PERIPHERAL INTERACTIONS BETWEEN ELEMENTARY PARTICLES

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Modern meson theory can describe in an adequate phenomenological way the peripheral interactions of strongly interacting particles. We propose to isolate the contribution of peripheral interactions, by deriving from experimental data the amplitudes for various processes which require large orbital angular momentum. Large orbital angular momentum corresponds to a large impact parameter. By comparing these amplitudes, derived from experiment, with theoretical calculations, we may obtain many important quantities characterizing the strong interactions of elementary particles, for example, the renormalized coupling-constant of the pion with nucleons and hyperons, the renormalized coupling-constant of two pions or of a pion with a K particle, etc.

1. INTRODUCTION

THE study of existing meson theories has shown that a "naive" extension of these theories to very small regions of space ($r \ll 10^{-13}$ cm) runs into serious contradictions.[1] The resolution of these contradictions will possibly require a change in the foundations of quantum mechanics and perhaps also of relativity. There is still every reason to believe that the present theoretical description is adequate for physical processes taking place at large distances. This paper is concerned with the analysis of strong interactions occurring at large distances ($r > 10^{-13}$ cm).

Suppose that two particles, for example two nucleons, are separated by a distance $r > 1/\mu$, where μ is the mass of a pion. We use units such that $\hbar = c = 1$. The interaction between the two particles depends on the properties of the meson clouds which surround them, and hardly at all on the properties of the inner regions inside the clouds. The nucleons touch each other with their surface regions. A large number of theoretical physicists[2] have understood that existing meson theory can describe this peripheral interaction phenomenologically. But to make use of this fact, one must know how to isolate the effect of the peripheral interaction. The experimental data obtained up till now on the interactions of elementary particles are very rarely suitable for determining peripheral interactions. The processes which have been studied hitherto depend essentially on the nature of the interaction at small distances, where we do not yet know how to make calculations.

A number of authors (Chew, Moravczik, Taylor, Uretskiĭ, Tsifra and others)[3] have proposed a method for isolating the contribution arising from the exchange of one virtual meson. The method of Chew is essentially based on the fact that the function which describes the angular distribution of a process can be extrapolated into the non-physical region $|\cos\theta| > 1$. It can be proved (and will be seen in the following argument), that for a particular value of $|\cos\theta| > 1$ the extrapolated angular distribution defines a one-meson amplitude proportional to g^2, where g is the renormalized pion-nucleon coupling constant. By comparing the values obtained by extrapolation, from the experimental data on N-P scattering and on photoproduction of pions, with the corresponding theoretical quantities, Chew and others could determine the value of g.

We here propose a second method which also allows us to isolate the contributions of peripheral interactions by an analysis of experimental data. Our method is based on the well-known fact that two particles, with a large relative orbital angular momentum l, effectively interact at a distance $\sim l\lambda\!\!\!^{-}$, where $\lambda\!\!\!^{-}$ is the particle wavelength. Penetration to smaller distances is hindered by the centrifugal barrier. It follows that, to study peripheral interactions, we need to separate from the experimental data that part which determines the amplitude of a process occurring with sufficiently large values of l.

The possibility of isolating and describing theoretically a peripheral interaction is based on the fact that two particles or systems of particles separated by a large distance exchange between them the smallest possible number of mesons. Thus pion-nucleon scattering at large distances is described by an exchange of two pions, nucleon-nucleon scattering is described by the exchange of one pion, etc. This allows us for example to describe nucleon-nucleon scattering for large l within the frame of a one-meson approximation, neglecting the exchange of two, three or more pions, and also the contributions of other mesons and baryon pairs, all such effects being included in the renormalized coupling-constant. We emphasize that in this case we may consider only the one-meson term, even though the coupling-constant is considerably larger than one $(g^2/4\pi \sim 15)$. The expansion parameter, as we shall see below, is proportional to $(g^2/4\pi)\, e^{-\mu\lambda l}$, which for sufficiently large l is much smaller than one. In a certain very academic sense, the one-meson approximation for large l is more accurate than the one-photon approximation in electrodynamics, since $e^2/4\pi = \text{const}$, but $(g^2/4\pi)\, e^{-\mu\lambda l} \to 0$, as $l \to \infty$.

Our method of analysis by orbital angular momentum allows us not only to determine the value of the constant g and other similar constants, but to find relations between various physical quantities, for example scattering phases, describing various processes.

In Sec. 2 we set up a simple model to justify our method. Sections 3 — 6 contain a study of experiments in which peripheral interactions of elementary particles might be observed. In Sec. 3 we consider pion-nucleon experiments, in Sec. 4 strange-particle experiments. In Sec. 5 we study interactions between photons and strongly interacting particles. In Sec. 6 we consider briefly experiments involving leptons. In Sec. 7 we discuss the correlation of the information which could be obtained from the various experiments.

2. STUDY OF A MODEL

To explain our method we consider the interaction of two scalar particles, exchanging neutral scalar mesons with each other. We ignore the symmetry of the amplitudes arising from the identity of the particles. This very unrealistic model allows us to forget the complications of the dependence of the amplitudes on spin and isotopic spin.

We shall compare the behavior for large l of the one-meson and two-meson approximations. We shall also examine to what extent the dependence of the vertex part and the meson Green's function on q^2 influences the one-meson approximation.

The amplitude arising from the exchange of one meson (Fig. 1) is

$$a^{(1)} = \mu f^{(1)}(k^2)\,\frac{d(q^2)\,\alpha^2(q^2)}{q^2-\mu^2}\,.$$

Here **k** is the momentum of one of the colliding particles in the center-of-mass system; $q^2 = -|\mathbf{q}|^2 = -2k^2(1-\cos\theta)$, θ is the scattering angle, μ is the mass of the virtual meson. The dimensionless function $f^{(1)}$, proportional to the square of the renormalized coupling-constant, depends on k^2 and not on q.

While we allow only one virtual meson to be exchanged between the two particles, we include all the virtual processes which contribute to the meson Green's function and to both the vertex parts. These effects appear in the expression for $a^{(1)}$ as the factors $d(q^2)$ and $\alpha(q^2)$ respectively. By definition $\alpha(\mu^2) = d(\mu^2) = 1$. In Fig. 1 and the other diagrams, blocks containing all possible virtual processes are represented by circles. Henceforth we shall call these block vertices.

The contribution of the one-meson diagram 1 to the scattering amplitude in a state with orbital angular momentum l can easily be obtained from the Legendre polynomial expansion:

$$a^{(1)} = \frac{1}{2ik}\sum_{l=0}(2l+1)\,\eta_l^{(1)}P_l(\cos\theta),$$

$$\eta_l^{(1)} = \exp(2i\delta_l^{(1)}) - 1,$$

This gives

$$\eta_l^{(1)} = \frac{i\mu f^{(1)}(k^2)}{2k}\int_0^{\pi}\frac{P_l(\cos\theta)\,d(\cos\theta)}{1+\mu^2/2k^2-\cos\theta} = \frac{i\mu f^{(1)}(k^2)}{k}\,Q_l\Big(1+\frac{\mu^2}{2k^2}\Big),$$

where Q_l is the Legendre polynomial of the second kind. Analytic expressions[4] and tables[5] for Q_l exist. When $l \gg k/\mu \gg 1$,

$$Q_l(1+\mu^2/2k^2) \sim \sqrt{\pi k/2\mu l}\; e^{-\mu l/k}.$$

When $k/\mu \ll 1$, the exponential behavior changes to a power law:

$$Q_l\Big(1+\frac{\mu^2}{2k^2}\Big) \sim \frac{l!\,2^{l+1}}{(2l+1)!!}\Big(\frac{k}{\mu}\Big)^{2l+2}.$$

We now estimate the contributions for large l from the two-meson amplitude corresponding to diagram 2. For large l the important effects come from the singularities in the meson Green's functions, and we therefore neglect the dependence of the remaining factors on p and q, and take these factors outside the integrals. We find then

$$a^{(2)} = \frac{f^{(2)}(k^2)}{\mu}\int\frac{d^4p}{[(p-q/2)^2-\mu^2]\,[(p+q/2)^2-\mu^2]}\,,$$

where $f^{(2)}$ is a dimensionless function of k^2. Combining the denominators and changing the variable of integration, we obtain

$$a^{(2)} = \frac{f^{(2)}(k^2)}{\mu}\int_0^1 dx\int\frac{d^4p}{[p^2+q^2(1-x)x-\mu^2]^2}\,.$$

We next rotate the integration with respect to p^0 onto the imaginary axis.[6,7] The denominator then becomes positive, and we find

$$\eta_l^{(2)} = -\frac{f^{(2)}(k^2)}{8\mu^2 k}\int_0^1 \frac{dx}{(1-x)x}\int_0^\infty \frac{\partial}{\partial\mu} Q_l\left(1+\frac{\mu^2+t^2}{2k^2(1-x)x}\right) t^2\, dt^2,$$

with $t^2 = |p|^2 + p_0^2$.

When $l \gg k/\mu \gg 1$, we use the asymptotic form of the function Q_l, and carry out the integration to obtain

$$\eta_l^{(2)} \approx -(\pi k f^{(2)}(k^2)/4\mu l^2)\, e^{-2\mu l/k}.$$

We thus see that for large l the two-meson amplitude is exponentially small compared with the one-meson amplitude. We will publish later a detailed analysis of the two-meson amplitude for large l. Here we wish only to remark that, in calculating the contribution for large l, convergence of the momentum integrals is always guaranteed by the presence of the factor $P_l(\cos\theta)$ in the integrand. Thus in the integrals which we considered above $t_{\text{eff}}^2 \sim \mu k/l$.

The contribution of the three-meson amplitude for large l $(l \gg k/\mu \gg 1)$ must obviously decrease like $\exp(-3\mu l/k)$. When $k/\mu < 1$ the exponential decrease is replaced by a decrease of the form 2^{-2l-2} for the two-meson amplitude and 3^{-2l-2} for the three-meson amplitude.

We now go back to the one-meson amplitude. In deriving the formula for the one-meson amplitude we assumed $\alpha(q^2) = 1$ and $d(q^2) = 1$, supposing that the important values of q^2 in the integral were around μ^2. We now verify the correctness of this approximation. The spectral representations of the Green's functions[8] and the vertex parts[9] give

$$\frac{\alpha^2(q^2)\, d(q^2)}{q^2-\mu^2} = \frac{1}{q^2-\mu^2}(1+S_1(q^2))(1+S_2(q^2))^2,$$

where

$$S_i = (q^2-\mu^2)\int_m \frac{\rho_i(\varkappa^2)\, d\varkappa^2}{q^2-\varkappa^2}.$$

Keeping only terms linear in S_i, we obtain

$$\eta_l^{(1)} \approx \frac{i\mu f^{(1)}(k)^2}{k}\left\{Q_l\left(1+\frac{\mu^2}{2k^2}\right) + \int_m^\infty Q_l\left(1+\frac{\varkappa^2}{2k^2}\right)(\rho_1(\varkappa^2)+2\rho_2(\varkappa^2))\, d\varkappa^2 + \ldots\right\}$$

The quantity m is the smallest sum of masses of particles into which a meson can be transformed. For example, for the pion $m = 3m_\pi$, and for the K particle $m = 2m_\pi + m_K$. For a scalar meson we would have $m = 2\mu$. When $l \gg k/\mu \gg 1$, the correction terms proportional to $e^{-2\mu l/k}$ are therefore exponentially small compared with $Q_l(1+\mu^2/2k^2)$ $\sim e^{-\mu l/k}$. The contribution which comes from the factors $\alpha^2(q^2)$ and $d(q^2)$ is seen to be of the same order as the contribution from exchange of two or more mesons. In the case of virtual pions, this contribution is of the same order as that arising from three-pion exchange. It is easy to verify that the terms of second and third order in S_i would not change this conclusion.

Effects of spin and isotopic spin, while essential in the discussion of actual processes, modify but do not qualitatively change our results.

3. PION-NUCLEON INTERACTION

Nucleon-nucleon scattering. A detailed study of this problem will be published elsewhere.* We therefore discuss here only the essential points.

It is clear from the preceding argument that the peripheral interaction between nucleons is described by the one-meson amplitude

$$M = 4\pi g^2\left\{(\bar{u}_1'\gamma_5\tau_\alpha u_1)(\bar{u}_2'\gamma_5\tau_\alpha u_2)\frac{\alpha^2(q^2)\, d(q^2)}{q^2-\mu^2} - (\bar{u}_2'\gamma_5\tau_\alpha u_1)(\bar{u}_1'\gamma_5\tau_\alpha u_2)\frac{\alpha^2(p^2)\, d(p^2)}{p^2-\mu^2}\right\},$$

as depicted in Fig. 3. Here g is the renormalized pion-nucleon coupling constant, $q = k - k'$, $p = k + k'$, and k and k′ are the momentum four-vectors of one of the particles before and after scattering. The factors α and d represent the departure of the values of the vertex part and the meson Green's function at $q^2 < 0$ and $p^2 < 0$ from their values at $q^2 = p^2 = \mu^2$.

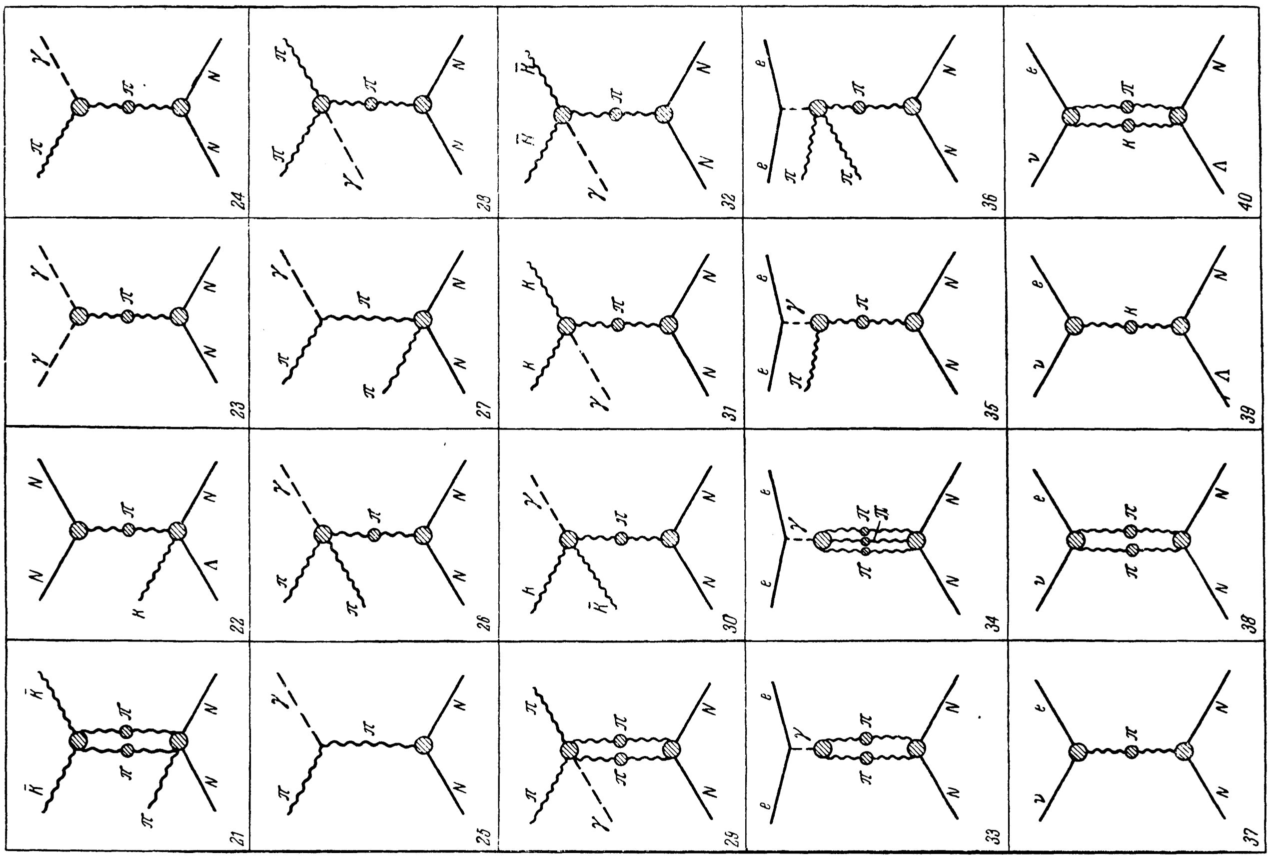
From the above expression for M we can calculate the scattering amplitudes with given values of the total angular momentum, just as we did for scalar particles. Estimates made from various models show that at energies of a few tens of Mev the F-wave nucleon-nucleon phase shifts will be accurately obtained from such a calculation.

In the non-relativistic approximation,[11] the nucleon-nucleon scattering amplitude can be written in the form

$$\alpha + \beta(\sigma_1+\sigma_2)\cdot\mathbf{n} + \gamma(\sigma_1\cdot\mathbf{n})(\sigma_2\cdot\mathbf{n}) + \delta(\sigma_1\cdot\mathbf{m})(\sigma_2\cdot\mathbf{m}) + \varepsilon(\sigma_1\cdot\mathbf{l})(\sigma_2\cdot\mathbf{l}),$$

Here $\alpha, \ldots, \epsilon$ are scalar functions of the scattering angles, and **n**, $\mathbf{l}$, **m** are axes along the directions of $\mathbf{k}\times\mathbf{k}'$, $\mathbf{k}-\mathbf{k}'$, $\mathbf{k}+\mathbf{k}'$. The amplitude M contains all the terms of this expression except the second; in the one-meson approximation $\beta = 0$. This means that spin-orbit interaction is not contained in the one-meson approximation, and must appear in the effective nucleon-nucleon potential[12] with a dis-

*Otsuki et al.[10] have calculated nucleon-nucleon scattering phase shifts with the one-meson potential.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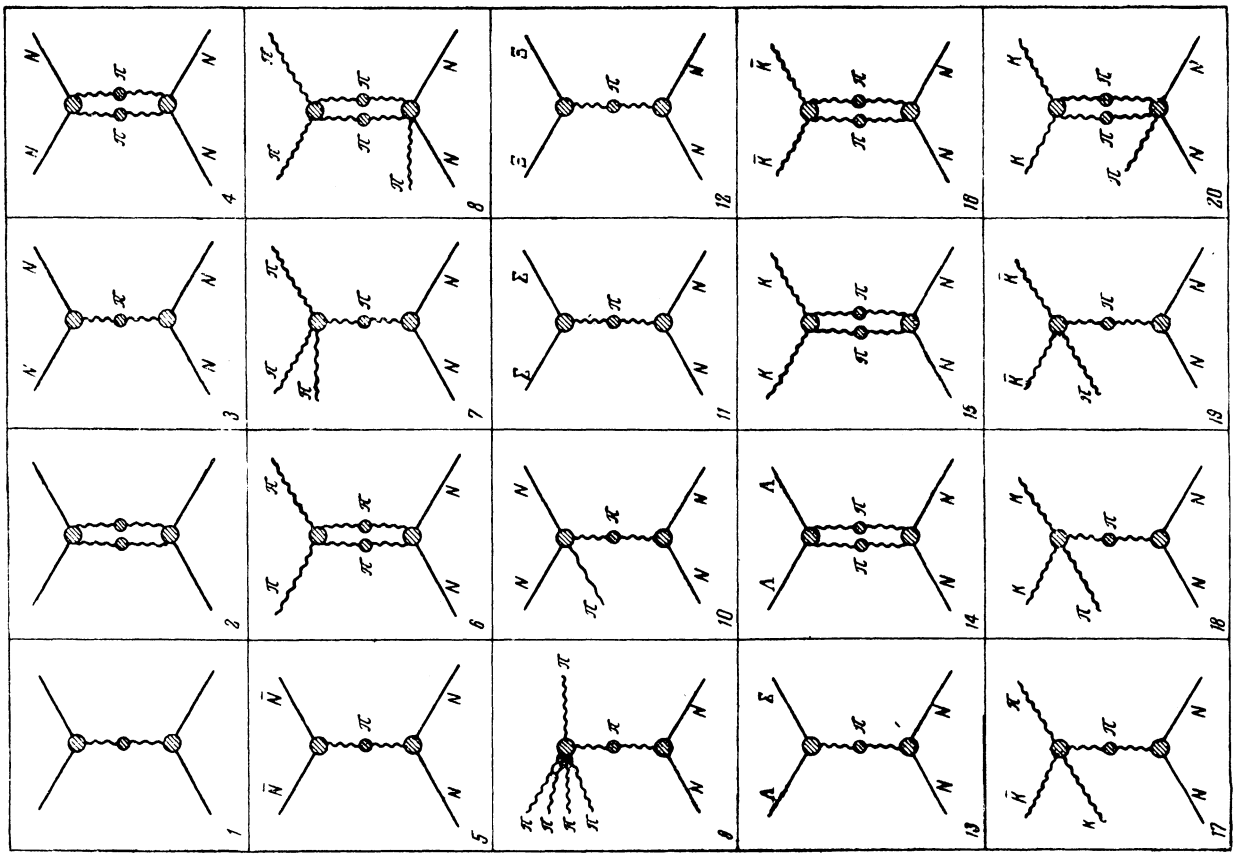
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tance-dependence at least as strong as $\exp(-2\mu r)$.

If we can separate from the experimental data a term proportional to β, and measure its contribution to an amplitude with large l, this allows us to determine the magnitude of the two-meson diagram 4. In the one-meson approximation, the spin-dependence of the nucleon-nucleon scattering amplitude is more complicated than nucleon-antinucleon or nucleon-hyperon scattering. In the latter processes there are no exchange forces and the corresponding antisymmetrization of the amplitude is absent.

Nucleon-nucleon scattering allows us to determine the quantity g. Since we may calculate several phases and compare them with different experiments, we can test the consistency of the method. Our procedure thus contains a criterion for estimating the accuracy of the result in each case, without any further theoretical estimates.

Nucleon-antinucleon scattering. Although nucleon-antinucleon scattering differs from nucleon-nucleon scattering in the size of the cross-section and in the character of the interaction, still for large l the nucleon-antinucleon scattering phases are represented by the one-meson diagram 5. Moreover, the phases for $\bar{p} + n \rightarrow \bar{p} + n$ and $\bar{n} + p \rightarrow \bar{n} + p$ must be equal and opposite to the phases for $p + p \rightarrow p + p$, in those states in which the latter process can occur. The phases for $\bar{p} + p$ and $\bar{n} + n$ must be equal and opposite to those for $p + n$ scattering, where however we must remember that the two-nucleon system possesses a symmetry which is absent in the nucleon-antinucleon system. The one-meson amplitude for antinucleon scattering has only one term, and this is of the form $\tau_1\tau_2(\sigma_1 l)(\sigma_2 l)$. Therefore the amplitudes with $T = 0$ and $T = 1$ are for large l related according to

$$a_{T=0} = -3a_{T=1}.$$

The diffraction scattering caused by inelastic nucleon-antinucleon interactions may greatly complicate the isolation of the one-meson contribution.

Pion-nucleon scattering. Pion-nucleon scattering with large l is described by the two-meson diagram 6. The corresponding one-meson diagram is forbidden, since the pion is pseudoscalar and the transition $\pi \rightarrow 2\pi$ is impossible.

Lee and Yang[13] proved that a transition in the vacuum from any even number to any odd number of pions is forbidden, by virtue of the invariance of the strong interactions under the combined operation of charge-conjugation and a rotation in isotopic spin space. Therefore in pion-nucleon scattering there are contributions only from diagrams representing the exchange of an even number of pions.

Diagram 6 shows that for large l the pion-nucleon phases are determined by factors arising from two vertices, a pion-pion vertex, and a pion-nucleon vertex in which both pions are virtual.

Pion production in pion-nucleon and nucleon-nucleon collisions. In the production of mesons by mesons $(\pi + N \rightarrow N + 2\pi)$, the pion-pion vertex which was mentioned above appears in the one-meson diagram 7. In this case the angular-momentum analysis is more complicated, since there are three particles in the final state. For a given angular momentum j_0 of the colliding pion and nucleon, we may choose a variety of angular momenta j_1 and j_2, referring respectively to the relative motion of the pions and to the motion of the nucleon in the final state. Diagram 7 describes a process occurring with a large value of j_2. An experimental determination of these amplitudes would allow us to measure pion-pion scattering. Further, by studying the dependence of the amplitude on j_1, the relative angular momentum of the two pions, one could find out in which orbital states the pions interact most strongly with each other. To carry out this program, a large number of experimental points would be needed. Possibly the simplest case to analyze would be the dependence on j_2 of scattering events in which the two pions emerge with almost equal velocities. Depending on which value of j_1 preponderates, the outgoing pions will be in a state of isotopic spin $T = 1$ if j_1 is odd, or $T = 0, 2$ if j_1 is even. The two-meson diagram 8 gives for large l a contribution small compared with that from diagram 7 but large compared with the other diagrams. For this diagram 8, the large angular momentum by which one should analyze the process is the angular momentum of one pion with respect to the center of mass of the second pion and the nucleon.

Processes of production of larger numbers of pions can be analyzed in the same way. For example, the process $\pi + N \rightarrow 4\pi + N$, (diagram 9), may give information about the vertex with six external meson lines.

The production amplitude in a nucleon-nucleon collision $N + N \rightarrow N + N + \pi$, when the two nucleons in the final state have large relative angular momentum, is described by the one-meson diagram 10. This contains, besides the pion-nucleon scattering vertex, also the $NN\pi$ vertex.

When any inelastic process is being analyzed into angular momentum states, one must work sufficiently far from the threshold for the process, since near to the threshold the angular momentum carried away by the nucleon cannot be small.

4. INTERACTION OF STRANGE PARTICLES WITH PIONS AND NUCLEONS

We now turn to processes involving strange particles. We must emphasize that our analysis is much more difficult for processes in which K particles are exchanged. The reason is that the K particle mass is more than three times greater than the pion mass. This makes it hard to separate the contribution of K particles from the contribution of diagrams in which one or two pion lines run parallel to a virtual K-particle line. We therefore consider in what follows only those processes in which the amplitudes for large l are produced by the exchange of pions.

Hyperon-nucleon scattering. Diagrams 11 and 12 represent the scattering of Σ and Ξ hyperons respectively by nucleons. Diagram 13 describes the process $\Sigma + N \rightarrow \Lambda + N$, and diagram 14 describes Λ-nucleon scattering. In the last case the one-meson diagram is forbidden, since the $\Lambda\Lambda\pi$ vertex is not isotopically invariant. For all these processes the analysis is similar to that which we gave for nucleon-antinucleon scattering. The antihyperon-nucleon scattering for large l is simply related to the hyperon scattering amplitudes. The form of the $\Sigma + N \rightarrow \Lambda + N$ amplitude depends on the relative parity of the Σ and Λ hyperons.

K-particle-nucleon scattering. In K-particle scattering, as in pion scattering, the one-meson diagram is forbidden because the pion is pseudoscalar. At large angular momenta the process is described by the two-meson diagram 15. This diagram is similar to diagram 6 and differs from it only by the replacement of a pion-pion scattering vertex by a K-particle-pion scattering vertex. It is therefore of great interest to compare directly the phases of the processes $\pi + N \rightarrow \pi + N$ and $K + N \rightarrow K + N$ for large l. For large l the scattering of $\overline{K}$ particles by nucleons (diagram 16) is also determined by a $\pi\pi KK$ vertex.

Production of K particles and pions by K particles. The interaction vertex $\pi\pi KK$, which occurred in the K-particle-nucleon scattering, appears also in many other diagrams. In particular it appears in diagram 17, describing the process $\pi + N \rightarrow K + \overline{K} + N$, although the momenta of the incoming particles are now different. This last process must be analyzed in the same way as the process $\pi + N \rightarrow \pi + \pi + N$, considering the relative motion of the K particles on the one hand and the motion of their center of mass relative to the nucleon on the other. When the latter angular momentum is large, this process is described by diagram 17. The same vertex $\pi\pi KK$ appears in the one-meson diagrams 18 and 19, describing the production of pions by K particles and by $\overline{K}$ particles: $K + N \rightarrow K + \pi + N$ and $\overline{K} + N \rightarrow \overline{K} + \pi + N$. The corresponding two-meson diagrams 20 and 21 are similar to diagram 5, and differ from it in having $\pi\pi KK$ instead of $\pi\pi\pi\pi$ at the upper vertex. The creation of K particles in nucleon-nucleon collisions, $N + N \rightarrow N + \Lambda(\Sigma) + K$, is described for large l by diagram 22, which is similar to diagram 10 describing the creation of pions.

5. INTERACTION OF PHOTONS WITH MESONS AND BARYONS

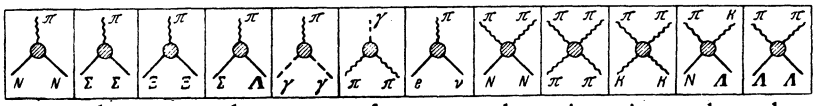
Scattering of photons by nucleons. For large l the phases for the Compton scattering $\gamma + N \rightarrow \gamma + N$ are represented by diagram 23. The relevance of this diagram was noticed already by Low.[14] The vertex $\pi\gamma\gamma$, which appears in this diagram, describes the decay of a neutrol pion into two photons. Thus a measurement of the Compton scattering of photons by nucleons for large l would be equivalent to a measurement of the lifetime of the neutral pion. The scattering phases defined by diagram 23 are equal and opposite for proton and neutron.

Photoproduction of pions. Peripheral production of charged pions is described by diagram 24. There is a generalized Ward's theorem for the interaction of bosons with real photons.[15] This has the consequence that diagrams 24 and 25 give identical effects, provided that the vertex $\gamma\pi\pi$ and the Green's function of the virtual meson are not renormalized. Thus a measurement of the amplitude for the process $\gamma + N \rightarrow \pi + N$ for large l gives a clean determination of the vertex πNN.

The contributions of diagram 25 for the proton and neutron differ only in sign. The process inverse to photoproduction, the radiative capture of a pion by a nucleon, is also described by diagram 25, reading the diagram from right to left.

Diagram 26, containing the vertex $\gamma\pi\pi\pi$, describes the photoproduction of a pair of pions ($\gamma + N \rightarrow 2\pi + N$) when the angular momentum of the pair relative to the nucleon is large. Diagram 27 will give a contribution to the same process, when the angular momentum of one meson relative to the center of mass of the other meson and the nucleon is large. The vertex $\gamma\pi\pi\pi$, which appears in diagram 26, appears again in the one-meson diagram 28, which describes the bremsstrahlung of a pion scattered by a nucleon. The bremsstrahlung at large impact parameters (large l) comes

Processes	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r
$N+N\to N+N$	1,1							2,2										3,3
$\overline{N}+N\to\overline{N}+N$	1,1							2,2										3,3
$\pi+N\to\pi+N$								2	2									
$\pi+N\to 2\pi+N$	1								1,2									2
$N+N\to N+N+\pi$	1							1,2										2
$\Sigma+N\to\Sigma+N$	1	1						2					2					
$\Xi+N\to\Xi+N$	1		1					2						2				
$\Sigma+N\to\Lambda+N$	1			1														
$\Lambda+N\to\Lambda+N$								2				2						
$\overline{\Sigma}+N\to\overline{\Sigma}+N$	1	1						2					2					
$\overline{\Sigma}+N\to\overline{\Lambda}+N$	1			1														
$K+N\to K+N$								2		2								
$\overline{K}+N\to\overline{K}+N$								2		2								
$\pi+N\to K+\overline{K}+N$	1									1								
$K+N\to K+\pi+N$	1									1								
$\overline{K}+N\to\overline{K}+\pi+N$	1									1								
$N+N\to N+\Lambda+K$	1										1							
$\gamma+N\to\gamma+N$	1				1													
$\gamma+N\rightleftarrows\pi+N$	1					1												
$\gamma+N\to 2\pi+N$	1					1		1							1			
$\gamma+N\to K+\overline{K}+N$	1															1		
$\pi+N\to\pi+\gamma+N$	1														1			
$K+N\to K+\gamma+N$	1															1		
$e+N\to e+N$						2		2							3			3
$e+N\to\nu+N$	1						1	2									2	



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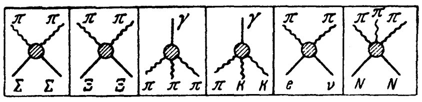

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mainly from diagram 28. Because the pion is pseudoscalar, the ordinary classical bremsstrahlung is given by the two-meson diagram 29, and becomes negligible at large l. Bremsstrahlung can also accompany charge-exchange scattering of pions.

Photoproduction of K particles. Diagram 30 describes the photoproduction of a pair $K+\overline{K}$, and contains the vertex $\gamma\pi KK$, which can be determined by the process $\gamma+N\to K+\overline{K}+N$. The same vertex appears in diagrams 31 and 32, which describe the bremsstrahlung emitted by K particles and $\overline{K}$ particles scattered by nucleons. Bremsstrahlung can also accompany charge-exchange scattering of K particles.

Scattering of electrons by nucleons, and production of pions and K particles by electrons. The same vertices which were considered above, with different values of the momenta, appear in the diagrams which describe the interaction of electrons with nucleons. The electron-nucleon scattering amplitude contains two terms, one proportional to τ_3, and the other an isotopic scalar. Goldberger[16] proved that the first of these terms is described by diagram 33, and the second by diagram 34. The vertices $\gamma\pi\pi$ and $\gamma\pi\pi\pi$, which appear in these diagrams with virtual photons having $k^2\neq 0$, also appear in diagrams 35 and 36, which describe the production of pions by electrons, $e+N\to e+\pi+N$ and $e+N\to e+2\pi+N$.

The diagram for the process $e+N\to e+N+K+\overline{K}$ is similar to diagram 36, and differs from it only in the replacement of the vertex $\gamma\pi\pi\pi$ by $\gamma\pi KK$.

Interactions involving neutrinos. The process $e^-+p\to n+\nu$, involving the weak beta-decay interaction, can occur in electron-proton collisions. If the angular distribution of the neutrons from this process could be analyzed into angular momentum states, then for large angular momentum the main contribution comes from diagrams 37 and 38. These diagrams contain the vertices $\pi e\nu$[17] and $\pi\pi e\nu$,[18] the former carrying an axial vector interaction, the latter a vector interaction. When high-energy electrons collide with protons, hyperon production, for example $e^- + p \rightarrow \Lambda + \nu$, is also possible. Diagrams 39 and 40 describe this process and contain the vertices $Ke\nu$ and $K\pi e\nu$. Like the vertices $\pi e\nu$ and $\pi\pi e\nu$, the vertices $Ke\nu$ and $K\pi e\nu$ can be measured independently by studying the decays $\pi \rightarrow e + \nu$, $\pi \rightarrow \pi + e + \nu$, $K \rightarrow e + \nu$, $K \rightarrow \pi + e + \nu$. All the remarks in this section apply equally to collisions of μ mesons with nucleons.

6. CORRELATION OF THE VARIOUS PROCESSES

We have enumerated in the table all the processes considered above, and the strongly interacting vertices which can be determined by studying these processes. The numbers 1, 2, 3 denote the number of virtual mesons linking the vertices in the corresponding diagram. Thus for example, in the row corresponding to the process $\pi + N \rightarrow 2\pi + N$, the number 1 stands in the column corresponding to the vertices $NN\pi$ and $\pi\pi\pi\pi$. These two vertices appear in diagram 7, and are linked by one virtual pion. In the same row, the numbers 2 correspond to the diagram in which the vertices $\pi\pi\pi\pi$ and $\pi\pi\pi NN$ are linked by two mesons. By examining the one-meson diagrams we shall show how the vertices determined by the various processes can be correlated with one another.

The vertex $NN\pi$, the renormalized coupling constant, can be determined independently from the processes $N + N \rightarrow N + N$, $\overline{N} + N \rightarrow \overline{N} + N$, $\gamma + N \rightarrow \pi + N$. These processes do not contain other unknown vertices. Knowing the vertex $NN\pi$, we can determine the following vertices:

$\pi\pi\pi\pi$	from the process	$\pi + N \rightarrow 2\pi + N$,
$NN\pi\pi$	"	$N + N \rightarrow N + N + \pi$,
$\Sigma\Sigma\pi$	"	$\Sigma + N \rightarrow \Sigma + N$,
$\Sigma\Lambda\pi$		$\Sigma + N \rightarrow \Lambda + N$,
$\Xi\Xi\pi$	"	$\Xi + N \rightarrow \Xi + N$,
$\pi\pi KK$	independently from the three different processes	$\pi + N \rightarrow K + \overline{K} + N$, $K + N \rightarrow K + \pi + N$, $\overline{K} + N \rightarrow \overline{K} + \pi + N$,
$\pi NK\Lambda$	from the process	$N + N \rightarrow N + \Lambda + K$,
$\pi\gamma\gamma$	"	$\gamma + N \rightarrow \gamma + N$,
$\gamma\pi\pi\pi$	"	$\gamma + N \rightarrow 2\pi + N$, $\pi + N \rightarrow \pi + N + \gamma$,
$\gamma\pi KK$	"	$\gamma + N \rightarrow \overline{K} + K + N$, $K + N \rightarrow \gamma + K + N$, $\overline{K} + N \rightarrow \gamma + \overline{K} + N$.

When we determine a vertex from a one-meson diagram, we obtain its value for the special case in which one of the lines incident at the vertex corresponds to a virtual pion.

In a similar way, the two-meson diagrams can determine values of the vertices involving two virtual pions. In many cases the same vertex can be determined from both one-meson and two-meson or three-meson diagrams, for example the vertex $\gamma\pi\pi\pi$. This allows us to find the value of the vertex when the momenta of the incident mesons are in the nonphysical region. It is to be emphasized that the same vertex, when determined from one-meson and two-meson diagrams, appears with different momenta for the incident particles.

We have discussed the amplitudes for various processes with orbital angular momentum $l \gg 1$. Experimental measurement of amplitudes with very large l is of course impossible in practice. However, in case of nucleon-nucleon scattering, we can show that at low energy the experimental amplitude can be described by a peripheral one-meson interaction already when $l = 3$. The shape of the experimental angular distribution of the nucleons in the process $\pi + N \rightarrow 2\pi + N$ indicates that the peripheral one-meson interaction can be very large, and can dominate the amplitude for quite small values of l. Thus an experimental study of the processes enumerated above, and a determination of the data necessary for their analysis, may in many cases be possible in the comparatively near future.

Relations, similar to those which exist between the various processes involving nucleons, will also hold for processes in which a deuteron or a more complicated nucleus replaces the nucleon. In particular, by comparing the scattering of nucleons and hyperons by nuclei, we can find the relation between the pion coupling-constants with nucleons and with hyperons.

We emphasize, in conclusion, that all our arguments about peripheral interactions do not depend upon the nature of the strong interactions at short distances. In particular, the arguments are unchanged, whether the pion is an elementary or a compound particle.

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