ON THE SHAPE OF ALPHA-ACTIVE NUCLEI

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The shape of a heavy nucleus can be determined from the rate of alpha decay to successive levels of the main rotational band of the daughter nucleus. The quantities α_2 and α_4 which are the coefficients in the expansion of the nuclear shape in Legendre polynomials, are computed. The calculations are performed for four even and three odd nuclei. The results of the calculations agree satisfactorily with each other and indicate that the contribution of the term $\alpha_4 P_4(\cos\vartheta)$ to the nuclear shape is significant.

SEVERAL recent theoretical papers[1-5] are devoted to the calculation of the intensity of the rate of α decay to levels belonging in the same rotational band. Calculations show that the rates of decay are very sensitive to the shape of the nucleus. It is therefore natural to use the experimental rates of α decay to determine not only the dimensions of the atomic nuclei, but also their configurations. In the present article we use for the calculation the intensities of α transitions only to levels that belonged to the main rotational bands, since the theory has been derived precisely for these transitions. By main rotational band we understand the band of the daughter nucleus, beginning with the level that is characterized by the same momentum I and the same parity as the main level of the parent nucleus. The α transitions to these levels are not connected with a change in K (the projection of I on the symmetry axis of the nucleus) and are favored transitions.

It was shown in reference 5 that the intensities of the α transitions to the levels of the main rotational bands are given by the formula

$$W_I = 4\pi^2 \frac{\hbar k_I}{\mu} \sum_{l=I-I_0}^{I+I_0} |C^{I\,I_0}_{I_0 I_0;\, l0}|^2 \left| \int_1^1 \frac{R(\vartheta)\chi(\vartheta)}{\varphi_{Il}[R(\vartheta)]} Y_{l0}(\vartheta)\, d(\cos\vartheta) \right|^2, \tag{1}$$

where W_I is the probability of α decay to a level with spin I (I_0 is the spin of the parent nucleus), k_I is the wave number of the α particles whose emission leads to the excitation of this level, μ is the mass of the α particle, $C^{IM}_{I_1M_1,I_2M_2}$ are the Clebsch-Gordan coefficients, l is the angular momentum carried away by the α particle, $R = R(\vartheta)$ is the equation for the surface of the nucleus (outside of which the nuclear forces are assumed to vanish) in a coordinate system fixed at the nucleus, $\chi(\vartheta)$ is the wave function of the α particle on the surface of the nucleus, $Y_{l_0}(\vartheta)$ is a spherical harmonic, and $\varphi_{Il}(r)$ is the radial eigenfunction of an α particle with a momentum l (for a transition to a level with spin I).

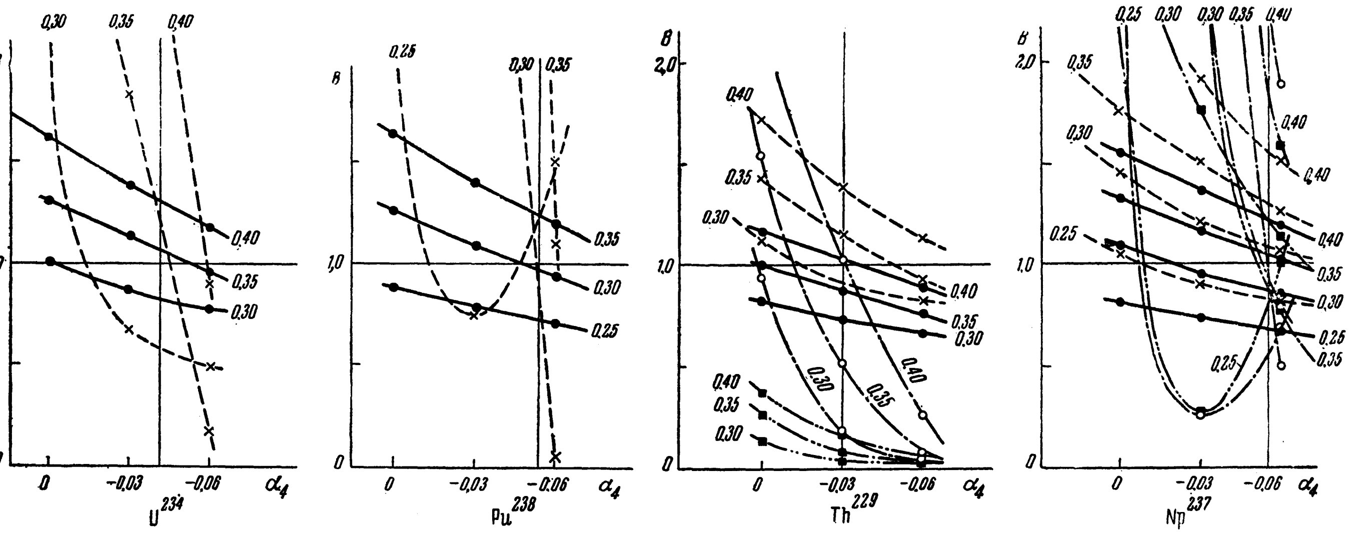
The theoretical problem is to find a nuclear shape $R(\vartheta)$ compatible with the experimental values of W_I. It is natural to assume in the calculation that the unknown function $\chi(\vartheta)$ is constant on the surface of the nucleus.

The greatest difficulties are involved in the calculation of $\varphi_{Il}(r)$. Exact calculations of $\varphi_{Il}(r)$ have been carried out thus far only for even nuclei, and necessitate the use of high-speed electronics computers. It was shown in reference 5 that the use of an approximate function $\varphi^{(0)}_{Il}(r)$ in Eq. (1), at least in the case of even nuclei, produces no substantial error in the calculation of the probability of the α decay to the rotational levels. The function $\varphi^{(0)}_{Il}(r)$ is the radial eigenfunction, calculated without allowance for the quadrupole term (in the higher multipoles) in the Coulomb potential of the nonspherical nucleus, and is determined with sufficient accuracy from a simple analytical formula. Although a comparison of the exact functions $\varphi_{Il}(r)$ and the approximate ones $\varphi^{(0)}_{Il}(r)$ has never been performed for odd nuclei, one can hardly expect the odd nuclei to behave in this respect substantially differently from the even ones. This has caused the authors of reference 5 to propose the substitution of $\varphi^{(0)}_{Il}(r)$ for $\varphi_{Il}(r)$ in the formulas, as is indeed done in the present paper.

In the calculations we have computed the first two terms that describe the deviation of the nuclear shape from spherical

$$R(\vartheta) = r_0[1 + \alpha_2 P_2(\cos\vartheta) + \alpha_4 P_4(\cos\vartheta)], \tag{2}$$

where P_2 and P_4 are Legendre polynomials, and



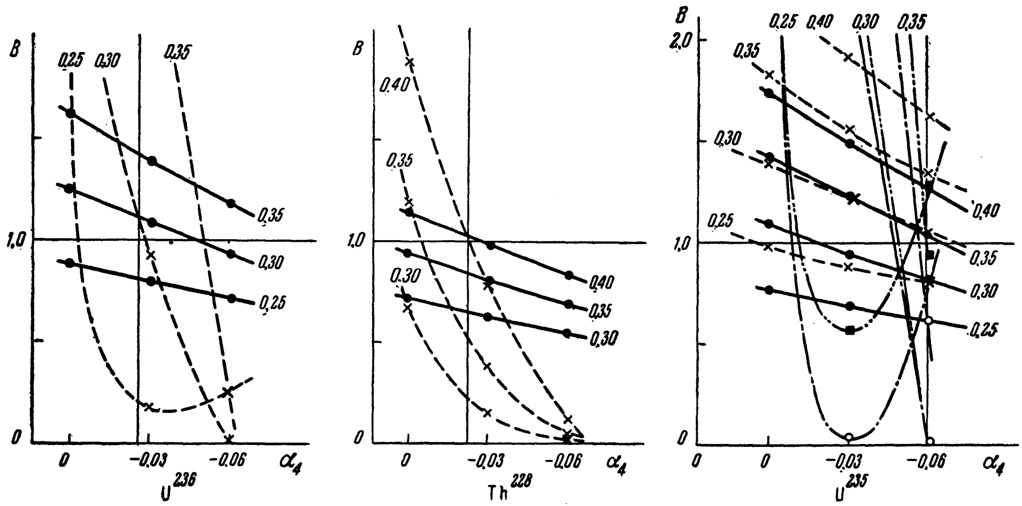


Fig. 1. Effect of the shape of the nucleus on the probability of α decay in transitions to excited levels of the main rotational bands. The solid and dotted lines join points corresponding to α decay to a level I_0+1 and I_0+2 in odd nuclei and to a level +2 and +4 in even-even nuclei; the dash-single-dot lines and the dash-double-dot lines correspond to decays to levels I_0+3 and I_0+4 in odd nuclei. The numbers along the curves designate the assumed value of u^2. The vertical lines correspond to the best value of α_4. The symbols for the daughter nuclei are below the curved.

α_2 and α_4 are coefficients that must be determined. We have preferred to use instead of α_2 the quantity $u^2 = (a^2 - b^2)/a^2 \approx 2\Delta R/R$ (a is the major semiaxis of the nucleus and b is the minor one).

Figure 1 shows the results of the calculations, performed for four even-even and three odd nuclei. The calculations were made under differing assumptions concerning the shape of the nucleus: u^2 assumed the values 0.25, 0.30, 0.35, and 0.40 while α_4 was assigned values 0, −0.03, and −0.06. The optimum values of u^2 and α_4 were found by interpolation.

To determine the size of the nucleus we used the rate of α decay to the lower level of the band. In determining the shape, the most useful quantities were the ratios W_I/W_{I_0} of the α-decay rates to the level with spin I and to the lowest level of the band. Figure 1 shows the quotients of these ratios divided by the experimental values, as functions of α_4, i.e.,

$$B = (W_I / W_{I_0})_{\text{calc}} / (W_I / W_{I_0})_{\text{exptl}}.$$

The table contains the the theoretical curves for the optimum values of u^2 and α_4 and a comparison of the calculated and experimental values of W_I/W_{I_0} for these parameters. Since the nuclear spin in the transition to a higher level increases successively, the levels are identified in the table by the spin.

As can be seen from the table, the intensities of the α transitions at $l_{\min} \leq 4$ agree well with the theory and lead to an almost identical shape for all the considered nuclei. The values listed in the table show a striking discrepancy for the levels I_0+4 and I_0+5 in Th^{229}. The rotational nature of these levels is not established with full assurance, however, since it merely follows from the fact that the energies of the levels fit well the rotational formula. The experiments specially set up by E. F. Tretyakov to investigate the internal-conversion electrons that accompany the α decay of U^{233} have confirmed the rotational nature of the I_0+3 level, but have contributed nothing so far for the I_0+4 and I_0+5 levels.

The calculated rate of α decay to the levels of

Nucleus	u^2	α_2	α_4	B: $l_{min}=2$, I_0+1	B: $l_{min}=2$, I_0+2	B: $l_{min}=4$, I_0+3	B: $l_{min}=4$, I_0+4	B: $l_{min}=6$, I_0+5	B: $l_{min}=6$, I_0+6	Q_0 (barns)	Q_4 (bn²)	$h^2/2J_{en}$ (kev)	J_{en} (10^{-46} g-cm²)	J_{sol} (10^{-46} g-cm²)	J_{irr} (10^{-46} g-cm²)
U^{235}*	0.34	0.161	−0.058	1	1.3	1	1			13.7	−3.6	5.29	0.66	1.18	0.034
Np^{237}*	0.34	0.160	−0.056	1	1.2	1	1			14.5	−3.6	6.20	0.57	1.20	0.035
Th^{229}	0.39	0.177	−0.030	1	1.3	1	0.1**	$2\cdot10^{-4}$**		14.7	+2.0	6.20	0.57	1.16	0.044
Pu^{238}	0.31	0.138	−0.052		1		1		4	12.3	−3.9	7.5	0.47	1.21	0.027
U^{236}	0.28	0.119	−0.026		1		1		26	10.9	−0.7	7.5	0.47	1.29	0.025
U^{234}	0.33	0.148	−0.041		1		1		23	11.7	−0.9	7.3	0.48	1.10	0.030
Th^{228}	0.39	0.173	−0.025		1		1		29	14.1	−3.0	9.6	0.37	1.13	0.043

*** All the nuclear characteristics listed in the table pertain to the lower level of the main rotational band, which does not coincide with the ground state of the nucleus.**

**** The rotational nature of this level follows only from the excitation energy and is not confirmed by other data.**

the $I_0 + 6$ type is in substantial disagreement with experiment. This discrepancy must, however, not be given any particular significance. The intensity of α radiation with $l = 6$ depends both on u^2 and α_4, and also on the term α_6 which has not been calculated. The contributions of the terms that depend on u^2 and on α_4 to the probability amplitude are almost equal in magnitude and opposite in sign when $l = 6$. The accuracy of calculations under such cancellation is naturally quite unsatisfactory. The discrepancy noted is apparently merely an example of the roughness of the theory.

Knowledge of the shape of the nucleus makes it possible to calculate its multipole moments. Column 11 of the table lists the values of the intrinsic quadrupole moment Q_0, calculated under the assumption that the protons are uniformly distributed. The calculations were carried out using the approximate formula

$$Q_0 = {}^6/_5\, Z r_0^2 \alpha_2 (1 + {}^4/_7\, \alpha_2), \qquad (3)$$

which follows directly from the usual definition

$$Q_0 = \frac{2Z}{V} \int r^2 P_2 (\cos \vartheta)\, d\tau$$

with allowance for Eq. (2). Direct measurements of Q_0 with which to compare our results are unfortunately lacking for these nuclei. The measurements available for U^{235} and Np^{237} pertain to the ground states of these nuclei and not to the lower level of the main rotational bands, for which our calculations were made. We give below for comparison data by Newton,[6] which show that our calculations give the correct order of magnitude for the quadrupole moments:

Nucleus:	U^{233}	U^{235}	U^{238}	Np^{237}	Pu^{238}
Q_0 (barns):	13.7	10.1	10.3	9.0	9.2

That the theoretical values of Q_0 exceed somewhat the experimental ones is apparently the result of our using in our calculations the nuclear radii obtained from the α decay (from the intensity of the zero level). These radii are known to be some 20% greater than the electrical radii of the nuclei measured by scattering of fast electrons; this should yield, approximately, a 40% increase in the calculated values over the true ones.

Column 12 of the table gives the values of the 2^4-pole moment, calculated from the formula

$$Q_4 = \frac{Z}{V} \int r^4 P_4 (\cos \vartheta)\, d\tau \simeq \frac{1}{3} Z r_0^4 (\alpha_4 + 1.54\, \alpha_2^2). \qquad (4)$$

In formula (4) the negative term α_4 partially offsets the positive term $1.54\,\alpha_2^2$, thus reducing substantially the accuracy of the results. Direct experimental measurements of Q_4 for heavy nuclei are unfortunately still not available.

Column 13 of the table gives the values of $h^2/2J_{en}$ calculated from the excitation energy of the rotational levels, while the next column gives the values of the moment of inertia J_{en} thus determined. In the last two columns of the table are listed the values of the moment of inertia, calcu-

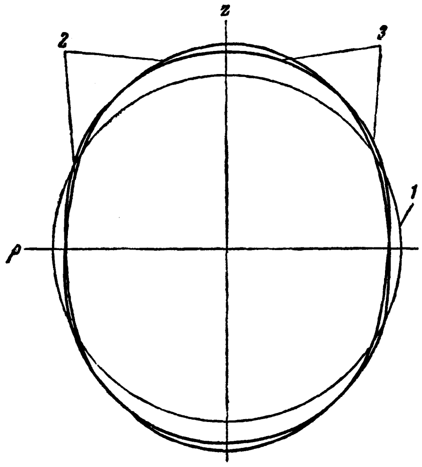


FIG. 2. Shape of U^{234} nucleus. 1 – circle, 2 – without allowance for α_4, 3 – with allowance for α_4.

lated from the model of the "solid nucleus," J_{sol}, and for the "drop nucleus" with irrotational motion of the liquid, J_{irr}.

As expected, J_{sol} is several (2 or 3) times greater the experimentally observed value of J_{en}. We cannot establish here a connection between the variations of J_{sol} and J_{en} from nucleus to nucleus. J_{irr} is found to be substantially smaller than J_{en}, so that the "irrotational" model is apparently too rough.

In conclusion we show in Fig. 2 the shape of the U^{234} nucleus, calculated with and without the term α_4. For comparison the figure shows a circle representing a fully spherical nucleus. As follows from the figure, the contribution of the term $\alpha_4 P_4$ is far from small.

We are glad to make use of this opportunity to thank G. M. Adel'son-Vel'skiĭ and A. P. Brizgal for the mathematical calculations.

[1] J. O. Rasmussen and B. Segal, Phys. Rev. **103**, 1298 (1956).

[2] V. M. Strutinskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 411 (1956) and **32**, 1412 (1957), Soviet Phys. JETP **3**, 450 (1956) and **5**, 1150 (1957).

[3] V. G. Nosov, Dokl. Akad. Nauk SSSR **112**, 414 (1957), Soviet Phys. "Doklady" **2**, 48 (1957); J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 226 (1957), Soviet Phys. JETP **6**, 176 (1958).

[4] Gol'din, Adel'son-Vel'skiĭ, Birzgal, Piliya, and Ter-Martirosyan, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 184 (1958), Soviet Phys. JETP **8**, 127 (1959).

[5] J. O. Newton, Nucl. Phys. **3**, 345 (1957); Preprint, 1958.

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86