from Eqs. (1) and (2) a formula that has no zeroes in the denominator.

In the extreme relativistic case, setting $\lambda\!\!\!^{-}\epsilon^{-1/2} \ll a \ll R$, we get for the radiation emitted backward the expression

$$W''_{\mathrm{Cer}} = 2\frac{e^2}{\pi c}\left(\ln\frac{2}{1-v/c}-1\right)(\omega_2-\omega_1)\left(\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}\right)^2, \qquad (5)$$

if $\epsilon(\omega)$ is constant in the frequency interval (ω_1, ω_2) In this same case we get for the radiation emitted forward the formula (5), but without the last factor, on the assumption $\lambda\!\!\!^{-}/|\epsilon^{1/2}-1| \ll a \ll R$ ($\lambda\!\!\!^{-}$ is the wavelength of the radiation divided by 2π.

It can be seen from the formulas (1) and (2) that there will be no Cerenkov radiation if $a \ll \lambda\!\!\!^{-}$. If, on the other hand, $\lambda\!\!\!^{-} < a \lesssim R$, then in finding the paths of steepest descent one must take into account the exponents appearing in Eqs. (1) and (2), and the result is that at a given point in the field we shall have bands of Cerenkov frequencies given by the relations

$$\frac{v}{c}\sin\theta\left(1-\frac{a}{R}\frac{s\cos^2\theta}{\sqrt{\varepsilon(\omega)-\sin^2\theta}}\right)\leqslant\sqrt{(v/c)^2\varepsilon(\omega)-1}\leqslant\frac{v}{c}\sin\theta\left(1-\frac{a}{R}\frac{t\cos^2\theta}{\sqrt{\varepsilon(\omega)-\sin^2\theta}}\right), \qquad (6)$$

where θ is the angle between R and the perpendicular to the plate, while $s = 2n + 2$, $t = 2n + 1$ for the Cerenkov radiation emitted backward, and $s = 2n + 1$, $t = 2n$ for the radiation emitted forward (n is a nonnegative whole number). The backward flux of Cerenkov radiation through the area between ρ and $\rho + d\rho$ will be given by

$$\frac{dW''_{\mathrm{Cer}}}{d\rho} = \frac{4e^2}{v^2}\sum_{n=0}^{\infty}\int_{\Delta\omega''_n}\frac{[1-\varepsilon\sqrt{1+(v/c)^2(1-\varepsilon)}]^{4n}}{[1+\varepsilon\sqrt{1+(v/c)^2(1-\varepsilon)}]^{4n+4}}\times\sqrt{\left(\frac{v^2}{c^2}\varepsilon-1\right)\left(1+\frac{v^2}{c^2}(1-\varepsilon)\right)}\times\left(\sqrt{1+\frac{v^2}{c^2}(1-\varepsilon)}-1\right)^2\omega d\omega, \qquad (7)$$

and for the forward radiation we have

$$\frac{dW'_{\mathrm{Cer}}}{d\rho} = \frac{4e^2}{v^2}\sum_{n=0}^{\infty}\int_{\Delta\omega'_n}\frac{[1-\varepsilon\sqrt{1+(v/c)^2(1-\varepsilon)}]^{4n}}{[1+\varepsilon\sqrt{1+(v/c)^2(1-\varepsilon)}]^{4n+2}}\times\sqrt{\left(\frac{v^2}{c^2}\varepsilon-1\right)\left(1+\frac{v^2}{c^2}(1-\varepsilon)\right)}\,\omega d\omega. \qquad (8)$$

For $a \ll R$ the intensity of the Cerenkov radiation goes to zero.

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*V. E. Pafomov has informed us that there are misprints in Eq. (2) of reference 2: the exponent of the second term in square brackets should have the plus sign, and the exponent of the third term the minus sign.

[1] G. M. Garibian, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1403 (1957), Soviet Phys. JETP **6**, 1079 (1958).

[2] V. E. Pafomov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1074 (1957), Soviet Phys. JETP **6**, 829 (1958).

Translated by W. H. Furry
265

RADIATIVE DECAY OF $\pi^{\pm}$ MESONS AND EFFECTS OF PARITY NON-CONSERVATION

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LONGITUDINAL polarization of particles is a consequence of the nonconservation of parity in weak interactions. A study of the radiative decay $\pi^{\pm} \to \mu^{\pm} + \nu + \gamma$ shows that parity is not conserved also in mixed interactions.

In the four-component-neutrino theory the equation for the decay has the form

$$D\psi_{\nu} = (eg/hc)\,\psi_{\pi}D^{-1}(\gamma_{\mu}A^{+}_{\mu})\,\psi_{\mu}. \qquad (1)$$

Here ψ_{μ}, ψ_{ν}, ψ_{π}, A_{μ} are the respective wave functions of the μ meson, the neutrino, the π meson, and the γ-ray quantum, and D is the Dirac operator. The longitudinal polarization of the μ meson and the neutrino is taken into account by means of the projection operator $\boldsymbol{\sigma}\hat{\mathbf{p}}/\mathbf{p}$; its characteristic values (s_{μ} or s_{ν}) describe the longitudinal polarizations. $s_{\mu} = 1$ ($s_{\nu} = 1$) corresponds to spin in the direction of motion of the μ meson (neutrino), and $s_{\mu} = -1$ ($s_{\nu} = -1$) to the opposite spin direction. The circular polarization of the γ-ray quantum is described by means of the polarization vector

$$\mathbf{a}_l = \{\boldsymbol{\beta} + il\,[\mathbf{n}\times\boldsymbol{\beta}]\}/\sqrt{2},$$

where β is a unit vector perpendicular to $\mathbf{n} = \boldsymbol{\kappa}/\kappa$;

$\hbar\kappa$ is the momentum of the quantum. $l = 1$ corresponds to right-circular polarization (spin directed parallel to the motion) and $l = -1$ to left-circular polarization (spin opposite to motion) of the γ-ray quantum.

In the case of a π meson at rest we get the following expression for the decay probability:

$$dW = \frac{e^2 g^2}{\hbar^2 c\pi} \frac{k_\mu^2 dk_\mu \sin\theta\, d\theta}{k_{0\pi} K_\mu} (f_1 \pm s_\mu s_\nu f_2 \pm s_\mu l f_3 + s_\nu l f_4). \tag{2}$$

Here the upper sign is for decay occurring with the emission of a neutrino, and the lower sign is for decay with emission of an antineutrino. The notations used are:

$$\begin{aligned} f_1 &= k_\mu^2 \frac{q(1-\cos^2\theta)}{Q-K_\mu} + \frac{Q-K_\mu}{k_{0\pi}-\gamma}\gamma k_{0\pi}, \\ f_2 &= -\left\{k_\mu(1-\cos^2\theta)\left(K_\mu + \frac{k_\mu^2}{Q-K_\mu}\right) + \frac{Q-K_\mu}{k_{0\pi}-\gamma}(k_\mu - K_\mu\cos\theta)\right\}, \\ f_3 &= -\frac{K_\mu}{k_{0\pi}-\gamma}\left\{k_\mu(1-\cos^2\theta)\left(k_{0\pi} - \frac{k_{0\mu}^2}{K_\mu}\right) - (Q-K_\mu)k_{0\pi}\cos\theta + \frac{k_{0\pi}(Q-K_\mu)}{K_\mu(k_{0\pi}-\gamma)}k_\mu^2\cos\theta\right\}, \\ f_4 &= -\frac{k_{0\pi}}{k_{0\pi}-\gamma}\{\gamma^2 - \gamma(Q+K_\mu) + k_{0\mu}^2\}, \end{aligned} \tag{3}$$

where $\hbar k_\mu$ is the momentum and $\hbar c K_\mu$ the energy of the μ meson, $\hbar c k_{0\pi}$ is the rest energy of the π meson, θ is the angle between the directions of motion of the μ meson and the γ-ray quantum, and

$$Q = (k_{0\pi}^2 + k_{0\mu}^2)/2k_{0\pi}, \quad q = (k_{0\pi}^2 - k_{0\mu}^2)/2k_{0\pi},$$
$$\gamma = K_\mu - k_\mu\cos\theta.$$

In the expression (2) for the decay probability the last three terms are due to parity nonconservation, i.e., the longitudinal polarizations of the μ meson, the neutrino, and the γ-ray quantum. If in Eq. (2) we carry out a summation over the directions of polarization of the μ meson and the γ-ray quantum, we get the well known expression[1] for the decay probability of the π meson.

Summation only over the spin states of the μ meson leads to the result of Bund and Ferreira.[2]

To simplify the analysis of the formula (2) we suppose that the momentum of the μ meson is very small (close to zero); then the momenta of the γ-ray quantum and the neutrino will be antiparallel. In this limit $(k_\mu \to 0)$

$$\begin{aligned} f_1 = f_4 &= k_{0\mu}(k_{0\pi} - k_{0\mu})/2, \\ f_2 = f_3 &= {}^1/_2\, k_{0\mu}(k_{0\pi} - k_{0\mu})\cos\theta, \end{aligned} \tag{4}$$

where θ is the angle between the spin vector of the μ meson and the direction of motion of the γ-ray quantum. The analysis leads to the following results:

(a) if the spin of the μ meson is directed opposite to the motion of the γ-ray quantum ($s_\mu = -1$ and $\cos\theta = -1$), then the decay probability is different from zero only in the case $s_\nu = 1$ and $l = 1$, i.e., when the decay involves emission of a neutrino and the quantum emitted has right-circular polarization;

(b) if the spin of the μ meson is directed along the direction of motion of the γ-ray quantum ($s_\mu = 1$), then we must permit decay of the π meson with emission of an antineutrino ($s_\nu = -1$) and a γ-ray quantum with left-circular polarization ($l = -1$). We note that in this limit the probabilities of the two types of decay are equal.

From the above it follows that if the π meson decays with emission of a neutrino, then for small momenta of the μ meson its spin must make an angle close to 180° with the direction of the quantum. In the case of antineutrino decay this angle is close to zero. Obviously this conclusion can be checked by measurement of the μ-γ correlation.

[1] B. Ioffe and A. Rudik, Dokl. Akad. Nauk SSSR **82**, 359 (1952). W. A. Fry, Phys. Rev. **83**, 1268 (1951). T. Eguchi, Phys. Rev. **85**, 943 (1952). H. Primakoff, Phys. Rev. **84**, 1255 (1951).

[2] G. W. Bund and P. L. Ferreira, Nuovo cimento **7**, 246 (1958).

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266

ON THE THEORY OF THE "SECOND MOMENT" IN THE NUCLEAR MODEL OF LANE, THOMAS, AND WIGNER

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THE "second moment" was first introduced in a paper of Lane, Thomas, and Wigner[1] as a quantitative criterion for the error committed when the nuclear Hamiltonian is replaced by the Hamiltonian of the shell model.

Let H be the nuclear Hamiltonian and H_0 the shell model Hamiltonian. Then $H = H_0 + H_1$, where H_1 is an operator which gives rise to correlations