TWO CASES OF UNSTABLE COMBUSTION

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A criterion has been derived for instability of the flame zone of a plane detonation wave; this criterion determines the conditions for the occurrence of a spin detonation. Conditions have been derived for the occurrence of single-headed spin detonations, and for oscillations of the combustion front in a detonation wave. It is shown that the criterion for unstable combustion which is derived for detonation waves is also applicable to forced combustion chambers.

The instability of the flame zone plane is considered as a source of high-frequency oscillations of the flame. The order of magnitude of the fundamental frequency of these oscillations has been determined, and the conditions for the appearance of higher harmonics have been found. The origin of resonance vibrations in a furnace is explained qualitatively, and an estimate is made of the maximum pressure during the oscillations.

1. INSTABILITY OF THE FLAME FRONT PLANE IN A DETONATION WAVE

LET us consider a stationary detonation wave. Gas, originally in a state represented by point O in the P-V diagram (Fig. 1), is rapidly compressed (by the shock wave) along the dynamic adiabatic to the state A. The high density and temperature of the gas in state A gives rise to chemical combustion reactions in the gas, whose final state is the Jouguet point J in the P-V diagram — the point where the Hugoniot adiabatic H is tangent to the line OA. At the point J, the sound velocity a is just equal to the velocity of the detonation front with respect to the compressed gas (the Jouguet condition):

$$a = D - W_J ,$$

where D is the velocity of detonation and W_J is the velocity of the gas in the pressure wave.

The chemical reactions going on in the detonation wave require a certain amount of time. Therefore the plane at which the combustion is completed does not coincide geometrically with the density discontinuity at A. Figure 2 shows schematically the distribution of pressure behind the front in a detonation wave. From A to J the pressure decreases in accordance with the rate at which heat is being generated. All the intermediate states on curve AJ in Fig. 2 are represented by points on the straight line AJ in Fig. 1; this follows from the requirement that all parts of the burning zone travel with the same velocity. The distribution of the states and velocities of the gas behind a detona-

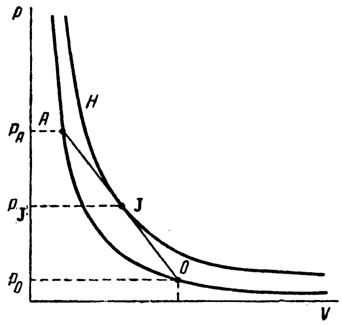

FIG. 1

tion front has been dealt with exhaustively by Ya. B. Zel'dovich.[1]

The time required for a chemical reaction depends, as a rule, on the temperature and pressure according to the relation

$$\tau \sim p^{-n} e^{E/RT}, \qquad (1)$$

where E is the activation energy for the reaction, R is the gas constant, and n is a constant.

To simplify the discussion, it will be assumed in what follows that during the induction period τ there is no reaction at all, but that all the heating occurs instantaneously at the time τ. The distribution of pressure in the detonation wave for this approximation is shown in Fig. 3. The temperature

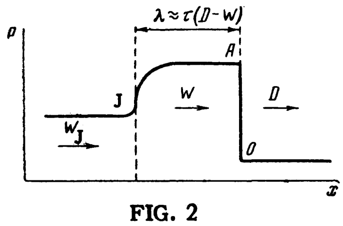

FIG. 2

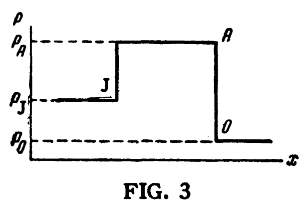

FIG. 3

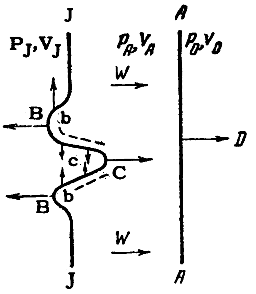

FIG. 4

and density distributions for the gas in the wave are analogous.

In Fig. 4 the compression shock front of the detonation wave is shown as a plane AA, perpendicular to the plane of the paper, and the heating front is represented by the surface JJ. On the heating front there is a perturbation BCB. This will arise if the flame induction period is lengthened in the regions marked b, and shortened in the region c — as a result of the non-uniformity of the burning mixture entering the flame zone, for example. To the right of this wavy surface JBCBJ, the gas is compressed to the pressure P_A, while to the left the pressure is equal to the lower value P_J. The values of P_A and P_J can be found from the P-V diagram, Fig. 1.

During the first few moments after the formation of a perturbation, the Jouguet conditions are fulfilled at points B, C, and B; along the direction of wave motion, the state and velocity of the gas remain the same as they were before the perturbation. However, in the direction perpendicular to the direction of propagation, the gas is set into motion and the pressure discontinuity that has been built up across the combustion surface begins to break up. The gas at the locations bb, being at the higher pressure, will expand adiabatically, and rarefaction waves will move into these regions, as shown by the dashed lines. The gas expanding out of bb will flow into the region c, compressing the gas in it. On the plane J-J, the pressure will be smoothed out to some value intermediate between P_A and P_J. In the neighborhood of the points B, B the gas b expands to the pressure P_J, while near point C the gas c is compressed to the pressure P_A.

In the regions b the detonation wave expands as if it were in a divergent cone. The compression at c, on the other hand, is equivalent to the concentration of a wave in a converging cone. To the right of the region c the induction period is decreased, because of the overcompression of the detonation wave; while in the regions bb the induction period is lengthened because of the reduced pressure. The original perturbation BCB spreads out, as shown by the arrows in Fig. 4, and the ignition front loses its stability.

It should be noted that, when the gas in the regions bb begins to expand and the gas at c is compressed, the Jouguet conditions at the points BB and C are destroyed and waves of pressure and rarefaction travel up to the plane AA and distort the shock-wave front. For a strong wave in a diatomic gas, the lateral extension of the disturbance along JJ is approximately three times as wide as the length of the burning zone by the time the initial disturbance has reached the shock front AA.

Even if an initial disturbance of the type BCB is suddenly formed, the burning zone will become unstable only if the perturbation grows sufficiently rapidly; otherwise the gas, in spite of its initial heterogeneity, will be able to escape from the vicinity of the ignition front, and the perturbation BCB will disappear.

The relationship can be formulated quantitatively as follows: if the adiabatic expansion of the gas from zone b into zone c, which lowers the temperature of the gas, increases the induction period of the reaction by an amount of the same order of magnitude as the induction period itself, or more, then any initial curvature of the front will be increased, and a plane front will not be stable. Reasoning in this way, and neglecting the dependence of the induction period on pressure or density, one is led to the expression

$$(d\tau/dT)|_{T_A}(T - T_A) \geqslant \tau, \qquad (2)$$

where T is the temperature of the unburned gas in region b after its expansion.

From (1) and (2) we can obtain the criterion for the instability of a plane flame zone in a detonation wave:

$$(E/RT_A)(1 - T/T_A) \geqslant 1. \qquad (3)$$

It has already been mentioned that near the point B the gas b expands to the pressure P_J of point J (Figs. 1 and 2), while near the point C the burning gas is compressed to the pressure P_A. Formula (3) can therefore be re-written as

$$(E/RT_A)[1 - (P_J/P_A)^{(\gamma-1)/\gamma}] \geqslant 1. \qquad (4)$$

The quantities occurring in equation (4) can be calculated readily for any concrete case.

As an example, let us estimate the value of expression (4) for a detonation wave in a diatomic gas with an activation energy of 40,000 cal/mole, propagating with a velocity of 1700 m/sec (M = 5).

For an initial temperature of $T_0 = 290°K$, the temperature of the unburned gas behind the pressure front will be equal to 1650°K. In the detonation wave, the ratio of the pressure of the burned gas to the pressure of the unburned gas, P_J/P_A, is close to one-half, so that

$$\frac{E}{RT_A}\left[1-\left(\frac{P_J}{P_A}\right)^{(\gamma-1)/\gamma}\right] = \frac{40\,000}{2\cdot 1650}\left[1-(1/2)^{0.4/1.4}\right] = 2.2 > 1.$$

The loss of stability of the flame front, as we have said, leads to disturbances in the shock wave front AA. This results in the formation of wrinkles in the compression front. Details of the surface irregularities and the formation of spin detonations can be found in references 1 to 4.

If the width of the detonation wave front λ (the distance from the plane AA to the plane JJ in Figs. 2, 3, and 4) is small is comparison with the tube diameter d, then a large number of wrinkles may form in the compression front, over the whole cross-section of the tube. A total of $(d/3\lambda)^2$ wringles may be formed in the whole area. Because there is no a priori tendency for the disturbances to move in any one direction, they will propagate randomly in different directions with respect to the shock front surface. They will mutually interfere and will ignite the gas, particularly in the regions of constructive interference. As a result, the detonation wave front will take on the appearance of a pulsating brush. As the diameter of the tube decreases, or as the chemical reaction time increases (i.e., as the composition or the pressure of the mixture approach the detonation limit) the tube cross-section will contain fewer and fewer irregularities, until finally only one remains — a classical single-headed spin detonation.

Intermediate between the single-headed spin detonation and the brush-like detonation front are the cases of spin detonation with two, three, and more heads.

From the above considerations it is possible to derive the conditions for the formation of a single-headed spin detonation in a diatomic gas: the reaction zone thickness must be of the order of one-third of the tube diameter, or greater.

$$3\lambda/d = 3\tau(D-W)/d \gg 1, \qquad (5)$$

where W is the gas velocity in the density discontinuity, and d is the tube diameter.

Instability of the ignition zone and a "brushlike" fine structure of the detonation front can also occur in the detonation of condensed explosives. Spin detonations are not observed in condensed explosives, apparently because the conditions for a single-headed spin are very close to the conditions for the failure of detonation near the limiting diameter for an unconfined charge. It is possible that spin

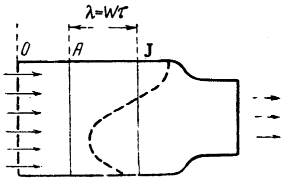

FIG. 5

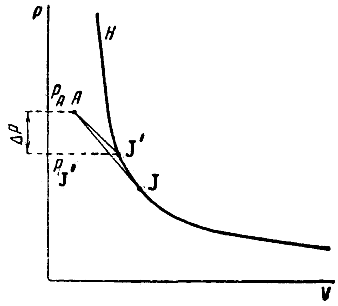

FIG. 6

might be observed in a small-diameter charge encased in a very strong confining tube.

Some years ago Apin[5] put forward a "jet" mechanism for the propagation of detonations in condensed explosives. He, too, proposed a brush-like structure for the front of his jet detonations. Apin ascribed the formation of the "brush" to the projection of gas jets into the unburned material. The jets were assumed to be formed either by microscopic non-uniformities in the material or by microcavitation in blisters or bubbles.

It is probably more natural to explain the "brush-like" structure of the burning or detonation zone of condensed explosives by the instability of the plane ignition zone, and the formation of many irregularities in the wave front, which interfere with each other and ignite the unreacted explosive, just as in the case of gaseous detonations.

To conclude this section, it should be mentioned that the instability of a plane ignition front in a detonation wave was first considered by the author[4] in 1949. Zel'dovich and Kompaneets,[1] speaking of the conditions for the existence of spin detonations, remarked that during the ignition of a gas the burning should be faster in a convex compression wave than in a plane wave. On the other hand, a convex front will not be concentrated as the detonation proceeds. The existence of a spin is therefore possible only if the magnitude of E/RT is sufficiently large. The critical value of this quantity is not to be found in the cited references.

2. INSTABILITY OF THE IGNITION ZONE AS THE SOURCE OF HIGH-FREQUENCY VIBRATIONS IN FORCED COMBUSTION CHAMBERS

Forced combustion chambers, e.g., in rockets, may (very schematically, of course) be represented as shown in Fig. 5. In the region O-A the components are mixed and preheated. In the space from A to J the mixture reacts by self-ignition; at the plane J the combustion reaction has been completed.

The combustion process in a furnace is hydrodynamically and thermodynamically analogous to the combustion in a detonation wave. The state of the gas at the plane A can be described by some point A in Fig. 1. Its state when it reaches J is given by the point J in the same figure, lying below A on a branch of the Hugoniot adiabatic, or (depending upon the burning rate of the mixture) by some other point J′ (Fig. 6) which lies above J but below A. The gas thermodynamics of furnace combustion have been considered in detail by Troshin.[6]

For the case in which a preheated mixture is subsequently ignited in a furnace, the reaction time (the induction period for ignition) is found from an expression of the type (1). Because of the similarity between combustion in a furnace and combustion in a detonation, we may carry over to a furnace all the considerations and conclusions of the preceding section about the instability of a plane ignition front.

We can write down the same criterion for the instability of a plane ignition zone:

$$\frac{E}{RT}\left[1-\left(\frac{P_A-\Delta P}{P_A}\right)^{(\gamma-1)/\gamma}\right]\geqslant 1. \quad (6)$$

Here T is the temperature of the heated mixture, P is the pressure drop along the furnace due to the burning. In Fig. 6, if the state of the combustion products is characterized by the point J′, the pressure drop $P = P_A - P_{J'}$.

Just as in the previous case, the loss of stability in the plane front leads to the formation of a pulsating combustion zone. In view of the fact that in furnaces the velocity of the unburned mixture is always considerably less than the speed of sound, the width of the disturbances in the combustion front will be of the order of twice the width of the burning zone. We may therefore have

$$(d/2\lambda)^2 = (d/2\tau W)^2 \quad (7)$$

disturbances of the combustion front over the cross-sectional area of the furnace.

We can now estimate the frequency of the pulsations. Since this frequency will depend on the ratio (7), it is useful to begin by estimating the lowest possible frequency. This is the case where the expression (7) is numerically equal to unity, i.e., when the length of the burning zone is about equal to, or greater than, half the diameter of the chamber. This can be considered to be the fundamental frequency of the furnace oscillations.

The time required to develop a single pulse is of the order of $\lambda/a = \tau W/a$, since the instability is propagated with the speed of sound, a, because the disturbances which lengthen or shorten the induction period are propagated with the speed of sound. However, the recovery of the perturbed front varies with the rate at which the unignited gas replenishes the combustion zone:

$$t_{rec} \approx \lambda/W = \tau W/W \approx \tau,$$

i.e., the time taken for the front to recover from a perturbation is of the order of the ignition induction period, τ. Since W is always small compared to a, and the time interval $\tau W/a$ can be neglected, the order of magnitude of the fundamental frequency is

$$\nu_0 = 1/\tau. \quad (8)$$

If the length of the combustion zone $\lambda = \tau W$ is noticeably less than the chamber diameter, and if we consider the pulsations to be random and unsynchronized over the cross section of the chamber, then the upper limit of frequency can be obtained by multiplying (7) and (8):

$$\nu_{max} = \frac{1}{\tau}\left(\frac{d}{2\tau W}\right)^2 = \frac{1}{\tau}\left(\frac{d}{2\lambda}\right)^2. \quad (9)$$

It is easy to estimate the order of magnitude of these frequencies. For a light gasoline, for example, which we may take to be composed entirely of heptane, in a furnace where the mixture is preheated to 700°K, the induction period is

$$\tau = 10^{-14.4}e^{38000/2\cdot 700} = 2.5\cdot 10^{-3}\ \text{sec}.$$

The fundamental frequency is therefore 400 cycles per second. The length of the burning zone for this case, if the velocity of the unburned gases is assumed to be 50 m/sec, is equal to $\lambda = 50 \times 2.5 \times 10^{-3} = 0.125$ m. In a chamber one meter in diameter the maximum attainable frequency is:

$$\nu_{max} = \frac{1}{2.5\cdot 10^{-3}}\left(\frac{1}{2\cdot 0.125}\right)^2 = 6400\ \text{cps}.$$

Each pulse in the combusion front produces a pressure wave in the chamber with an amplitude of the order of ΔP (Fig. 6). This wave will spread out from the location of the disturbance, decaying rapidly. In themselves, waves such as these apparently constitute no danger to the furnace. However, if the oscillation of the flame front resonates with one of the proper frequencies of the gas in the furnace (either longitudinal or transverse) dangerous vibrational forces may be set up in the chamber, with a large pressure drop across the wave fronts.

The resonant frequencies are of the order of magnitude of

$$\nu_p = k\bar{a}/L, \quad (10)$$

where a is a quantity close to the mean speed of sound in the chamber, L is a characteristic dimension (length or diameter) of the chamber, and k is the order of the harmonic.

It can be seen from a consideration of the instability condition (6) that resonance could produce dangerous results. Expression (6) shows that the flame will begin to oscillate if a rarefaction wave with an amplitude of the order of ΔP would noticeably increase the induction period for combustion; a compressive wave of the same strength would decrease the induction period. Consequently, compressive waves with amplitude ΔP moving through the hot gas will accelerate its combustion. As they pass through the unburned mixture, they "ignite" it in their paths, picking up the energy of combustion as they go, and so increasing in amplitude. This provides a mechanism for increasing the amplitude of resonance oscillations.

We can now attempt to estimate the order of magnitude of the pressure drop attainable in a compression wave during resonance oscillation in a combustion chamber. For this purpose we may use a crude, almost unreasonable model: waves travelling in the region of hot, unignited gas will be assumed to compress the mixture at almost constant volume, while waves in the remaining parts of the chamber will be treated as simple decaying shock waves.

The maximum pressure drop in a travelling wave can be approximated (very roughly, of course) by

$$\Delta P_{\max} = P_A (\pi - 1)\, W/a, \qquad (11)$$

where π is the increase in pressure which would result from combustion at constant volume. One of the crudest approximations made in the derivation of equation (11) is the assumption that the flow rate of gas out of the chamber, and consequently the pressure in the chamber, are not affected by the oscillation. A number of other factors are also ignored.

Using formula (11) with $\pi = 10$, for the same numerical example used above, one obtains

$$\Delta P_{\max} = P_A (10 - 1)\, {}^{50}/_{600} = 0.75\ P_A \sim P_A.$$

During resonance oscillations, therefore, the pressure drop in the compression waves can, if the chamber is strong enough, reach a value of the same order of magnitude as the total chamber pressure itself.

It must be emphasized again that throughout this entire second section, we have been considering the combustion very schematically, and that the conclusions are of a qualitative rather than a quantitative nature.

In conclusion, it must be remembered that qualitative evidence for the role which the induction period for combustion plays in setting up high-frequency oscillations in combustion chambers has already been presented. See, for instance, the work of Crocco which is mentioned in reference 7.

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Translated by D. C. West
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