c}{\sqrt{\varepsilon}}k. \tag{2}$$

In Eq. (1), n is the number of electrons in unit volume, and ϵ is the dielectric constant of the medium. The region of integration in the last term of the left member of Eq. (1) is restricted by the condition $|\mathbf{q}-\mathbf{q}'| \le c(t-t_0)/\epsilon^{1/2}$, which comes from the fact that we are considering the problem in which the distribution functions of the electrons and field oscillators are prescribed at the initial time t_0; therefore, owing to the finite speed of propagation of the interaction, the change of the distribution function f_1 at the point $\mathbf{q}$ at the time t can be affected only by the states of the electrons at distances less than or equal to $c(t-t_0)/\epsilon^{1/2}$ from the point $\mathbf{q}$. In Eq. (1), only the interaction of the electrons with the transverse electromagnetic waves has been expressed separately.

The coefficients $B_{\alpha\beta}$ and $\mathbf{A}$ are nonvanishing only when the condition for Cerenkov radiation is satisfied. The coefficient $\mathbf{A}$ gives the deceleration of a particle with momentum $\mathbf{p}$ by the field. After integration the expression for the decelerating force takes the form

$$(e/c)^2\int(1 - c^2 m^2/p^2\varepsilon)\,\omega_k\,d\omega_k$$

and agrees with the well known expression of the theory of Cerenkov radiation.[3,4]

We note further that in the equilibrium case the Maxwell distribution satisfies Eq. (1).

Under the inverse conditions, with the electrons in equilibrium and the oscillators close to an equilibrium state at the initial time, we get for the distribution function of the oscillators an equation of the Fokker-Planck type in the phase space of the coordinates and momenta of the field oscillators. In the equilibrium case the solution of this equation has the form:

$$F_1^{(0)} = A\exp\{-P_\mathbf{k}^2/2\varkappa T - \omega_\mathbf{k}^2 Q_\mathbf{k}^2/2\varkappa T\}.$$

By means of this equation we get the averaged equation for the coordinates of the oscillators. For example, for a uniform distribution of the electrons this equation has the form:

$$\ddot{\bar{Q}}_\mathbf{k} + 2\gamma\dot{\bar{Q}}_\mathbf{k} + \Omega_k^2\bar{Q}_\mathbf{k} = 0; \quad \bar{Q}_\mathbf{k} = \int Q_\mathbf{k} F_1\,(dQ_\mathbf{k}\,dP_\mathbf{k}), \tag{3}$$

where

$$\Omega_k^2 \approx \omega_L^2 + \omega_k^2; \qquad \gamma = \left(\frac{\pi m}{8\varkappa T}\right)^{1/2}\frac{\omega_L^2}{k}\exp\left\{-\frac{m\omega_k^2}{2k^2\varkappa T}\right\}.$$

We have also considered the more general case in which neither of the subsystems (electrons and electromagnetic oscillations) is in a state of thermal equilibirum. The results obtained will be applied in the study of the radiation emitted from electron beams passing through decelerating systems.

We take occasion to express our gratitude to Academician N. N. Bogoliubov for his interest in this work.

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[3] B. M. Bolotovskii, Usp. Fiz. Nauk **62**, 201 (1957).
[4] L. D. Landau and E. M. Lifshitz Электродинамика сплошных сред (The Electrodynamics of Continuous Media), Gostekhizdat 1957.

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THE SCATTERING OF SPIN 2 PARTICLES BY A COULOMB FIELD

A. I. BEDRITSKII

Vitebsk Pedagogical Institute

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THE effective cross section for the elastic scattering of particles with spin 2 by a heavy Coulomb center can be found by analogy with the theory for electrons (see reference 1). The relativistic wave equations for free particles with spin 2 can be written, taking into account their irreducibility to the Hamiltonian form, (see, for example, references 2 and 3)

$$(\gamma_l \nabla_l + \varkappa)\psi = 0, \tag{1}$$

and in first order perturbation theory, the effective cross section for elastic scattering of particles with spin 2 and charge e from a heavy nucleus with charge ze becomes

$$\sigma = \frac{4z^2e^4k_0}{c^2h^2k}\int \frac{G(k_0 + k_0')}{(\mathbf{k}-\mathbf{k}')^4(k+k')}\delta(k'-k)\,d^3k', \tag{2}$$

$$G = \frac{1}{5}\sum_s \sum_{s'} B^+\gamma^+{}_4 B' \cdot B'^+\gamma_4 B \,/\, B'^+\gamma_4^+ B' \cdot B'^+\gamma_4 B'. \tag{3}$$

Here and in the following primed quantities refer to the final state of the particle, after scattering.

The matrix γ_4 and the matrix A of the invariant bilinear form for spin 2 particles are known;[3] they are of the thirtieth degree. The other three matrices γ_α are easily found from the "coupling" formulas found by Fedorov,[3] and the irreducible representations of the Lorentz group,[2] use being made of the relations $\gamma_\alpha = i[I^{0\alpha}, \gamma_4]$, where the $I^{0\alpha}$ are the infinitesimal operators of the representation.

In calculating the quantity G, it is not necessary to know the matrices γ_l explicitly. The wave functions can be classified by their spin projections and normalized by their charge, $\psi^* A\gamma_4\psi = 1$ (the normalization $\psi^* A\psi = 1$ leads to the same final result) with the help of the method proposed by Fedorov,[4] using an invariant form. In this method, we do not use the functions B_{rs}, which describe the state with rest mass m_r and projection of spin on momentum s, but use instead the matrices $T = \alpha_r\beta_s$, where $\alpha_r = \alpha_+ = P_+(\alpha)/P_+(\kappa)$, $\beta_s = Q_s(S)/Q_s(s)$ can be determined by the minimal polynomials of the operator $\alpha = ik_l\gamma_l$ and the operator $S = (i/2|\mathbf{k}|)\,\delta_{\alpha\beta\sigma}I^{\beta\sigma}k_\alpha$ ($\alpha, \beta, \sigma = 1, 2, 3$) which projects the spin on the momentum of the particle:

$$P(\alpha) = \alpha^3(\alpha^2 - \varkappa^2) = (\alpha \mp \varkappa)P_\pm(\alpha) = 0, \tag{4}$$

$$Q(S) = S(S^2-1)(S^2-4) = Q_s(S)(S-s) = 0. \tag{5}$$

The method of reference 4 leads to the formula

$$G = (k_0'/5k_0)\sum_{s'} \mathrm{Sp}(\alpha_+ A\gamma_4^+\alpha'_+ A\gamma_4\beta'_{s'})/\mathrm{Sp}(\alpha'_+ A\gamma_4^+\alpha'_+ A\gamma_4\beta'_{s'}). \tag{6}$$

Even with this method, the calculations involved in finding G are tedious. In calculating the traces which occur in the numerators and denominators of the terms in G, it is helpful to remember the structure of the matrices $\beta'_{s'}$ and of the other factors. The matrices $\beta'_{s'}$ are quasi-diagonal, while each of the matrices $\alpha'_+ A\gamma_4^+$, $\alpha'_+ A\gamma_4$, $\alpha_+ A\gamma_4^+$ can be written as the sum of two matrices in such a way that each of the terms arising from multiplication have zero in the quasi-diagonal part corresponding to the quasi-diagonal part of $\beta'_{s'}$. All five denominators in the fractions of (6) turn out to be different (in the nonrelativistic case they are all equal to $5k_0$), but it is much easier to calculate them than the numerators.

Equations (2) and (6) give the following formula for the differential elastic scattering cross section:

$$d\sigma = \frac{z^2r_0^2}{960\varepsilon^4(1-5\varepsilon^2+4\varepsilon^4)^2\sin^4(\theta/2)}\{(7-14\varepsilon^2-166\varepsilon^4 + 722\varepsilon^6 - 41\varepsilon^8 + 832\varepsilon^{10} + 756\varepsilon^{12} + 64\varepsilon^{14}) + (\varepsilon^2-1) \times (3-15\varepsilon^2+15\varepsilon^4+41\varepsilon^6+660\varepsilon^8-576\varepsilon^{10}-128\varepsilon^{12})\cos^2\theta + 4(\varepsilon^2-1)^2(-1+9\varepsilon^2-28\varepsilon^4+49\varepsilon^6-45\varepsilon^8+16\varepsilon^{10}) \times \cos^4\theta + 4(\varepsilon^2-1)[(1+2\varepsilon^2-44\varepsilon^4+140\varepsilon^6+211\varepsilon^8+50\varepsilon^{10}) + (\varepsilon^2-1)(1-9\varepsilon^2+31\varepsilon^4+27\varepsilon^6-50\varepsilon^8)\cos^2\theta]\cos\theta\}\,d\Omega, \tag{7}$$