NONLINEAR INTERACTION OF RADIO WAVES PROPAGATING IN A PLASMA

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WHEN high-power radio waves propagate in a plasma, in particular, in the earth's ionosphere, several nonlinear effects are observed: cross-modulation (the Luxemburg effect) and the interaction of unmodulated radio waves with their "self-effect" (cf. for example reference 1, §64 and references 2 and 3. In the work cited above consideration has been given only to the nonlinearity associated with the dependence of the effective number of collisions ν_{eff} on the field strength E_1 of a strong radio wave, say 1. In the simplest case, in which radio wave 1 is unmodulated, $E_1 = E_0 \cos(\omega t - k \cdot r) = E_0 \cos \varphi$; the condition $\omega^2 \gg \nu_{eff}^2$ is satisfied and we can neglect the constant magnetic field. In this case the nonlinear effect is simply related to the change in electron velocity. The velocity associated with the forced oscillations is $v_1 = (e/m\omega) E_0 \sin \varphi$ and the change in the absolute magnitude of the velocity is $\Delta v \approx e^2 E_0^2 \sin^2 \varphi / 2m^2 \omega^2 v_0$ where $v_0 \sim \sqrt{\kappa T/m}$ is the velocity associated with the thermal motion (e and m are the charge and mass of the electron and it is assumed that $\Delta v \ll v = v_0 + \Delta v$).

The corresponding change in electrical conductivity is

$$\Delta\sigma / \sigma = \Delta\nu_{eff} / \nu_{eff} = \Delta v / v_0 = e^2 E_0^2 \sin^2 \varphi / 2m^2 v_0^2 \omega^2, \quad (1)$$

since $\sigma = e^2 N \nu_{eff} / m\omega^2$, where N is the electron concentration. For simplicity, it is assumed that the basic role is played by collisions of electrons with molecules, so that ν_{eff} is proportional to the velocity v. When the electron velocity distribution, modulation etc. are taken into account the picture becomes more complicated. These effects, however, will not be important in what follows, since we wish to point out the existence of a nonlinear effect of another kind.

In a non-uniform isotropic plasma (also in a uniform magneto-plasma), in contrast with an isotropic uniform plasma, the electron concentration depends on the field associated with the radio wave. In the absence of transverse charges, when div $\epsilon' E = 0$, in an isotropic non-uniform medium we have

$$\text{div } E = 4\pi\bar{\rho} = - E \cdot \nabla\epsilon' / \epsilon',$$

where $\epsilon' = \epsilon - i4\pi\sigma/\omega$ is the complex dielectric permittivity. If the ion motion is neglected in the plasma, we have $\bar{\rho} = e\Delta N$ where ΔN is the change in electron concentration. Thus wave 1 produces a change

$$\Delta\epsilon' = - 4\pi e^2 \Delta N / m\omega (\omega - i\nu_{eff})$$
$$= e E \cdot \nabla\epsilon' / m\epsilon'\omega (\omega - i\nu_{eff}), \quad (2)$$

since

$$\epsilon' = 1 - 4\pi e^2 N / m\omega (\omega - i\nu_{eff}).$$

In a magneto-plasma, which for simplicity we also consider uniform and nonabsorbing, for div $D = \partial(\epsilon_{ik}E_k)/\partial x_i = 0$ generally speaking div $E = 4\pi e\Delta N \neq 0$. We consider a plane wave in a magneto-active medium, having in mind elliptical polarization of the general form

$$E_1 = E_{0a} \cos \varphi + E_{0b} \sin \varphi,$$

obviously

$$\frac{\Delta N}{N} = - \frac{\omega n / c}{4\pi e N} \{E_{0a} \cos \theta_a \sin \varphi + E_{0b} \cos \theta_b \cos \varphi\}, \quad (3)$$

where $\varphi = \omega t - k \cdot r$, $n = ck/\omega$ is the refractive index for the given wave, which is assumed to be normal (ordinary and extraordinary components) and θ_a and θ_b are the angles between E_{0a}, E_{0b} and k. All these quantities can be found from the general formulas (cf. reference 1, §§ 62 and 75). The change in electron concentration ΔN is proportional to the change in the complex permittivity tensor for the magneto-active plasma ϵ'_{ik}. If the external magnetic field is strong and we neglect unusual directions of propagation (for example, along the pole) $\cos \theta_a$ and $\cos \theta_b$ can be quantities of order unity. In such cases the effect given by (3) is larger than the effect given by (2) if the wavelength $\lambda = cn/\omega$ is smaller than the characteristic distance L, the distance in which there is a significant change in the properties of the medium.

It is clear from (1) and (3) that the relation between the present nonlinear effect and the one usually considered is characterized by the parameter

$$\xi = m^2 v^2 \omega^3 n / 2\pi c e^3 N E_0. \quad (4)$$

with $n \sim 1$, $v \sim 10^7$ (T $\sim 300°$K), $\xi \sim 10^{-23} \omega^3 / NE_0$. It is apparent that with $N \sim 10^5$ and $\omega \sim$

10^7 ($\lambda = 2\pi c/\omega \sim 180$ m), $\xi \sim 1$ for a field E_0 $\sim 1/300$ v/m.

It is important to note that the usual effect (1) is a quadratic effect and leads to the appearance of combination frequencies $\omega' \pm 2\omega$ (ω' is the frequency of the field of a weak radiowave which propagates in a medium which is perturbed by a strong wave at frequency ω). On the other hand, the effects in (2) and (3) are linear* with respect to field 1 and the combination frequencies are $\omega' \pm \omega$.

It should be noted that effects (2) and (3) are, by their nature, the same as occur in the scattering of transverse (radio) waves on plasma waves in an isotropic medium (cf. references 4 and 5).†

The concrete role played by the effect described in (2) and (3) on the propagation of radio waves in the ionosphere of the earth and the solar corona require special investigation.

*Obviously we are considering here the nonlinear dependence of the tensor ε'_{ik} on the field. In this case the field equations are clearly nonlinear.

†We note that the existence, of plasma oscillations excited by various perturbations in the ionosphere can be verified experimentally by observing on earth radio waves in which combination frequencies are produced as a result of scattering in the ionosphere.

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324

ELECTRON-POSITRON PAIRS FROM THE DECAY $\pi^0 \rightarrow e^- + e^+ + \gamma$

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FOURTEEN events of charge-exchange scattering of π^- mesons on hydrogen with the subsequent decay of the π^0 meson into a photon and a Dalitz electron-positron pair[2] were observed in a diffusion chamber filled with hydrogen at 25 atmos and placed in the 150 Mev π^--meson beam of the synchrocyclotron of the Joint Institute for Nuclear Research.[1] The sensitive volume of the chamber measured 380 mm in diameter. A constant magnetic field of 9000 Oe intensity was used. The 14 events mentioned were found in the scanning of 45,000 stereophotographs and were identified by π^--meson track endings in hydrogen accompanied by electron-positron pairs. Photographs of two such events are shown in Fig. 1. It is estimated that other processes giving rise to similar pairs (internal conversion of photons in the reaction $\pi + p \rightarrow n + \gamma$, pair creation by a photon near the

decay point of $\pi^0 \rightarrow 2\gamma$, etc) are negligible under our experimental conditions.

The number of conventional π^0-meson decays (into two photons) must be known in order to determine the relative probability for the decay process $\pi^0 \rightarrow e^- + e^+ + \gamma$. Direct counting of the number of charge exchange events is extremely difficult in a diffusion chamber, owing to the existence of local insensitive regions and to edge effects. However the number of π^0 mesons can be determined from data on the cross sections for charge-exchange and elastic π^--p scattering,[3] whose ratio is 1.8 at energies ~ 150 Mev. Since we observed 600 events of elastic π^--p scattering it follows that the number of π^0 mesons was 1080. Consequently the ratio of the probabilities for the

Pair No.	E^- (Mev)	E^+ (Mev)	$E = E^- + E^+$ (Mev)	α° (lab. frame)	θ° (lab. frame)
1	—	—	—	2.5	118
2	>134	70	>204	16	50
3	19	41	60	36	95
4	106	>24	>130	7	91
5	20	111	131	8	145
6	27	6	33	22	117
7	89	67	156	6	50
8	>14	>96	>110	>5	100
9	—	—	—	2	110
10	>58	103	>161	53	140
11	36	>134	>170	3	99
12	10	24	34	38	86
13	166	20	186	28	65
14	22	27	49	46	118.5