

MEASUREMENT OF THE ELECTRICAL RESISTANCE OF METALS IN A MAGNETIC FIELD
AS A METHOD OF INVESTIGATING THE FERMI SURFACE

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Results are given of an investigation of the polar diagrams of resistance for single crystal specimens of Sn, Pb, Tl, Ga, and Na. It is found that for Sn and Pb (as for Au and Cu) the variation of resistance with magnetic field changes from a quadratic relation to complete saturation, as the angle between the field and the crystallographic axes changes. A strong anisotropy of resistance in a magnetic field is also found for Tl and Ga. These results are ascribed to the existence of open Fermi surfaces in these metals.

It has recently been proposed by Lifshitz, Azbel' and Kaganov¹ and also Chambers² that the iso-energetic surfaces for the conduction electrons in metals (the Fermi surfaces) may be complicated topologically, with open sections. Fermi surfaces can be constructed on the basis of data from measurements on the de Haas-van Alphen effect, the anomalous skin effect, and cyclotron resonance.³ These methods, however, cannot give unambiguous results in cases where one must study directions near to open sections.

It follows from the work of Lifshitz et al. that for open Fermi surfaces it is possible to have a quadratic increase in resistance for some directions of the field, and saturation for others. We showed previously⁴ that the increase of the resistance of gold and copper in a magnetic field, $r(H)$, changes considerably with the angle between the field and the crystallographic axes of the specimen: the resistance either increases quadratically with field in the direction corresponding to a sharp maximum on the polar diagram or reaches saturation in the direction of a minimum.

The linear growth law for the resistance of polycrystalline specimens in a magnetic field (Kapitza's

law⁵), which has so far no theoretical explanation, can probably be considered⁴ as the result of an averaging of the different laws of increase of $r(H)$, observed in single crystals.

We note also that according to the theory of Lifshitz et al., the sharp maxima in the polar diagrams of gold and copper can be attributed to the existence of open Fermi surfaces in these metals.

The appropriate theoretical treatment, explaining these results, was recently given by Lifshitz and Peschanskii.⁶

It seemed of interest to study in detail the angular dependence of $r(H)$ for single crystals of other metals, and we measured specimens of Sn, Pb, Tl, Ga, and Na, as detailed in the table, at 4.2 and 1.5°K.

Figure 1 shows the polar diagrams of the variation of resistance at constant H for specimens Sn-I, Pb-I and Tl. All measurements refer to the magnetic field perpendicular to the measuring current J . Figure 2 shows the variation of $\Delta r_H/r_0 = [r(H) - r_0]/r_0$ (where r_0 is the resistance in zero field) in Sn-I and Pb-I for two fixed directions of the field.

It can be seen from the figures that in the direc-

Specimen	Sn-I	Sn-II	Sn-III	Sn-IV	Sn-V	Pb-I	Pb-II	Tl	Ga	Na
Direction of axis of specimen	[001]	[010]	[110]	[111]	[011]	[111]	[110]	—	—	—
Dimensions, mm: length*	22.7	22.1	24.25	18.05	21.28	11.75	6.26	11.61	10.82	—
diameter	1.95	2.04	1.63	2.07	1.53	1.01	0.74	1.97	0.68	—
$r_{4.2^\circ K}/r_{300^\circ K}$	$8.9 \cdot 10^{-5}$	$10.5 \cdot 10^{-5}$	$11.4 \cdot 10^{-5}$	$9.4 \cdot 10^{-5}$	$4.4 \cdot 10^{-5}$	$7.2 \cdot 10^{-5}$	$14 \cdot 10^{-5}$	$2.6 \cdot 10^{-4}$	$3.6 \cdot 10^{-4}$	$2.4 \cdot 10^{-4}$

*Distance between potential leads.

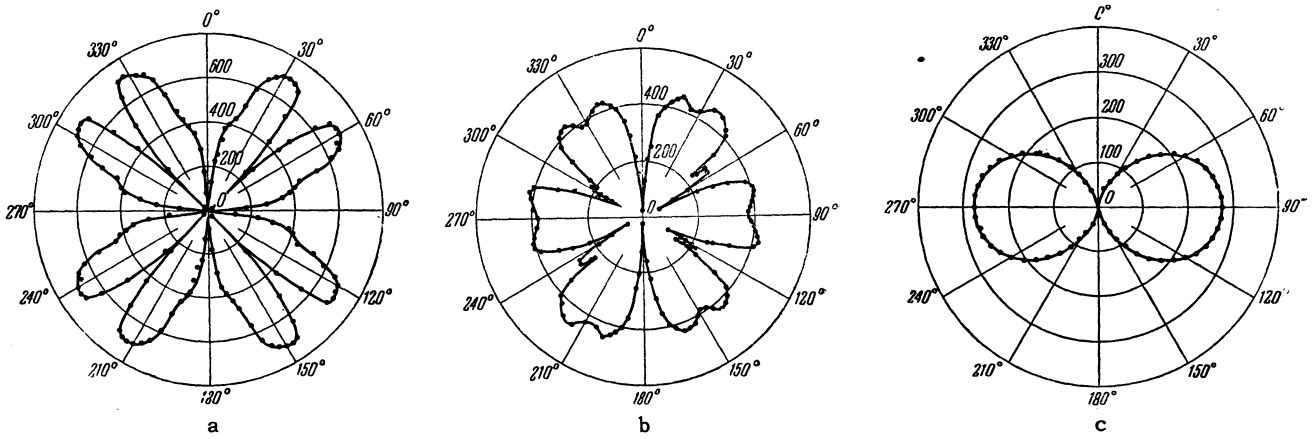


FIG. 1. Polar diagrams of the resistance change $\Delta r_H/r_0$ for constant H as a function of the angle, φ , between the crystallographic axes of the specimen and the direction of the magnetic field. $T = 4.2^\circ\text{K}$. a – specimen Sn-I, $H = 23$ kOe; b – Pb-I, $H = 22.3$ kOe; c – Tl, $H = 22.3$ kOe.

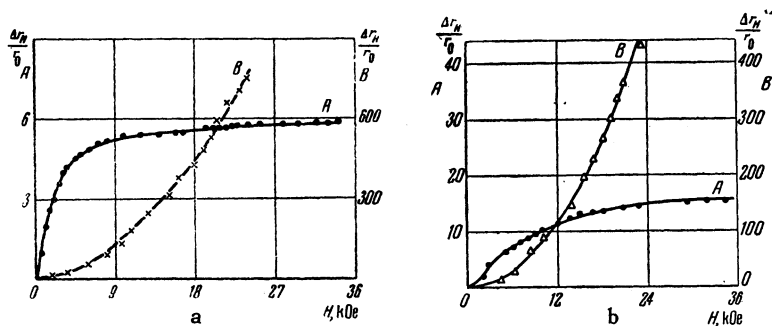


FIG. 2. The resistance change, $\Delta r_H/r_0$, in a magnetic field for fixed angle φ . $T = 4.2^\circ\text{K}$. a – specimen Sn-I; b – Pb-I. Curve A – measurements at the minimum ($\varphi = 0^\circ$), curves B – at the maximum ($\varphi = 30^\circ$) of the corresponding polar diagrams.

tion of the minima of the polar diagrams there is complete saturation, while in the direction of the maxima the resistance continuously increases according to a quadratic relation. A similar behavior was found with Sn-III.

We also found considerable anisotropy in the resistance of Tl and Ga specimens in a magnetic field, and the law of increase, $\Delta r_H/r_0 = AH^n$, for Tl changes at different crystallographic directions from quadratic to linear. For Ga the exponent n , decreases from 2 to 1.6. There appears to be no anisotropy in the resistance of Na in a magnetic field. $r(H)$ is very weakly dependent on the field and tends to saturation ($\Delta r_H/r_0 \approx 1$ in a field $H \approx 55,000$ Oe).

On analyzing our results and those of other authors⁷ we can conclude that Au, Cu, Sn, Pb, Tl (and possibly Ga) have open Fermi surfaces, while Al, In, and Na, which show saturation and a very weak anisotropy of $r(H)$, have closed surfaces.

According to the data of Lüthi and Olsen,⁸ the resistance of Al increases again on increasing the magnetic field beyond the saturation region. It is not impossible that such an increase may be a consequence of the openness of the Fermi surface. If the dependence of resistance on field in large fields can be represented by the relation

$$\Delta r_H/r_0 = A(H/H_0)^2 + B,$$

(where the first term refers to a narrow region of open sections, and the second to closed sections) then for $0 < A \ll B$, when all possible directions are averaged, the resistance will increase again with field after saturation for $H \gg H_0$ [i.e., $A(H/H_0)^2 > B$]. Measurements on aluminum single crystals, which we intend to make soon, will enable us to test these ideas.*

The absence of saturation in the direction of a minimum of the polar diagram for Tl and Ga is probably a consequence of the averaging of $r(H)$ over a range of angles close to the minimum. Such an averaging can result from an imperfection of the single crystal or from inhomogeneities of the field. However, strong anisotropy of resistance in a field and different variations of $r(H)$ in the maximum and minimum of the polar diagram are a sufficiently convincing indication of the existence of open Fermi surfaces.

*We should point out that measurements of $r(H)$ on polycrystalline specimens in extremely strong magnetic fields will enable us to show the existence of open sections for all metals in which $r(H)$ shows little anisotropy and rapidly reaches saturation for $H > H_0$ (H_0 is determined from the condition $1/R = 1$, where l is the mean free path and R the radius of curvature of the trajectory of an electron in the magnetic field).

On the basis of the data obtained for tin we conclude that the directions of the open sections coincide with the [010], [110] and [001] directions.

Our results on one specimen of copper are not in conflict with the form of Fermi surface proposed by Pippard.⁹ Here the directions of the open sections coincide with the main diagonals. It is most likely that gold has a Fermi surface analogous to copper.

The results obtained give us reason to consider that, in addition to the three well known methods for studying the Fermi surface, the measurement of the variation of $r(H)$ for various orientations of single crystals in a magnetic field can serve as a simple and convenient means of showing up open sections.

It is a pleasure to express our thanks to Academician P. L. Kapitza for his constant interest in this work.

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