electromagnetic oscillations) is in a state of thermal equilibirum. The results obtained will be applied in the study of the radiation emitted from electron beams passing through decelerating systems.

We take occasion to express our gratitude to Academician N. N. Bogoliubov for his interest in this work.

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THE SCATTERING OF SPIN 2 PARTICLES BY A COULOMB FIELD

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THE effective cross section for the elastic scattering of particles with spin 2 by a heavy Coulomb center can be found by analogy with the theory for electrons (see reference 1). The relativistic wave equations for free particles with spin 2 can be written, taking into account their irreducibility to the Hamiltonian form, (see, for example, references 2 and 3)

$$(\gamma_l \nabla_l + \varkappa)\psi = 0, \tag{1}$$

and in first order perturbation theory, the effective cross section for elastic scattering of particles with spin 2 and charge e from a heavy nucleus with charge ze becomes

$$\sigma = \frac{4z^2e^4k_0}{c^2h^2k}\int \frac{G(k_0 + k_0')}{(\mathbf{k}-\mathbf{k}')^4(k+k')}\delta(k'-k)\,d^3k', \tag{2}$$

$$G = \frac{1}{5}\sum_s\sum_{s'} B^+\gamma^+{}_4 B'\cdot B'^+\gamma_4 B \,/\, B'^+\gamma_4^+ B'\cdot B'^+\gamma_4 B'. \tag{3}$$

Here and in the following primed quantities refer to the final state of the particle, after scattering.

The matrix γ_4 and the matrix A of the invariant bilinear form for spin 2 particles are known;[3] they are of the thirtieth degree. The other three matrices γ_α are easily found from the "coupling" formulas found by Fedorov,[3] and the irreducible representations of the Lorentz group,[2] use being made of the relations $\gamma_\alpha = i[I^{0\alpha}, \gamma_4]$, where the $I^{0\alpha}$ are the infinitesimal operators of the representation.

In calculating the quantity G, it is not necessary to know the matrices γ_l explicitly. The wave functions can be classified by their spin projections and normalized by their charge, $\psi^* A\gamma_4\psi = 1$ (the normalization $\psi^* A\psi = 1$ leads to the same final result) with the help of the method proposed by Fedorov,[4] using an invariant form. In this method, we do not use the functions B_{rs}, which describe the state with rest mass m_r and projection of spin on momentum s, but use instead the matrices $T = \alpha_r\beta_s$, where $\alpha_r = \alpha_+ = P_+(\alpha)/P_+(\kappa)$, $\beta_s = Q_s(S)/Q_s(s)$ can be determined by the minimal polynomials of the operator $\alpha = ik_l\gamma_l$ and the operator $S = (i/2|\mathbf{k}|)\,\delta_{\alpha\beta\sigma} I^{\beta\sigma} k_\alpha$ (α, β, σ = 1, 2, 3) which projects the spin on the momentum of the particle:

$$P(\alpha) = \alpha^3(\alpha^2 - \varkappa^2) = (\alpha \mp \varkappa)P_\pm(\alpha) = 0, \tag{4}$$

$$Q(S) = S(S^2-1)(S^2-4) = Q_s(S)(S-s) = 0. \tag{5}$$

The method of reference 4 leads to the formula

$$G = (k_0'/5k_0)\sum_{s'}\mathrm{Sp}(\alpha_+ A\gamma_4^+\alpha'_+ A\gamma_4\beta'_{s'})/\mathrm{Sp}(\alpha'_+ A\gamma_4^+\alpha'_+ A\gamma_4\beta'_{s'}). \tag{6}$$

Even with this method, the calculations involved in finding G are tedious. In calculating the traces which occur in the numerators and denominators of the terms in G, it is helpful to remember the structure of the matrices $\beta'_{s'}$ and of the other factors. The matrices $\beta'_{s'}$ are quasi-diagonal, while each of the matrices $\alpha'_+ A\gamma_4^+$, $\alpha'_+ A\gamma_4$, $\alpha_+ A\gamma_4^+$ can be written as the sum of two matrices in such a way that each of the terms arising from multiplication have zero in the quasi-diagonal part corresponding to the quasi-diagonal part of $\beta'_{s'}$. All five denominators in the fractions of (6) turn out to be different (in the nonrelativistic case they are all equal to $5k_0$), but it is much easier to calculate them than the numerators.

Equations (2) and (6) give the following formula for the differential elastic scattering cross section:

$$d\sigma = \frac{z^2r_0^2}{960\varepsilon^4(1-5\varepsilon^2+4\varepsilon^4)^2\sin^4(\theta/2)}\{(7 - 14\varepsilon^2 - 166\varepsilon^4 + 722\varepsilon^6 - 41\varepsilon^8 + 832\varepsilon^{10} + 756\varepsilon^{12} + 64\varepsilon^{14}) + (\varepsilon^2-1) \times (3 - 15\varepsilon^2 + 15\varepsilon^4 + 41\varepsilon^6 + 660\varepsilon^8 - 576\varepsilon^{10} - 128\varepsilon^{12})\cos^2\theta + 4(\varepsilon^2-1)^2(-1 + 9\varepsilon^2 - 28\varepsilon^4 + 49\varepsilon^6 - 45\varepsilon^8 + 16\varepsilon^{10}) \times \cos^4\theta + 4(\varepsilon^2-1)[(1 + 2\varepsilon^2 - 44\varepsilon^4 + 140\varepsilon^6 + 211\varepsilon^8 + 50\varepsilon^{10}) + (\varepsilon^2-1)(1 - 9\varepsilon^2 + 31\varepsilon^4 + 27\varepsilon^6 - 50\varepsilon^8)\cos^2\theta]\cos\theta\}\,d\Omega, \tag{7}$$

where $\epsilon = E/E_0$, E is the initial total energy of the particle, $E_0 = mc^2$ is the rest energy, $r_0 = e^2/mc^2$ and $d\Omega = \sin\theta d\theta d\varphi$.

In the nonrelativistic case ($k \ll k_0, \kappa$) (7) gives the classical Rutherford formula, while in the extreme-relativistic case ($\kappa \ll k, k_0$) only transitions in which the final states have spin projections $s' = \pm 2$ are important and

$$d\sigma = \frac{z^2}{15} r_0^2 \varepsilon^2 \cos^4 \frac{\theta}{2} d\Omega. \quad (8)$$

The effect of the field due to the electron cloud around the nucleus can be evaluated with the Thomas-Fermi method;[1] for no screening ($qa \gg 1$) the result is

$$d\sigma = \frac{z^2 r_0^2}{960\varepsilon^4 (1 - 5\varepsilon^2 + 4\varepsilon^4)^2 \sin^4(\theta/2)} (7 - 14\varepsilon^2 - 166\varepsilon^4 + 722\varepsilon^6 - 41\varepsilon^8 + 832\varepsilon^{10} + 756\varepsilon^{12} + 64\varepsilon^{14}) d\Omega, \quad (9)$$

while for maximum screening we have

$$d\sigma = \frac{0.32 z^{2/3} \hbar^4}{r_0^2 \varepsilon^4 (1 - 4\varepsilon^2)^2 m_e^4 c^4} (7 - 14\varepsilon^2 - 166\varepsilon^4 + 722\varepsilon^6 - 41\varepsilon^8 + 832\varepsilon^{10} + 756\varepsilon^{12} + 64\varepsilon^{14}) d\Omega. \quad (10)$$

At low velocities ($p \ll \hbar z^{1/3}/2a_0$, $a_0 = \hbar^2/m_e e^2$, m_e = rest mass of the electron) there is total shielding at all scattering angles and we must use formula (10), but can neglect the small quantities ϵ:

$$d\sigma = (2.24 z^{2/3} \hbar^4 / r_0^2 \varepsilon^4 m_e^4 c^4) d\Omega. \quad (11)$$

At high velocities ($p \gg \hbar z^{1/3}/2a_0$) the screening effect is noticeable only at small angles θ, defined by[1]

$$\sin\frac{\theta}{2} < \hbar z^{1/3}/2a_0 p. \quad (12)$$

Hence for large velocities and at angles satisfying (12), formula (10) should be used, while at angles not satisfying (12) the proper formula to use is (9).

In the extreme relativistic case (when the condition $p \gg \hbar z^{1/3}/2a_0$ will be satisfied if $m \geq m_e$) we need consider only higher order terms in ϵ, and for small angles satisfying (12), Eq. (10) gives

$$d\sigma = (1.28 z^{2/3} \varepsilon^6 \hbar^4 / r_0^2 m_e^4 c^4) d\Omega, \quad (13)$$

while for angles which do not satisfy (12), Eq. (9) reduces to

$$d\sigma = \frac{z^2 r_0^2 \varepsilon^2}{240 \sin^4(\theta/2)} d\Omega. \quad (14)$$

For ϵ close to unity, Eq. (9) gives the classical Rutherford formula (for $p \gg \hbar z^{1/3}/2a$, and $\sin\theta/2 > \hbar z^{1/3}/2a_0 p$).

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HYDRODYNAMIC ANALYSIS OF THE COMPRESSION OF A RAREFIED PLASMA IN AN AXIALLY-SYMMETRIC MAGNETIC FIELD

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We consider a totally ionized plasma (neglecting collisions) confined in a narrow magnetic tube by an axially-symmetric magnetic field (the axis of the tube is the axis of symmetry of the magnetic field). Assuming quasi-neutral motion of the plasma along the tube, we can write the hydrodynamic equations:[1,2]

$$m\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = -\frac{W_\perp}{H}\frac{\partial H}{\partial x} - \frac{1}{n}\frac{\partial}{\partial x}(2W_\parallel n),$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \quad (1)$$

$$m = m_e + m_i,\ W_\perp = W_{\perp e} + W_{\perp i},\ W_\parallel = W_{\parallel e} + W_{\parallel i}.$$

Here the subscript e refers to the electrons and the subscript i refers to the ions, $W_\perp$ and $W_\parallel$ are the mean energies due to thermal motion of the electrons (ions) (perpendicular and parallel to the tube); $n \equiv N/H$ is a quantity proportional to the number of electrons (ions) per unit length of the tube, and N is the density per unit volume. We consider two modes of longitudinal compression of the plasma along the magnetic tube.

(a) Isothermal compression. Let $W_\perp$ and $W_\parallel$ be constant in time and over the length of the tube, with $W_\perp > 2W_\parallel$ (the latter condition can be rea-