ELECTROACOUSTIC PHENOMENA IN A DEGENERATE ELECTRON-ION PLASMA

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Electrical phenomena occurring in an electron-ion plasma in which acoustic waves are propagated are considered. The space attenuation coefficient (absorption coefficient) of the waves is computed.

THE spectrum of characteristic oscillations of an electron-ion plasma consists of two modes: plasma or electronic oscillations with the limiting Langmuir frequency, and acoustic or ion oscillations. Thermal motion excites in the plasma only acoustic oscillations, which largely determine the temperature dependence of the physical properties of the plasma (for example, electric conductivity, thermal conductivity, absorption of sound, etc.).

Certain properties of real metals can be explained on the basis of the so-called plasma model,[1,2] in which the metal is regarded as an isotropic electron-ion plasma. This model is useful for describing those phenomena in which the characteristic length is considerably greater than the average distance between the particles of the metal. In particular, such phenomena include effects, considered below, which are associated with the propagation of supersonic waves in the metal.

In the first section of this paper, the problem is formulated and the fundamental equations are discussed, on the second section the potential of the electric field of supersonic waves is calculated, and in the third section the absorption coefficient of a longitudinal supersonic wave in the plasma is calculated.

1. When a longitudinal supersonic wave is propagated in an electron-ion plasma the amplitudes of the ion and electron densities will differ in magnitude due to the large difference in the volume compressibility of electron and ion fluids. This circumstance will result in the appearance of a space charge that produces a longitudinal electric field which varies at the supersonic frequency. The interaction of the spreading supersonic wave with thermal acoustic vibrations will lead to absorption and dissipation of energy of the supersonic wave. These effects can be taken into account by introducing a finite value for the electrical conductivity σ of the plasma, i.e., by taking into account the collisions between the electrons and the thermal vibrations of the plasma. For finite values of σ, the energy of the longitudinal electric field excited by the supersonic wave will be dissipated as Joule heat. We shall show that this mechanism of dissipation of supersonic vibrations in the metal gives the correct order of magnitude for the absorption coefficient.

We consider a dynamical system consisting of a large number of electrons and ions in which an ultrasonic wave is propagated. If the interactions between the particles are taken into account in the self-consistent field approximation, the initial equations may be written in the following form:

$$i\hbar\dot{\psi}_{\gamma j} = -(\hbar^2/2m_\gamma)\,\Delta\psi_{\gamma j} + e_\gamma\Phi\psi_{\gamma j},$$
$$\Delta\Phi = -4\pi\sum_{j\gamma} e_\gamma |\psi_{\gamma j}|^2. \qquad (1)$$

Here $\psi_{\gamma j}$ is the self-consistent wave function for the j-th particle. The subscript γ takes on two values, 1 for electrons, and 2 for ions. The equations in (1) are a system of Hartree's equations generalized to the case of nonstationary states. Our problem consists of evaluating the potential of the longitudinal electric field of ultrasonic waves in terms of constants characterizing the plasma and the amplitude of the supersonic wave.

Equations (1) have an exact solution for the system in a state of constant density $|\psi_{\gamma j}|^2 = \text{const}$. In states with a density that does not differ much from a constant, (1) can be linearized. To do this it is convenient to write (1) in the so-called hydrodynamic form,[3] by setting

$$\psi_{\gamma j} = \rho_{\gamma j}^{1/2}\exp\{is_{\gamma j}/\hbar\}.$$

If we seek solutions of the linearized equations in the form of a superposition of plane progressive waves $\exp\{i\mathbf{k}\cdot\mathbf{r} - i\omega(k)t\}$, then we obtain the following system of equations for the Fourier amplitudes of the quantities $\rho_{\gamma j}$ and $s_{\gamma j}$:

$$\{-i\omega + i\mathbf{k}\cdot\mathbf{v}_{\gamma j}\} s_{\gamma j} + e_\gamma \Phi_k + (\hbar^2/4m_\gamma) k^2 \rho_{\gamma j} = 0,$$

$$\{-i\omega + i\mathbf{k}\cdot\mathbf{v}_{\gamma j}\} \rho_{\gamma j} - (k^2/m_\gamma) s_{\gamma j} = 0, \quad (2)$$

$$\Phi_k = 4\pi k^{-2} \sum_{\gamma j} e_\gamma \rho_{\gamma j},$$

where $\mathbf{v}_{\gamma j}$ is the velocity of the j particle in the state with homogeneous density, equal to $\nabla s_{\gamma j}^{(0)}/m_\gamma$. The system of equations (2) leads to the dispersion equation[3]

$$1 = (k^2 G(k)/L^3) \sum_{j\gamma} (e_\gamma^2/m_\gamma)[(\omega - \mathbf{k}\cdot\mathbf{v}_{\gamma j})^2 - \hbar^2 k^4/4m_\gamma^2]^{-1}, \quad (3)$$

where

$$G(k) = \int \frac{1}{|r|} e^{i\mathbf{k}\mathbf{r}} d\mathbf{r} = \frac{4\pi}{k^2},$$

and L^3 is the volume of the system. If the velocity distribution of the electrons obeys Fermi statistics and the random motion of the ions can be neglected in the state with a spatially homogeneous density, then, taking into account the inequality $(\omega/k)^2 \ll v_f^2$ (v_F is the limiting velocity on the Fermi surface), which is justified by the result obtained, we obtain from (3)

$$\omega^2(k) = \omega_{02}^2 u_0^2 k^2/(\omega_{02}^2 + u_0^2 k^2), \quad (4)$$

where

$$\omega_{02}^2 = 4\pi n_{02} e_2^2/m_2, \quad u_0^2 = {}^1/_3 / m_1 e_2 / m_2 e_1 | v_F^2;$$

n_{02} is the average ion density. As $k \to 0$ expression (4) coincides with the corresponding formula of Silin's paper.[1]

2. From (2) we can easily obtain the Fourier component of the longitudinal electric field Φ_k, which can be conveniently expressed in terms of only the ion characteristics and $\omega(k)$. On using the approximations adopted in deriving Eq. (4) we obtain

$$\Phi_k = \frac{m_2}{e_2} \frac{\omega^2(k)}{k} A_k, \quad (5)$$

where A_k is the amplitude of displacement of the ions equal to $\sum_j \rho_{2,j}/(n_{02}k)$, while $\omega(k)$ is the frequency of the supersonic wave.

Let us consider the case of a standing supersonic wave. The position in space of the loops and nodes of the amplitude of the displacement of the ions A_k will not depend upon the time in this case. On the basis of (5) we can assert that between a node and a loop there exists a difference of potential Φ_k which varies in time with the frequency of the supersonic wave. Preliminary experiments support this conclusion.[4] In these experiments we measured the magnitude of the variable electromotive force that arises in a polycrystalline copper sample when stationary supersonic waves are excited in it. When the standing waves had a wavelength of 5 cm and the amplitude of oscillations was $\sim 10^{-5}$ cm the magnitude of the electromotive force was of the order of 10 microvolts, which agrees with the estimate in Eq. (5). The distribution of the variable electromotive force along the sample corresponded to the nodes and loops of the standing wave.

3. The potential (5) can also be obtained by considering oscillations in a system containing not two, but only one kind of particles of mass m_2 with the interaction potential between these particles being given by

$$V(r) = (e_2/r) \exp\{-\omega_{02}/u_0) r\}.$$

From this it follows that the effect of electrons on the interaction between ions reduces to screening the ionic charge. Since the average velocity of the random motion of the electrons is large compared with the same quantity for the ions, the Debye electron cloud practically does not differ in shape from a sphere. At the same time the velocities of the ordered (hydrodynamic) motions of the electrons and the ions are the same, since the Debye polarization cloud consisting of electrons moves practically together with the ion which is at the center of this cloud. The ratio of the kinetic energies of ordered motion of electrons and ions will be equal to the ratio of their mass densities $T_1/T_2 = m_1 z/m_2$ (where $z = |e_2/e_1|$ is the number of electrons per ion). This conclusion follows directly from Eq. (2).

We now proceed to evaluate the coefficient of sound absorption α in an electron-ion plasma. We consider the absorption due to the viscosity of the electron gas which arises as a result of collisions between the electrons and the thermal vibrations of the lattice. The existence of viscosity leads to the dissipation of kinetic energy of the ordered (hydrodynamic) motion. Consequently, the quantity q (the energy converted into heat per unit time per unit volume) is a function of the kinetic energy T_1 of the ordered motion of the electrons. By expanding this function into a series in powers of T_1, and by restricting ourselves to the first term of the expansion, we obtain

$$q = \beta (m_1 z/m_2) \left\{ \frac{1}{2} n_{02} m_2 v_2^2 \right\}. \quad (6)$$

(β is the expansion constant). Since the average density of kinetic energy in a supersonic wave is equal to the average density of potential energy, which in turn is equal to the energy density of the longitudinal electric field with the potential given by (5), formula (6) may be written in the form

$$q = (\beta/8\pi)(m_1 z/m_2) |\nabla \Phi_k|^2. \quad (7)$$

The quantity $\{(m_1 z/m_2) |\nabla\Phi_k|^2\}^{1/2}$ can be interpreted as a certain effective electric field which gives rise to the ordered motion of the electrons. If formula (7) is interpreted in such a way, the constant $\beta/8\pi$ can be identified with the electrical conductivity of the electron-ion plasma.

By equating q to the divergence of the energy flux density in the sound wave, which is equal to $2\alpha u_0 m_2 n_0 \omega^2(k) |A_k|^2$, we obtain the following expression for the absorption coefficient:

$$\alpha = (\sigma m_1 z/2m_2) |\nabla\Phi_k|^2/u_0 m_2 n_{02}\omega^2(k) |A_k|^2. \qquad (8)$$

By utilizing (5), Eq. (8) can be rewritten as

$$\alpha = (m_1 \sigma z/2e_2^2 n_{02} u_0)\,\omega^2. \qquad (9)$$

This expression for the absorption coefficient coincides with the corresponding expression of references 5, 6, and 7, in which the formula for α was obtained by other methods. The numerical value of the quantity α and its dependence on the electrical conductivity are apparently confirmed by experimental data.[5]

We note that in the electron-ion plasma of a metal other mechanisms of absorption of supersonic waves also exist in addition to the mechanism of absorption considered above. The absorption of supersonic waves may be due, for example, to thermal conductivity. In this case the correction to Eq. (9), according to reference 8, will be of order $(1 - c_V/c_p)$. Since the ratio of specific heats c_V/c_p for metals at not very high temperatures is close to unity the corrections to formula (9) due to thermal conductivity will be small.

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