SPIN STRUCTURE OF THE SCATTERING MATRIX FOR REACTIONS INVOLVING GAMMA RAYS

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Submitted to JETP editor June 23, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1485-1487 (December, 1958)

The scattering matrix for reactions involving γ rays (such as $\gamma + b \rightarrow a' + b'$ and $\gamma + b \rightarrow \gamma' + b'$) is expanded in terms of spin operators Q_i which are invariant under rotations. A method of constructing the Q_i is given and the number of independent ones is found. If the scattering matrix is invariant under time inversion, the corresponding conditions on the form and number of the Q_i are found. Example are given for the representation of the matrix S by operators Q_i in the case that neither the spin of b, nor the spin of the system $a' + b'$ $(\gamma + b \rightarrow a' + b')$, nor the spins of the particles b, b' $(\gamma + b \rightarrow \gamma' + b')$ are more than one.

In an earlier paper[1] the author has considered the expansion of the scattering matrix $S(\mathbf{k}', \mathbf{k})$ for the reaction $a + b \rightarrow a' + b'$ in terms of spin operators Q_i which are invariant under rotation and reflection:*

$$S(\mathbf{k}', \mathbf{k}) = \sum_i A_i(\mathbf{k}' \cdot \mathbf{k}) Q_i(\mathbf{k}', \mathbf{k}, \mathbf{T}) \tag{1}$$

A method was given for obtaining the independent spin operators and for determining their number. Also considered were the conditions imposed on the form and number of the independent spin operators if the scattering matrix were invariant under time inversion.

In the present paper, these considerations are extended to scattering matrices $S(\mathbf{k}', \mathbf{k})$ for reactions involving γ rays, i.e., reactions like $\gamma + b \rightarrow a' + b'$, $\gamma + b \rightarrow \gamma' + b'$, where the particles b, b' and the system $a' + b'$ can have any spin and parity.

Using the expansion of the scattering matrix in terms of angle operators L_α,[2] it is not difficult to show that the independent spin operators for the reaction $\gamma + b \rightarrow \gamma' + b'$ will be the operators

$$(\mathbf{k}' \cdot \mathbf{e}) Q_i^{(\pm)}(\mathbf{k}', \mathbf{k}, \mathbf{T}), \; i(\mathbf{k}' \cdot \mathbf{s}) Q_i^{(\mp)}(\mathbf{k}', \mathbf{k}, \mathbf{T}), \tag{2}$$

where $\mathbf{e}$ is the polarization vector for the incident photon, $\mathbf{s} = \mathbf{k} \times \mathbf{e}$, $Q_i^{(\pm)}$ are the spin operators for the reaction $a + b \rightarrow a' + b'$ which differs from the reaction $\gamma + b \rightarrow a' + b'$ in that the γ-quantum has been replaced by a particle a with spin zero. The operator $Q_i^{(+)}$ corresponds to the case that the inner parity of the system does not change, while $Q_i^{(-)}$ corresponds to the case where the inner parity does change. The upper sign in $Q_i^{(\pm)}$ is to be used in formula (2) when the intrinsic parity of particle b in the reaction $\gamma + b \rightarrow a' + b'$ is the same as the intrinsic parity of the system $a' + b'$, while the lower sign is to be used in the opposite case. From formula (2) and the number of independent spin operators $Q_i^{(\pm)}$ for the reaction $a + b \rightarrow a' + b'$[1] it follows that the number of independent spin invariants for the reaction $\gamma + b \rightarrow a' + b'$ is $(2S' + 1)(2S + 1)$, where S is the spin of particle b and S′ is the spin of the system $a' + b'$. Summing on the possible values of S′, we obtain $(2s'_a + 1)(2s'_b + 1)(2s_b + 1)$ invariants, where s'_a, s'_b, and s_b are the spins of the particles a′, b′, and b. One can similarly show that the independent spin invariants for the reaction $\gamma + b \rightarrow \gamma' + b'$ are given by

$$(\mathbf{e}' \cdot \mathbf{e}) Q_i^{(\pm)}, \; (\mathbf{s}' \cdot \mathbf{s}) Q_i^{(\pm)}, \; i(\mathbf{s}' \cdot \mathbf{e}) Q_i^{(\mp)}, \; -i(\mathbf{e}' \cdot \mathbf{s}) Q_i^{(\mp)}, \tag{3}$$

where the $Q_i^{(\pm)}$ are the invariant spin operators for the reaction $a + b \rightarrow a' + b'$, in which a, a′ are particles with spin zero. The signs (±) on the operators $Q_i^{(\pm)}$ mean the same as in the preceding. In formula (3), the upper sign in the operator $Q_i^{(\pm)}$ is to be used if the intrinsic parity of the particles b, b′ in the reaction $\gamma + b \rightarrow \gamma' + b'$ are the same, while the lower sign is to be used in the opposite case. It is clear that the total number of independent spin invariants for the reaction $\gamma + b \rightarrow \gamma' + b'$ is $2(2S' + 1)(2S + 1)$, where S, S′ are the spins of particles b, b′. It

*The notation used here is the same as in reference 1.

should be noted that in reference 3 the number of spin invariants for the photo-production of mesons with spins 0 and 1 and in the scattering of photons from spin $\frac{1}{2}$ particles was found to be, respectively, 4, 14, and 10, while the present considerations lead to 4, 12, and 8. A detailed examination of the last mentioned case shows that the ten invariants mentioned in reference 3 reduce to eight.

It was shown in reference 1 that if the scattering matrix for elastic scattering of particles with spin is invariant under time inversion, then the form of the invariant spin operators Q_i changes, while the number of independent ones decreases. Similar statements hold for the elastic scattering of photons from particles with spin, $\gamma + b \rightarrow \gamma + b$. The proof is the same as in reference 1. We find that the spin operators enter the scattering matrix in the combinations

$$(\mathbf{e}'\cdot\mathbf{e})\,[Q_i^{(+)}(\mathbf{k}',\ \mathbf{k},\ \mathbf{T}) + Q_i^{(+)+}(\mathbf{k},\ \mathbf{k}',\ \mathbf{T})],$$

$$(\mathbf{s}'\cdot\mathbf{s})\,[Q_i^{(+)}(\mathbf{k}',\ \mathbf{k},\ \mathbf{T}) + Q_i^{(+)+}(\mathbf{k},\ \mathbf{k}',\ \mathbf{T})],$$

$$i\,[(\mathbf{s}'\cdot\mathbf{e})\,Q_i^{(-)}(\mathbf{k}',\ \mathbf{k},\ \mathbf{T}) - (\mathbf{e}'\cdot\mathbf{s})\,Q^{(-)+}(\mathbf{k},\ \mathbf{k}',\ \mathbf{T})],$$

$$-i\,[(\mathbf{e}'\cdot\mathbf{s})\,Q_i^{(-)}(\mathbf{k}',\ \mathbf{k},\ \mathbf{T}) - (\mathbf{s}'\cdot\mathbf{e})\ Q_i^{(-)+}(\mathbf{k},\ \mathbf{k}',\ \mathbf{T})],$$

i.e., those combinations which are symmetric under hermitian conjugation and simultaneous interchange $\mathbf{k} \rightleftarrows \mathbf{k}'$, $\mathbf{e} \rightleftarrows \mathbf{e}'$. The number of independent spin operators decreases by the number of relations between elements of the scattering matrix for which $j \neq j'$;

$$S^{ii}_{Jj'S\lambda',\,jS\lambda} = S^{ii}_{JjS\lambda,\,jS\lambda'}, \tag{4}$$

In (4), λ and λ' are numbers characterizing the type of radiation ($\lambda = 1$ for electric and 0 for magnetic radiation), while j, j' are the moments. There are $2S(2S+1)$ relations (4), where S is the spin of particle b. Hence the number of independent spin invariants for elastic scattering of photons from particles with spin S is $2(2S+1) \times (S+1)$.

As an example, let us write out the matrix $S(\mathbf{k}', \mathbf{k})$ in terms of independent operators Q_i for some reactions like $\gamma + b \rightarrow a' + b'$ and $\gamma + b \rightarrow \gamma' + b'$. If the intrinsic parity of the particle b and of the system $a' + b'$, or of the particles b, b′, are the same, then the reaction will be marked +, while if they are opposite, then the reaction will be marked −. We note that the matrices $S(\mathbf{k}', \mathbf{k})$ of such reactions differ only by the change $\mathbf{e} \rightleftarrows -i\mathbf{s} \equiv -i\mathbf{k} \times \mathbf{e}$ or the change $\mathbf{e}' \rightleftarrows i\mathbf{s}'$.

(1) $\gamma + b \rightarrow a' + b'$.

$S = S' = 0.\ (+).\qquad S(\mathbf{k}',\ \mathbf{k}) = (\mathbf{k}'\cdot\mathbf{e})\,A_1.$

$S = S' = 1/2.\ (-).\qquad S(\mathbf{k}', \mathbf{k}) = i\,(\mathbf{n}\cdot\mathbf{e})\,A_1 + (\sigma\cdot\mathbf{e})\,A_2 + (\sigma\cdot\mathbf{k}')\,(\mathbf{k}'\cdot\mathbf{e})\,A_3 + (\sigma\cdot\mathbf{k})\,(\mathbf{k}'\cdot\mathbf{e})\,A_4.$

$S = S' = 1.\ (+).$

$$S(\mathbf{k}',\ \mathbf{k}) = (\mathbf{k}'\cdot\mathbf{e})\,A_1 + i\,(\mathbf{S}\cdot[\mathbf{k}'\times\mathbf{e}])\,A_2 + i\,(\mathbf{S}\cdot[\mathbf{k}\times\mathbf{e}])\,A_3$$
$$+ i\,(\mathbf{S}\cdot\mathbf{n})\,(\mathbf{k}'\cdot\mathbf{e})\,A_4 + [(\mathbf{S}\cdot\mathbf{k}')\,(\mathbf{S}\cdot\mathbf{e}) + (\mathbf{S}\cdot\mathbf{e})\,(\mathbf{S}\cdot\mathbf{k}')]\,A_5$$
$$+ [(\mathbf{S}\cdot\mathbf{k})\,(\mathbf{S}\cdot\mathbf{e}) + (\mathbf{S}\cdot\mathbf{e})\,(\mathbf{S}\cdot\mathbf{k})]\,A_6 + [(\mathbf{S}\cdot\mathbf{k}')\,(\mathbf{S}\cdot\mathbf{k}) + (\mathbf{S}\cdot\mathbf{k})\,(\mathbf{S}\cdot\mathbf{k}')]\,A_7$$
$$+ (\mathbf{S}\cdot\mathbf{k}')^2\,(\mathbf{k}'\cdot\mathbf{e})\,A_8 + (\mathbf{S}\cdot\mathbf{k})^2\,(\mathbf{k}'\cdot\mathbf{e})\,A_9.$$

$S = 0,\ S' = 1$ or $S = 1,\ S' = 0.\ (+).$

$$S(\mathbf{k}',\ \mathbf{k}) = i\,(\mathbf{T}\cdot[\mathbf{k}'\times\mathbf{e}])\,A_1 + i\,(\mathbf{T}\cdot[\mathbf{k}\times\mathbf{e}])\,A_2 + i\,(\mathbf{T}\cdot\mathbf{n})\,(\mathbf{k}'\cdot\mathbf{e})\,A_3.$$

(2) $\gamma + b \rightarrow \gamma' + b'$.

$S = S' = 0.\ (+).\ S(\mathbf{k}',\ \mathbf{k}) = (\mathbf{e}'\cdot\mathbf{e})\,A_1 + (\mathbf{s}'\cdot\mathbf{s})\,A_2.$

$S = S' = 1/2.\ (+).$

$$S(\mathbf{k}'\cdot\mathbf{k}) = (\mathbf{e}'\cdot\mathbf{e})\,A_1 + (\mathbf{s}'\cdot\mathbf{s})\,A_2 + i\,(\sigma\cdot[\mathbf{e}'\times\mathbf{e}])\,A_3 + i\,(\sigma\cdot[\mathbf{s}'\times\mathbf{s}])\,A_4$$
$$+ i\,(\sigma\cdot\mathbf{k}')\,(\mathbf{e}'\cdot\mathbf{s})\,A_5 + i\,(\sigma\cdot\mathbf{k})\,(\mathbf{s}'\cdot\mathbf{e})\,A_6$$
$$+ i\,(\sigma\cdot\mathbf{k}')\,(\mathbf{s}'\cdot\mathbf{e})\,A_7 + i\,(\sigma\cdot\mathbf{k})\,(\mathbf{e}'\cdot\mathbf{s})\,A_8,$$

with $A_5 = -A_6$ and $A_7 = -A_8$ for elastic scattering.

$S = S' = 1.\ (+).$

$$S(\mathbf{k}',\ \mathbf{k}) = (\mathbf{e}'\cdot\mathbf{e})\,\{A_1 + i\,(\mathbf{S}\cdot\mathbf{n})\,A_2$$
$$+ [(\mathbf{S}\cdot\mathbf{k}')\,(\mathbf{S}\cdot\mathbf{k}) + (\mathbf{S}\cdot\mathbf{k})\,(\mathbf{S}\cdot\mathbf{k}')]\,A_3 + (\mathbf{S}\cdot\mathbf{k}')^2\,A_4 + (\mathbf{S}\cdot\mathbf{k})^2\,A_5\}$$
$$+ (\mathbf{s}'\cdot\mathbf{s})\,\{A_6 + i\,(\mathbf{S}\cdot\mathbf{n})\,A_7 + [(\mathbf{S}\cdot\mathbf{k}')\,(\mathbf{S}\cdot\mathbf{k}) + (\mathbf{S}\cdot\mathbf{k})\,(\mathbf{S}\cdot\mathbf{k}')]\,A_8$$
$$+ (\mathbf{S}\cdot\mathbf{k}')^2\,A_9 + (\mathbf{S}\cdot\mathbf{k})^2\,A_{10}\} + i\,(\mathbf{s}'\cdot\mathbf{e})\,[(\mathbf{S}\cdot\mathbf{k}')\,A_{11} + (\mathbf{S}\cdot\mathbf{k})\,A_{12}$$
$$+ i\,(\mathbf{S}\cdot\mathbf{k}')\,(\mathbf{S}\cdot\mathbf{n})\,A_{13} + i\,(\mathbf{S}\cdot\mathbf{k})\,(\mathbf{S}\cdot\mathbf{n})\,A_{14}] - i\,(\mathbf{e}'\cdot\mathbf{s})\,[(\mathbf{S}\cdot\mathbf{k}')\,A_{15}$$
$$+ (\mathbf{S}\cdot\mathbf{k})\,A_{16} + i\,(\mathbf{S}\cdot\mathbf{k}')\,(\mathbf{S}\cdot\mathbf{n})\,A_{17} + i\,(\mathbf{S}\cdot\mathbf{k})\,(\mathbf{S}\cdot\mathbf{n})\,A_{18}],$$

and for elastic scattering, $A_4 = A_5$, $A_9 = A_{10}$, $A_{11} = A_{16}$, $A_{12} = A_{15}$, $A_{13} = A_{18}$, $A_{14} = A_{17}$.

In these formulas the A_i are functions of $(\mathbf{k}'\cdot\mathbf{k})$ and the energy of the system, while $\mathbf{n} = \mathbf{k}' \times \mathbf{k}$.

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[2] V. I. Ritus, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1536 (1957), Soviet Phys. JETP 5, 1249 (1958).

[3] M. Kawaguchi and N. Mugibayashi, Progr. Theoret. Phys. 8, 212 (1952).

Translated by R. Krotkov
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