FERROMAGNETIC RESONANCE IN A CIRCULARLY POLARIZED MAGNETIC FIELD OF ARBITRARY AMPLITUDE

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An analysis is given of the exact solutions of the magnetization equation of motion. The dependence of the magnetization components on the amplitude of the fixed magnetizing field and the amplitude of the radio-frequency field is found. A comparison is made with experimental results.

1. Ferromagnetic resonance is usually observed in a weak periodic radio-frequency field (amplitude h_0). Under these conditions, because of the width of the resonance absorption line the direction of the magnetization vector **M** departs so slightly from the direction of the magnetizing field $H_0 = H_z \gg h_0$ that its projection in this direction may be assumed constant. Even close to resonance the effect is extremely insensitive to the form of the damping terms in the equation of motion and can be satisfactorily described by the Bloch equation

$$\dot{\mathbf{M}} = \gamma\ \mathbf{M}\times\mathbf{H}\ + (\chi_0\mathbf{H} - \mathbf{M})/\tau, \qquad (1)$$

or the Landau-Lifshitz equation, which can be written in the form[1]

$$\dot{\mathbf{m}} = \gamma\ \mathbf{m}\times\mathbf{H}\ + \alpha\ \dot{\mathbf{m}}\times\mathbf{m}\ ,\ \alpha < 0, \qquad (2)$$

where $\mathbf{m} = \mathbf{M}/M_S$; $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$.

The situation is rather different in strong radio-frequency fields; in these fields the change in M_z cannot be neglected. In this case the variation of **M** becomes very sensitive to the actual form of the damping terms in the equation of motion.

Damon[2] has investigated experimentally effects connected with the change in the z component of the magnetization of a ferromagnet as a function of the field H_0 and the amplitude of the radio-frequency field. Bloembergen and Wang[3] repeated the Damon experiments on ferromagnetic and paramagnetic specimens at various temperatures. It was found in these experiments that at high amplitudes of the radio-frequency field the total magnetization in both paramagnetic and ferromagnetic material does not remain constant. In comparing the experimental results with theory, the authors started with the Bloch equation. It is found that this equation cannot explain a number of the effects which are observed.

Some of the results observed by Bloembergen and Wang have been explained by Suhl[1] on the basis of a number of assumptions concerning the redistribution of energy between spin waves.

Below we give an analysis of the exact solutions of Eqs. (1) and (2) for a circularly polarized radio-frequency field of arbitrary amplitude and of the role of paramagnetic effects in ferromagnets in a strong radio-frequency field.

2. In what follows it will be assumed that the sample is homogeneous and isotropic; the sample will also be assumed to be a sphere with radius less than the skin depth at the frequencies in question. It will also be assumed that the magnetizing field and the radio-frequency field are uniform.

In a coordinate system which rotates about the direction of $\mathbf{H}_0$ at the frequency of the radio-frequency field ω (for the steady-state motion) Eq. (1) assumes the form

$$\tau\ \mathbf{M}_0\times\Delta\boldsymbol{\omega}\ + \chi_0\mathbf{H} - \mathbf{M}_0 = 0, \qquad (3)$$

where $\Delta\boldsymbol{\omega} = \boldsymbol{\omega} + \gamma\mathbf{H}$ solving Eq. (3) for M_{0z}, we find

$$M_{0z} = \chi_0\left\{H_0 + \gamma\tau^2\frac{\omega h_0^2}{1+\tau^2(\Delta\omega^2+\gamma^2h_0^2)}\right\}, \qquad (4)$$

where $\Delta\omega = \Delta\omega_z$. When $\Delta\omega = 0$

$$M_{0z} = \chi_0 H_0\,(1+\gamma^2\tau^2h_0^2)^{-1}. \qquad (5)$$

Further, using the relation

$$M_{0x} + iM_{0y} = (\chi' - i\chi'')\,(h_x + ih_y) \qquad (6)$$

and the solution of Eq. (3) for M_{0x} and M_{0y}, we obtain

$$\chi' = \chi_0\left\{1 - \frac{\tau^2\omega\Delta\omega}{1+\tau^2(\Delta\omega^2+\gamma^2h_0^2)}\right\},\quad \chi'' = \chi_0\,\frac{\tau\omega}{1+\tau^2(\Delta\omega^2+\gamma^2h_0^2)}. \qquad (7)$$

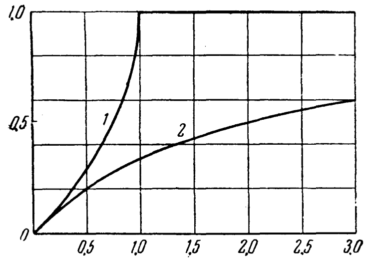

FIG. 1. The quantity $1 - M_z/M_0$ as a function of a^2 for the cases $M_0 = M_s$ (1) and $M_0 = \chi_0 H_0$ (2) with $\Delta\omega = 0$.

In a similar way, starting from Eq. (2), in the rotating coordinate system the steady-state motion is described by

$$[\mathbf{m}_0 \times \Delta\boldsymbol{\omega}] + \alpha(\mathbf{m}_0 \cdot \boldsymbol{\omega})\,\mathbf{m}_0 = \alpha\boldsymbol{\omega}, \tag{8}$$

whence the quantity m_{0z} is found from the following:

$$2m_{0z}^2 = 1 - a^2 - \xi^2 + [(1 - a^2 - \xi^2)^2 + 4\xi^2]^{1/2}, \tag{9}$$

where

$$a = \gamma h_0 / \alpha\omega;\quad \xi = \Delta\omega / \alpha\omega.$$

In the weak-field case $a \ll 1$ and $m_{0z} \approx 1$, as is usually assumed in the theory of ferromagnetic resonance. In fields such that $a \leq 1$ when $\Delta\omega = 0$

$$M_{0z} / M_s = (1 - a^2)^{1/2}. \tag{10}$$

When $a \geq 1$, the vector $\mathbf{m}_0$ is perpendicular to H_0 and we have total radio-frequency saturation ($M_{0z} = 0$). Computing $M_x + iM_y$ from Eq. (8) and using Eq. (6), we find in this case

$$\chi' = M_s \frac{\gamma}{\alpha\omega} \frac{\xi m_{0z}}{\xi^2 + m_{0z}^2}, \quad \chi'' = M_s \frac{\gamma}{\alpha\omega} \frac{m_{0z}^2}{\xi^2 + m_{0z}^2}, \tag{11}$$

where m_{0z} is determined from Eq. (9).

3. Bloembergen and Wang have investigated the quantity $\eta = 1 - M_{0z}/M_0$ as a function of the amplitude of the radio-frequency field h_0 for $\Delta\omega = 0$, where M_0 is the magnitude of the magnetization vector in the absence of the radio-frequency field ($\chi_0 H_0$ or M_s). In the relatively weak fields which obtain in experiments, η depends linearly on h_0^2, as follows from Eqs. (5) and (10). The slope of the line can be used to determine τ or α.

The dependence of η on h_0^2 for $\Delta\omega = 0$ which follows from Eq. (1) and Eq. (2) is shown in Fig. 1.

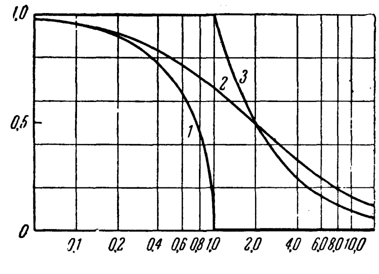

FIG. 2. The quantity $M_z M_0$ (curves 1 and 2) and χ'' (curve 3) as functions of a^2 for $\Delta\omega = 0$; curve 1 is for $M_0 = M_s$ curve 2 is for $M_0 = \chi_0 H_0$.

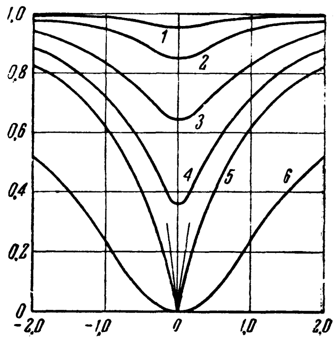

FIG. 3. The dependence of $(M_z/M_s)^2$ on ξ for the following values: 1) a = 0.2; 2) a = 0.4; 3) a = 0.6; 4) a = 0.8; 5) a = 1.0; 6) a = 2.0.

Using the Landau-Lifshitz equations we find $\eta = 1$ when $a \geq 1$.

If the length of the vector $\mathbf{M}$ remains constant, the ratio M_{0z}/M_0 obeys Eq. (10) for $\Delta\omega = 0$ and vanishes when h_0 equals the half-width of the resonance absorption line $\Delta H = |\alpha| H_0$. On the other hand, if we start from Eq. (1), in accordance with Eq. (5) M_{0z}/M_0 falls off gradually as h_0 increases, finally approaching zero. The quantity M_{0z}/M_0 is shown in Fig. 2 where, with $a \ll 1$, it is assumed that $\sqrt{2}\,\alpha\gamma\tau H_0 = 1$.

Experiments[3] on nickel-zinc and manganese-zinc ferrites at temperatures below the Curie point at $h_0 \leq 40$ oersteds have shown the reduction in M_{0z}/M_0 with increasing h_0. Unfortunately, an experiment does not allow us to choose between curve 1 and curve 2 of Fig. 2 since a is less than 0.1. In this case saturation is to be expected at $h_0 \approx 100$ oresteds. Total saturation can occur in relatively weak radio-frequency fields only in ferrites with narrow absorption lines.[4] In paramagnetic materials and ferrites above the Curie point, the dependence of M_{0z}/M_0 is shown by curve 2 of Fig. 2.

In the experiments performed by Damon,[2] carried out on single-crystal nickel ferrites at room temperature, the dependence of M_{0z}/M_0 on field strength H_0 was measured at various amplitudes of the radio-frequency field. The relations obtained are in good agreement with the theory if one starts from Eq. (2). The theoretical dependence of $(M_{0z}/M_s)^2$ on ξ is shown in Fig. 3 for various values of a. When $a \geq 1$ and $\Delta\omega = 0$, M_{0z}/M_s vanishes. It would seem that in the Damon experiments a was not greater than 0.2.

It follows from (4) and (8) that with $\Delta\omega = 0$, $\chi'' = |\gamma|\,\tau M_{0z}$, i.e., the change in χ'' with h_0 is proportional to the change in M_{0z}. The latter situation actually obtains for paramagnetic materials and ferromagnetic materials above the Curie temperature Θ. In ferromagnetic materials with $T < \Theta$, as has been shown in references 2 and 3, the proportionality between χ'' and M_{0z} no longer holds. Up to a certain threshold value of h_0, χ''

remains constant when $\Delta\omega = 0$; then it falls off approximately as a^{-2}.

In accordance with (11), in this case (curve 3, Fig. 2)

$$\chi'' = \gamma M_s / \alpha\omega \text{ for } a < 1$$
$$\text{and } \chi'' = \gamma M_s / \alpha\omega a^2 \text{ for } a > 1 \qquad (12)$$

the threshold field is determined by the value $a = 1$, which is larger than that observed experimentally.[1]

4. Equation (2) follows directly from Eq. (1) if we require that the magnitude of the magnetization vector remain constant: $M = M_S$. Equation (1) can be obtained under the assumption that the equilibrium magnetization obeys the Curie law.[5] Hence, it might be supposed that magnetic resonance effects in ferromagnets above the Curie point should obey Eq. (1); this conclusion is found to be in agreement with experiment. On the other hand, at temperature below the Curie temperature, if paramagnetic effects are neglected one would expect the Landau-Lifshitz equations to hold. In cases in which the paramagnetic effects cannot be neglected (for example, at temperatures close to the Curie temperature) one would expect that the ferromagnetic resonance effect would not be satisfactorily described by Eq. (2) or Eq. (1). In this case it would appear that the observed effects can be described by the joint application of Eqs. (1) and (2) if $\mathbf{M} = \mathbf{M}_S + \chi_0\mathbf{H}$.

[1] H. Suhl. Proc. I.R.E. **44**, 1270 (1956).

[2] R. W. Damon, Revs. Modern Phys. **25**, 239 (1953).

[3] N. Bloembergen and S. Wang, Phys. Rev. **93**, 72 (1954).

[4] J. F. Dillon Jr., Phys. Rev. **105**, 759 (1957).

[5] G. V. Skrotskii and V. T. Shmatov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 740 (1958), Soviet Phys. JETP **7**, 508 (1958).

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311