[1] Lane, Thomas, and Wigner, Phys. Rev. **98**, 693 (1955).

[2] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

[3] N. Bohr and F. Kalckar, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **14**, No. 10 (1937).

[4] Ya. I. Frenkel, Sov. Phys. **9**, 533, (1936).

[5] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **7**, 819 (1937).

[6] V. M. Agranovich and A. S. Davydov, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1429 (1957), Soviet Phys. JETP **5**, 1164 (1957).

[7] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, John Wiley and Sons, Inc., New York, 1952.

[8] V. M. Agranovich and V. S. Stavinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 700 (1958), Soviet Phys. JETP **34 (7)**, 481 (1958).

[9] E. Vogt, Phys. Rev. **101**, 1792 (1956).

[10] G. E. Brown and C. T. De Dominicis, Proc. Phys. Soc. **A70**, 668 (1957).

Translated by M. Hamermesh
267

ON THE LIMITS OF APPLICABILITY OF THE IMPACT-PARAMETER METHOD

M. L. TER-MIKAELIAN and B. V. KHACHATRIAN

Erevan State University

Submitted to JETP editor June 23, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1287-1289 (November, 1958)

In studies of radiative processes, use is made of ordinary perturbation theory, and also of the impact-parameter method (Weizsäcker-Williams method). The opinion is sometimes expressed that the latter gives insufficiently accurate results.[1] But let us compare the results given by these methods.

We consider deceleration radiation in collisions with atoms.

Following Überall[1] we write the bremsstrahlung cross section in the form

$$\sigma_t = \frac{Z^2 r_0^2}{137}\frac{2d\varepsilon}{\varepsilon_1^2}\int_0^\infty dq_\perp^2 \int_{\delta+q_\perp^2/2\varepsilon_1}^{q} dq_z \frac{[1-F(q)]^2}{q^4}$$
$$\times\left\{-\frac{1}{(q_z-q_\perp^2/2\varepsilon_1)^2}-\frac{q_z+q_\perp^2(\varepsilon-\varepsilon_2)/2\varepsilon_1\varepsilon_2}{[(q_z-q_\perp^2/2\varepsilon_2)^2+4\delta^2 q_\perp^2]^{3/2}}\right. \quad (1)$$
$$\left.+\frac{(1+\varepsilon\delta)q_\perp^2+2}{(q_z-q_\perp^2/2\varepsilon_1)[(q_z-q_\perp^2/2\varepsilon_2)^2+4\delta^2 q_\perp^2]^{1/2}}\right\}.$$

Here ϵ_1, ϵ_2 are the initial and final energies of the electron, ϵ is the energy of the photon (denoted by k in reference 1), $\delta = \epsilon/2\epsilon_1\epsilon_2$, $\hbar = m = c = 1$. On the other hand, the impact-parameter method gives[2,3]

$$\sigma_t = \frac{Z^2 r_0^2}{137}\frac{d\varepsilon}{\varepsilon_1^2}\int_0^{\sim 1} dk^2 \int_\delta^{\sim 1}\frac{k^2 dk_1}{k_1^2(k_1^2+k^2+R^{-2})^2}$$
$$\times\left[\frac{\varepsilon_1}{\varepsilon_1-\varepsilon}+1-\frac{\varepsilon}{\varepsilon_1}-\frac{2\varepsilon}{k_1\varepsilon_1(\varepsilon_1-\varepsilon)}+\frac{\varepsilon^2}{k_1^2\varepsilon_1^2(\varepsilon_1-\varepsilon)^2}\right]; \quad (2)$$

here $k^2 = k_2^2 + k_3^2$ (ϵ is the energy of the photon). In Eq. (1) let us replace the variable q_z by $q_z' = q_z - q_\perp^2/2\epsilon_1$ and expand the expression in square brackets in powers of $(q_\perp/mc)^2$:

$$\frac{1}{q_z'^2}\left(1+\varepsilon\delta-\frac{2\delta}{q_z'}+\frac{2\delta^2}{q_z'^2}\right)q_\perp^2$$
$$+\left[\frac{\delta(1+\varepsilon\delta)}{q_z'^3}-\frac{9\delta^2+2\varepsilon\delta^3}{q_z'^4}+\frac{24\delta^3}{q_z'^5}-\frac{18\delta^4}{q_z'^6}\right]q_\perp^4\cdots \quad (3)$$

Comparing Eqs. (3) and (2) we see that the first term of the expansion of the exact formula gives the result of the impact-parameter method (we have here $q_z' = k_1$, $q_\perp^2 = k^2$), and the second term is only a correction if $q_\perp^2 \ll 1$. Consequently, Eq. (2) agrees with the exact formula (1) only in the region $q_\perp^2 \ll 1$ (which corresponds to values of the impact parameter larger than $\hbar/mc$). A similar treatment can be given for pair production.

Let us now turn our attention to radiation and pair production in periodic structures. In reference 1 a problem of this sort is solved for a chain of atoms [Eqs. (33) and (23)]. These formulas differ from the corresponding formulas for collisions with a single atom (for example, Eq. (1)) by the factor:

$$\frac{2\pi}{a}N\sum_h 2\Big/\sqrt{q_\perp^2\vartheta^2-\left(q_z-\frac{2\pi}{a}h\right)^2}, \quad (4)$$

which gives the effect of interference in radiation or pair production in collisions with a chain of atoms. It is easy to obtain analogous formulas by the impact-parameter method. To do this we set $\mathbf{r}_i = h\mathbf{a}$ in Eq. (4) of reference 2, where h is an integer and $\mathbf{a}$ is the direction vector of the chain of atoms. Integrating the crystalline factor with respect to ψ [Eq. (4) of reference 2] we get

$$\int d\psi\left|\sum_i e^{i(\mathbf{k}\cdot\mathbf{r}_i)}\right|^2 = \frac{2\pi}{a}N\sum_h 2\Big/\sqrt{k^2\vartheta^2-\left(k_1-\frac{2\pi}{a}h\right)^2}, \quad (5)$$

where we have set $k_1a_1 + k_2a_2 + k_3a_3 = k_1a \cos \vartheta + k \sin \vartheta \cos \varphi$. We see that the factor obtained is similar. It must be noted that after the changes of variables $q'_z = q_z - q_\perp^2/2\epsilon_1$ or $q'_z = q_z - q_\perp^2/2\epsilon$, for bremsstrahlung and pair production, respectively, the factors (4) and (5) differ somewhat, but not to any important degree if $\delta \gg q_\perp^2/2\epsilon_1$, $\delta \ll q_\perp^2/2\epsilon_2$, and $\delta \ll q_\perp^2/2\epsilon_-$, $\delta \ll q_\perp^2/2\epsilon_+$ for the two respective cases.

We remark that when one takes into account thermal vibrations in the interference factor of the radiation these make the contribution[2] $q_\perp < h/(\overline{u^2})^{1/2}$, where $\overline{u^2}$ is the mean square deviation of the atoms from their equilibrium positions. Furthermore, on inserting the factor (5) under the integral in Eq. (2) (or in the corresponding formula for the case of pair production) we arrive at a formula for radiation or pair production in collisions with a chain of atoms obtained by the impact-parameter method (when thermal vibrations are included there is an additional factor $\exp\{-(k^2 + k_1^2)\overline{u^2}\}$).

If we now compare these formulas with the formulas for the inteference radiation obtained by perturbation theory [Eqs. (41) and (42) of reference 1], we see that they are completely identical. An analogous argument can also be carried through for the case of collisions of charged particles with atomic electrons (ionization losses). Here it can be shown that the exact quantum mechanical formulas go over into those obtained by the impact-parameter method when they are expanded in powers of the parameter $q_\perp R$, where R is the radius of the atom and $q_\perp$ is the change of momentum in the direction perpendicular to the motion.

[1] H. Überall, Phys. Rev. **103**, 1055 (1956).

[2] M. L. Ter-Mikaelian, J. Exptl. Theoret. Phys. (U.S.S.R.) **25**, 289 (1953).

[3] M. L. Ter-Mikaelian, Izv. Akad. Nauk ArmSSR **9**, 77 (1956).

Translated by W. H. Furry
268

DIFFRACTION BREAKUP OF LIGHT NUCLEI

A. G. SITENKO and Iu. A. BEREZHNOI

Khar'kov State University

Submitted to JETP editor June 28, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1289-1291 (November, 1958)

REFERENCE 1 treated the processes of diffractive interaction of deuterons with nuclei. Obviously, diffraction processes (scattering, disintegration, and stripping) can also occur in the collision of other loosely bound light nuclei with heavy nuclei. The possible occurrence of diffraction phenomena must be taken into account in studying the interaction with heavy nuclei of beams of light nuclei which have been accelerated to high energy.

There are several light nuclei whose binding energies against two-particle breakup are small. Thus, for example, Li^6 may be regarded as being made up of a deuteron and an α particle (with binding energy $\epsilon = 1.53$ Mev), Li^7 is a triton plus an α particle ($\epsilon = 2.52$ Mev), Be^9 is Be^8 plus a neutron ($\epsilon = 1.64$ Mev), B^{10} is Li^6 plus an α particle ($\epsilon = 4.36$ Mev), etc. Diffraction processes in the interaction with heavy nuclei of light nuclei which are made up of two relatively weakly bound particles can obviously be described in the same way as the diffractive interaction of deuterons with nuclei. However, the results of reference 1 cannot be used without change, since it was assumed in reference 1 that the deuteron radius R_d is considerably smaller than the nuclear radius R.

In the present note we calculate the total cross sections for various processes of diffractive interaction of a deuteron with a black nucleus, assuming an arbitrary ratio of the radii R_d and R. The Coulomb interaction is neglected.

To simplify the calculations, the wave function of the deuteron ground state is chosen to be Gaussian

$$\varphi_0(r) = (2/\pi R_d)^{3/2} \exp\left\{-2r^2/\pi R_d^2\right\}, \tag{1}$$

in which the constants are determined from the conditions $\int \varphi_0^2(r)\,d\mathbf{r} = 1$ and $\int r\varphi_0^2(r)\,d\mathbf{r} = R_d$.

Using the general formulas of reference 1, one easily obtains the following expressions for the total cross section for all processes σ_t, the cross section σ_n for stripping off a neutron, the cross section σ_p for stripping off a proton, and the elastic scattering cross section σ_e: