ON THE WEAK-INTERACTION TYPES POSSIBLE IN THE SCHEME OF FEYNMAN AND GELL-MANN

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It is shown that the β-decay current is uniquely determined by the requirement that the divergence of its vector part vanish; along with this we have the fact that the current responsible for the decay of hyperons is not conserved. If this were not so the Lagrangian of the strong interactions would have a symmetry that would be in contradiction with experimental results on the associated production of strange particles. As a consequence of this the reaction $\Sigma \rightarrow \Lambda + e + \nu$ can go only through the A-type interaction, and the result of the experiment proposed in reference 6 must be negative.

To explain the fact that the vector coupling constant for β decay does not undergo renormalization, despite the existence of the strong interactions, Feynmann and Gell-Mann[1] have suggested that the vector part of the β-decay Lagrangian involves only currents that are conserved in the presence of strong interaction (cf. also reference 2). In this connection the question arises as to the extent to which the current responsible for β decay is uniquely determined by the requirement that its divergence vanish. Furthermore it is not obvious a priori whether the current responsible for the decay of hyperons is conserved. At the beginning one can only assert that the existence of a current with a vanishing divergence means the conservation of a certain vector, and consequently means an additional symmetry of the interaction Lagrangian. Pais[3] has shown that certain classes of such symmetries lead to contradictions with the experiments on associated production of strange particles. It will be shown here that from the results of Pais it follows that the β-decay current is uniquely determined and the current responsible for the decay of hyperons is not conserved.

The strong-interaction Lagrangian can be written in the form (boldface letters denote isovectors, i.e., vectors in the isotopic space)

$$L = \{g_1(\overline{N}i\gamma_5\boldsymbol{\tau}N) + g_2(\overline{\Lambda}i\gamma_5\boldsymbol{\Sigma} + \overline{\boldsymbol{\Sigma}}i\gamma_5\Lambda) - ig_3[\overline{\boldsymbol{\Sigma}}i\gamma_5\boldsymbol{\Sigma}] + g_4(\overline{\Xi}i\gamma_5\boldsymbol{\tau}\Xi)\}\boldsymbol{\pi} + \{g_5(\overline{N}i\gamma_5\Lambda)K + g_6(\overline{N}i\gamma_5\boldsymbol{\tau\Sigma})K + g_7K(\overline{\Lambda}i\gamma_5\tau_2\Xi) + g_8K(\overline{\boldsymbol{\Sigma}}i\gamma_5\tau_2\boldsymbol{\tau}\Xi) + \text{Herm. conj.}\}. \quad (1)$$

The only difference from the notations of the paper of d'Espagnat, Prentki, and Salam[4] is that the sign of g_2 is changed, whereas in Pais's notation[3] $g_i = G_i$ $(i = 1, 2, 3, 4)$, $g_5 = F_1$, $g_6 = F_2$, $g_7 = -iF_3$, $g_8 = -iF_4$. The assumption that K has a parity different from that of Λ and Σ is immaterial.

It is not hard to see that in virtue of the conservation laws for baryon number, strangeness, and charge and the isotopic invariance of the theory with the Lagrangian (1), we have for arbitrary values of the g_j conservation of the isoscalar currents

$$J_\mu^S = (\overline{N}i\gamma_\mu N) + (\overline{\Lambda}i\gamma_\mu\Lambda) + (\overline{\boldsymbol{\Sigma}}i\gamma_\mu\boldsymbol{\Sigma}) + (\overline{\Xi}i\gamma_\mu\Xi), \quad (2)$$

$$J^\mu = -(\overline{\Lambda}i\gamma_\mu\Lambda) - (\overline{\boldsymbol{\Sigma}}i\gamma_\mu\boldsymbol{\Sigma}) - 2(\overline{\Xi}i\gamma_\mu\Xi) + i\left(K^*\frac{\partial K}{\partial x_\mu} - \frac{\partial K^*}{\partial x_\mu}K\right), \quad (3)$$

and also the isovector current

$$\mathbf{J}_\mu^V = (\overline{N}i\gamma_\mu\boldsymbol{\tau}N) - 2i[\overline{\boldsymbol{\Sigma}}i\gamma_\mu\boldsymbol{\Sigma}] + (\overline{\Xi}i\gamma_\mu\boldsymbol{\tau}\Xi) + 2\left[\boldsymbol{\pi}\frac{\partial\boldsymbol{\pi}}{\partial x_\mu}\right] + i\left(K^*\boldsymbol{\tau}\frac{\partial K}{\partial x_\mu} - \frac{\partial K^*}{\partial x_\mu}\boldsymbol{\tau}K\right). \quad (4)$$

Furthermore the Lagrangian for the electromagnetic interactions has the form $\frac{1}{2}(J_\mu^S + j_\mu^G + (J_\mu^V)_3) \times A_\mu$.

If the bare masses of the Λ and the Σ are equal, and the coupling constants satisfy the relations

$$g_2 = \varepsilon g_3, \quad g_5 = \varepsilon g_6, \quad g_7 = \varepsilon g_8; \quad m_\Lambda = m_\Sigma; \quad \varepsilon = \pm 1, \quad (5)$$

then in addition to (4) we can construct another conserved current isovector

$$\mathbf{J}_\mu^P = \varepsilon(\overline{\Lambda}i\gamma_\mu\boldsymbol{\Sigma} + \overline{\boldsymbol{\Sigma}}i\gamma_\mu\Lambda) + i[\overline{\boldsymbol{\Sigma}}i\gamma_\mu\boldsymbol{\Sigma}] - i\left(K^*\boldsymbol{\tau}\frac{\partial K}{\partial x_\mu} - \frac{\partial K^*}{\partial x_\mu}\boldsymbol{\tau}K\right). \quad (6)$$

The equations in (5) agree exactly with the conditions obtained in reference 4 for the invariance of the theory with respect to rotations in a four-dimensional Euclidean isotopic space. On the other hand, Pais[3] has shown that from precisely these conditions there follow consequences that are in contradiction with experiment. In fact, for $\epsilon = 1$* the vanishing of the divergence of the component $(J_\mu^P)_3$ corresponds to the conservation of the operator

$$\hat{N}_Y - \hat{N}_Z + \hat{N}_{K^+} - \hat{N}_{K^0}, \tag{7}$$

where

$$Y^+ \equiv \Sigma^+, \quad Y^0 \equiv (\Lambda^0 - \Sigma^0)/\sqrt{2};$$

$$Z^0 \equiv (\Lambda^0 + \Sigma^0)/\sqrt{2}, \quad Z^- \equiv \Sigma^-, \tag{8}$$

and $\hat{N}$ is the operator for a number of particles ($\hat{N}_Y = \hat{N}_{Y^+} + \hat{N}_{Y^0}$, $\hat{N}_Z = \hat{N}_{Z^0} + \hat{N}_{Z^-}$). Together with the vanishing of the divergence of the expression (3), i.e., the conservation of

$$-\hat{N}_Y - \hat{N}_Z - 2\hat{N}_\Xi + \hat{N}_{K^+} + \hat{N}_{K^0}, \tag{9}$$

Eq. (7) at once means also the conservation of the operators introduced by Pais,

$$\hat{S}_1 \equiv -\hat{N}_Z - \hat{N}_\Xi + \hat{N}_{K^+}, \quad \hat{S}_2 \equiv -\hat{N}_Y - \hat{N}_\Xi + \hat{N}_{K^0}, \tag{10}$$

which, for example, directly forbids the reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+ \equiv Y^+ + K^+$, which has been observed experimentally. Therefore we can regard it as established that the conditions (5) are not satisfied, and that there is no possibility of including J_μ^P and J_μ^V in the β-decay Lagrangian.

Direct calculation shows that it is impossible to construct any other conserved isovectors; consequently the β-decay current $(J_\mu^V)_+ = (J_\mu^V)_1 + i(J_\mu^V)_2$ is uniquely determined.

Since furthermore there is no term $(\bar{\Lambda} i\gamma_\mu \Sigma)$ in Eq. (4), the decays $\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu$, $\Sigma^- \rightarrow \Lambda^0 + e^- + \tilde{\nu}$ can occur only owing to the renormalized axial-vector interaction. For this reason there should be no polarization of the Λ in such reactions if the Σ are not polarized, and the Λ is emitted asymmetrically for polarized Σ.

Feynman and Gell-Mann suggested that the decay of hyperons is due to the existence of currents with change of the strangeness. Such currents will have half-integral isotopic spin, since from the quantities of the theory one can construct only expressions of the types $(\bar{\Lambda} i\gamma_\mu N)$ or $(\bar{\Lambda} i\gamma_\mu \tau N)$. It is easy to see tha, generally speaking, such currents are not conserved. Only under the conditions

$$g_1 = -\varepsilon g_2 = -g_3 = g_4 = \varepsilon'' g_5 = \varepsilon\varepsilon'' g_6 = \varepsilon'\varepsilon'' g_7 = \varepsilon\varepsilon'\varepsilon'' g_8,$$
$$m_N = m_\Lambda = m_\Sigma = m_\Xi; \quad m_K = m_\pi; \quad \varepsilon, \varepsilon', \varepsilon'' = \pm 1 \tag{11}$$

*If $\varepsilon = -1$, N_{Y^0} and N_{Z^0} are interchanged in Eqs. (7)–(10).

which are stronger than Eq. (5), do we get conservation of the current

$$J_\mu^\psi = (\bar{\Lambda} i\gamma_\mu \boldsymbol{\tau} N) + \varepsilon\,[(\bar{\boldsymbol{\Sigma}} i\gamma_\mu N) - i\,[\bar{\boldsymbol{\Sigma}} i\gamma_\mu \boldsymbol{\tau} N]] - \varepsilon'(\bar{\Xi} i\gamma_\mu \boldsymbol{\tau}\tau_2 \Lambda)$$
$$- \varepsilon\varepsilon'\,[(\bar{\Xi} i\gamma_\mu \tau_2 \boldsymbol{\Sigma}) - i\,[\bar{\Xi} i\gamma_\mu \boldsymbol{\tau}\tau_2 \boldsymbol{\Sigma}]] + 2i\varepsilon''\Big(K\frac{\partial \boldsymbol{\pi}}{\partial x_\mu} - \frac{\partial K}{\partial x_\mu}\boldsymbol{\pi}\Big), \tag{12}$$

and also of the currents

$$-\frac{i}{2}[\boldsymbol{\tau} J_\mu^\psi] = (\bar{\Lambda} i\gamma_\mu \boldsymbol{\tau} N) - \varepsilon(\bar{\boldsymbol{\Sigma}} i\gamma_\mu N) - \varepsilon'(\bar{\Xi} i\gamma_\mu \boldsymbol{\tau}\tau_2 \Lambda)$$
$$+ \varepsilon\varepsilon'(\bar{\Xi} i\gamma_\mu \tau_2 \boldsymbol{\Sigma}) + \varepsilon''\Big(\Big[\boldsymbol{\tau} K\frac{\partial \boldsymbol{\pi}}{\partial x_\mu}\Big] - \Big[\boldsymbol{\tau}\frac{\partial K}{\partial x_\mu}\boldsymbol{\pi}\Big]\Big), \tag{13}$$

$$\boldsymbol{\tau} J_\mu^\psi = -\frac{i}{2}\boldsymbol{\tau}[\boldsymbol{\tau} J_\mu^\psi] = 3(\bar{\Lambda} i\gamma_\mu N) - \varepsilon(\bar{\boldsymbol{\Sigma}} i\gamma_\mu \boldsymbol{\tau} N) + 3\varepsilon'(\bar{\Xi} i\gamma_\mu \tau_2 \Lambda)$$
$$- \varepsilon\varepsilon'(\bar{\Xi} i\gamma_\mu \boldsymbol{\tau}\tau_2 \boldsymbol{\Sigma}) + 2i\varepsilon''\Big(\boldsymbol{\tau} K\frac{\partial \boldsymbol{\pi}}{\partial x_\mu} - \boldsymbol{\tau}\frac{\partial K}{\partial x_\mu}\boldsymbol{\pi}\Big). \tag{14}$$

If there exist also other mesons besides π and K, other currents besides the expressions (12)—(14) can be conserved. For example, if one introduces into the theory a pseudoscalar ρ meson having zero charge and isotopic spin and interacting with the baryons through the additional Lagrangian terms

$$L' = g_\rho\,[(\bar{N} i\gamma_5 N) - (\bar{\Lambda} i\gamma_5 \Lambda) - (\bar{\boldsymbol{\Sigma}} i\gamma_5 \boldsymbol{\Sigma}) + (\bar{\Xi} i\gamma_5 \Xi)]\,\rho \tag{15}$$

and impose the conditions

$$g_1 = \varepsilon g_2 = g_3 = g_4; \quad g_5 = \varepsilon g_6 = \varepsilon' g_7 = \varepsilon\varepsilon' g_8 = \varepsilon'' g_\rho;$$
$$m_N = m_\Lambda = m_\Sigma = m_\Xi; \quad m_\rho = m_K; \quad \varepsilon, \varepsilon', \varepsilon'' = \pm 1, \tag{16}$$

we find that the current

$$J_\mu^\chi = (\bar{\Lambda} i\gamma_\mu N) + \varepsilon(\bar{\boldsymbol{\Sigma}} i\gamma_\mu \boldsymbol{\tau} N) + \varepsilon'(\bar{\Xi} i\gamma_\mu \tau_2 \Lambda)$$
$$+ \varepsilon\varepsilon'[\bar{\Xi} i\gamma_\mu \boldsymbol{\tau}\tau_2 \boldsymbol{\Sigma}) + 2i\varepsilon''\Big(\frac{\partial\rho}{\partial x_\mu}K - \rho\frac{\partial K}{\partial x_\mu}\Big). \tag{17}$$

is conserved.

The introduction of the ρ meson has the result that $\partial J_\mu^\psi/\partial x_\mu \neq 0$, so that the simultaneous conservation of J_μ^χ and the currents (12)—(14) is impossible. It can be noted that if one uses in Eq. (1) the same spatial parities for Λ, Σ, and K, the current (17) can be conserved as before (the ρ meson would then be scalar), and the currents (12)—(14) are then not conserved. The conditions (11), however, and also the conditions (16), contain the relations (5) which are in contradiction with experiment. Consequently both sets of conditions are incapable of fulfillment, the current responsible for the decay of hyperons is not conserved, and, accordingly, the decay coupling constants are subject to renormalization. This explains the fact that hyperon decays with lepton emission has so far not been observed, whereas in the absence of the renormalization several percent of the total number of Λ and Σ decays should be accompanied by the emission of electrons or μ mesons (cf. e.g., reference 5). In addition, because of the nonconservation of the pion-neutrino decay current the correlation in the $K_{\mu 3}$ decay cannot be described by the expression obtained in reference 6.

Up to now we have been speaking only about vector currents. Since in ordinary β decay the experimental ratio of the axial-vector and vector coupling constants[7] is $|C_A/C_V| = 1.14$, i.e., close enough to unity, the question of the conservation of axial-vector currents is also of interest. The construction of such currents is impossible, however, in the theory with the interaction (1) unless one introduces additional particles. If, for example, we introduce a σ meson that is scalar in the ordinary and isotopic spaces[8] with the interaction

$$L' = g_\sigma [(\overline{N}N) + (\overline{\Lambda}\Lambda) + (\overline{\mathbf{\Sigma}}\mathbf{\Sigma}) + (\overline{\Xi}\Xi)]\,\sigma, \qquad (18)$$

then with conditions on the bare masses and coupling constants given by

$$g_1 = g_2 = g_3 = g_4 = g_\sigma \equiv g;$$

$$m_N = m_\Lambda = m_\Sigma = m_\Xi \equiv m; \qquad m_\sigma = m_\pi = 0, \qquad (19)$$

and only apart from effects of the K-meson interactions, we have conservation of the current

$$(\overline{N} i\gamma_5\gamma_\mu \boldsymbol{\tau} N) + (\overline{\Lambda} i\gamma_5\gamma_\mu \mathbf{\Sigma}) + (\overline{\mathbf{\Sigma}} i\gamma_5\gamma_\mu \Lambda) - i\,[\overline{\mathbf{\Sigma}} i\gamma_5\gamma_\mu \mathbf{\Sigma}]$$

$$+ (\overline{\Xi} i\gamma_5\gamma_\mu \boldsymbol{\tau}\Xi) + 2\left(\frac{\partial\sigma}{\partial x_\mu}\boldsymbol{\pi} - \sigma\frac{\partial\boldsymbol{\pi}}{\partial x_\mu}\right) - \frac{2m}{g}\frac{\partial\boldsymbol{\pi}}{\partial x_\mu}. \qquad (20)$$

The conditions (19), which differ from the Gell-Mann scheme[9] by the addition of the σ meson, do not impose the limitations (5) on the K-meson interaction constants, and therefore do not contradict experiment. Nevertheless the addition (20) to the vector current J_μ^V would mean a violation of the assumption[1] that the weak-interaction Lagrangian involves only fermion operators of definite spirality.

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] S. S. Gershteĭn and Ya. B. Zel'dovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 698 (1955), Soviet Phys. JETP **2**, 576 (1956).

[3] A. Pais, Phys. Rev. **110**, 574 (1958).

[4] d'Espagnat, Prentki, and Salam, Nucl. Phys. **3**, 446 (1957).

[5] V. M. Shekhter, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 458 (1958), Soviet Phys. JETP **8**, 316 (1959).

[6] Weinberg, Marshak, Okubo, Sudarshan, and Teutsch, Phys. Rev. Let. **1**, 25 (1958).

[7] A. Winther and O. Kofoed-Hansen, Dansk. Mat.-Fys. Medd. (in press).

[8] J. C. Polkinghorne, Nuovo cimento **8**, 179 (1958).

[9] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

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