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ON THE ELECTROMAGNETIC RADII OF THE LIGHTEST NUCLEI IN THE GROUND AND LOWEST EXCITED STATES

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The radii of nuclei of mass numbers 5 to 8 are calculated for the ground and excited states from experimental data on isotopic multiplets and on the scattering of high energy electrons by nuclei. It is shown that for all the nuclei with the exception of the doublet He^5-Li^5 the radii increase monotonically with energy. One of the levels of Be^8 is identified with greater precision.

1. INTRODUCTION

At the present time nuclear radii have been measured only very roughly. The formula $R = r_0A^{1/3}$ is based on the model of a uniformly charged sphere and is very inaccurate, particularly in the region of the light nuclei.

Recently a number of sufficiently accurate experimental data has been accumulated which enable us to determine the mean square radius for the charge distribution in the lightest nuclei and to draw certain conclusions with respect to the dependence of the electromagnetic radii on the energy of excitation of the nuclei. For this purpose the present paper makes use of data on charge multiplets. The energy of isotopically similar levels must differ only by the Coulomb interaction. The nuclear forces are assumed to have isotopic invariance. The energy levels of the lightest nuclei have been measured with a sufficient degree of accuracy, many of them have been identified with respect to their angular momentum, parity and isotopic spin. We also make use of data on the scattering of fast electrons by nuclei which enable us to determine in a different way the mean square radii of the charge distribution for the ground states of certain light nuclei.

There exists also a third independent type of experiments: the energy levels of mesic atoms, but these experiments are not yet accurate and allow us only to conclude that the electromagnetic radii of the nuclei are considerably smaller than the values obtained from experiments on the scattering of nucleons by nuclei.

2. CALCULATIONS

The wave function for the nucleus is constructed in accordance with the shell model using jj coupling from oscillator single nucleon wavefunctions taking the Pauli exclusion principle into account and incorporating the total angular momentum J and its component M along the z axis, the isotopic spin T (as an approximate quantum number) and its component T_ζ, determined by the nuclear charge Z.

Such a function may be constructed explicitly if the symmetrization is carried out by means of Young's tableau and utilizing vector addition rules, but this is very complicated. This may be avoided if one uses the technique of fractional parentage coefficients. This enables us to reduce the calculation of the matrix elements of the Coulomb energy (and of any arbitrary pair operator) with respect to the nuclear wavefunctions to the calculation of matrix elements with respect to wavefunctions of a pair of particles. The latter may be easily computed since both the functions and the operators are known. The corresponding formulas have been derived by utilizing methods of second quantization as proposed by Balashov, Tumanov, and Shirokov.[1]

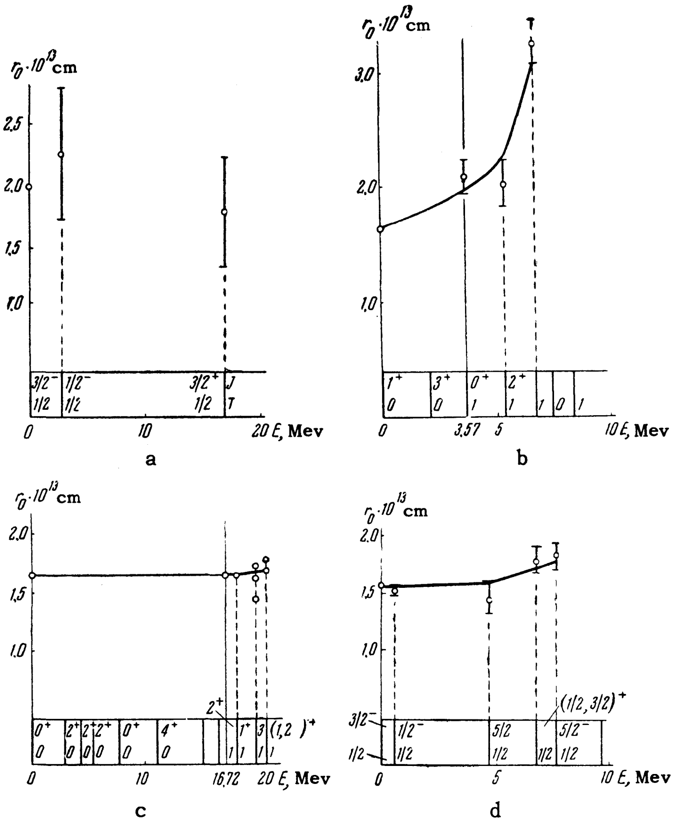
Making use of these formulas we have calculated the matrix elements of the Coulomb energy for all of the nuclear states up to Be^8.

The experimentally obtained binding energies have been taken from the review by Wapstra,[2] while the energies of the excited states of nuclei have been taken from the review article of Ajzenberg and Lauritsen.[3]

Theoretically the difference in Coulomb energies turns out to be equal to $ke^2/\sqrt{2\pi}r_0$, where k is a certain numerical factor, while r_0 is the parameter which determines the single nucleon wavefunctions, which are equal to

$$P_n(r/r_0)\exp(-r^2/2r_0^2),$$

where $P_n(r/r_0)$ are the generalized Laguerre polynomials; $r_0 = \sqrt{\hbar/m\omega}$, where m is the nucleon mass, while ω is the characteristic oscillator frequency; r_0 is proportional to the mean square radius a of the charge distribution in the nucleus. Similar calculations for the ground states of nuclei have been carried out under certain simplifying assumptions by Carlson and Talmi.[4] They assumed that there are no neutrons in the unfilled shell while the protons are in the state with the lowest "seniority," i.e., they have been paired as far as possible into pairs with zero angular momentum.



a – $He^5 - Li^5$; b – $He^6 - Li^6 - Be^6$, E – excitation energy in Li^6; c – $Li^8 - Be^8 - B^8$, E – excitation energy in Be^8; d – $Li^7 - Be^7$.

The dependence of r_0 (and therefore also of the mean square radius of the charge distribution in the nucleus) on the excitation energy of the nucleus is shown by means of graphs. The magnitude of the error in this is determined by two factors: the accuracy with which the energies of the corresponding levels have been measured, and the contribution which may be introduced by an admixture of other configurations within the limits of the p-shell (intermediate coupling).

In this way we can determine the values of r_0 for all the levels of the isotopic doublets (He^5-Li^5, Li^7-Be^7) and for the levels with T = 1 of isotopic triplets. However, it is not possible by this method to determine r_0 for the levels of Li^6 and Be^8 with T = 0 since there simply are no corresponding levels in neighboring nuclei.

To extrapolate the curves showing the energy dependence of r_0 to states with T = 0 we have used data on the scattering of fast electrons by nuclei given in the paper of Hofstadter.[5] Hofstadter discusses the relation between the various models of charge distribution in the nucleus and the scattering cross section for fast electrons by nuclei. For light nuclei the first Born approximation is applicable, according to which the effective cross section for elastic scattering of electrons by nuclei is equal to

$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2(\theta/2)}{\sin^2(\theta/2)} \left[\int_0^\infty \rho(r) \frac{\sin p_0 r}{p_0 r} 4\pi r^2 dr\right]^2,$$

i.e., $\sigma_S(\theta)$ represents the effective cross section for elastic scattering of electrons by a point charge multiplied by the square of the structure factor

$$F = \frac{4\pi}{p_0} \int_0^\infty \rho(r) \sin p_0 r \cdot r dr.$$

Here $\hbar p_0$ is the change in momentum on scattering:

$$p_0 = \frac{2E}{\hbar c} \sin\frac{\theta}{2} = \frac{2}{\lambda\!\!\!^{-}} \sin\frac{\theta}{2},$$

where E is the electron energy, $\lambda\!\!\!^{-} = \hbar/p$. The calculation is carried out in the center of mass system, i.e., the recoil of the nucleus is not taken into account.

Hofstadter[5] gives mean square radii a and the form factors F corresponding to different assumptions on the distribution of charge in the nucleus. For H and He the distribution assumed by us coincides with the Gauss distribution. In this case

$$a^2 = \frac{3}{2} r_0^2, \quad \rho(r) = \frac{Ze}{\pi^{3/2} r_0^3} \exp\left(-\frac{r^2}{r_0^2}\right),$$

$$F = \exp\left(-\frac{r_0^2 p_0^2}{4}\right) = \exp\left(-\frac{p_0^2 a^2}{6}\right).$$

For nuclei in which the p shell is being filled

$$a^2 = \frac{r_0^2(5Z-4)}{2Z},$$

$$\rho(r) = \frac{2e}{\pi^{3/2} r_0^3} \exp\left(-\frac{r^2}{r_0^2}\right) + \frac{2(Z-2)}{3\pi^{3/2} r_0^5} r^2 \exp\left(-\frac{r^2}{r_0^2}\right),$$

$$F = \left\{\frac{2}{Z} + \frac{Z-2}{Z}\left(1 - \frac{r_0^2 p_0^2}{6}\right)\right\} \exp\left(-\frac{p_0^2 r_0^2}{4}\right).$$

A direct evaluation of r_0 from Hofstadter's data gives an overestimate of r_0 compared to the value obtained from the difference in Coulomb energies by about 25%. We make use of the fact that the mean square radius a is proportional to r_0.

By utilizing the value of r_0 for the ground level of Li^7, obtained from the difference in Coulomb energies, and the value of a for the ground states of Li^6, Li^7, and Be^9 given by Hofstadter,[5] we can roughly determine r_0 for the ground states of the nuclei Li^6 and Be^8, and extrapolate the curves giving the dependence on the excitation energy to the state with T = 0.

If the experiments on the scattering of fast electrons by nuclei are accurate, then the difference in the magnitude of the radii determined on the basis of the data of these experiments and on the basis of the Coulomb differences indicates the amount by which the true charge distribution in the nucleus differs from the distributions assumed by us.

The theoretically calculated Coulomb energy and electromagnetic radius of the nucleus depends on the choice of wave functions which, in particular, determines the distribution $\rho(r)$ of the electric charge in the nucleus. For levels with very great widths, for example, for the nuclei He^5-Li^5, the oscillator function is a very rough approximation and allows us to investigate the nature of the increase in radius with energy only qualitatively.

The agreement between experimental data on the scattering of fast electrons by nuclei and the values of nuclear radii obtained from differences in Coulomb energy may be improved if we assume that r_0 has different values for s and p shells (r_s and r_p). This may be done, since levels with even and odd values of l are orthogonal. Then for nuclei in which the p-shell is being filled $\rho(r)$ is of the form:

$$\rho(r) = \frac{Ze}{\pi^{3/2} r_s^3} \exp\left(-\frac{r^2}{r_s^2}\right) + \frac{2(Z-2)}{3\pi^{3/2} r_p^5}\, r^2 \exp\left(-\frac{r^2}{r_p^2}\right).$$

We have determined r_s and r_p from the difference in the Coulomb energies of the ground state of Li^7 and from the mean squared radius of the charge distribution Li^7 which is equal to 2.71 $\times 10^{-13}$ cm. They turned out to be equal to:

$$r_s = 2.42 \cdot 10^{-13}\ \text{cm}, \qquad r_p = 1.32 \cdot 10^{-13}\ \text{cm}.$$

The differences in Coulomb energies calculated with these values of r_s and r_p for all the nuclei with $A \leq 8$ differ from the experimental ones by not more than 10%.

Hofstadter notes[6] that such a $\rho(r)$ with $r_s = 2.65 \times 10^{-13}$ cm, $r_p = 1.07 \times 10^{-13}$ cm gives satisfactory agreement between experiment and the theoretical angular dependence of the scattering cross section for fast electrons by the Li^6 nucleus.

3. ANALYSIS OF THE CURVES

For the nuclear pair H^3-He^3 we obtain $r_0 = 1.48 \times 10^{-13}$ cm.

In contrast to all other nuclei the radius of the nuclei He^5 and Li^5 does not increase monotonically with energy. The radius of the nucleus in the second excited level with E = 16.69 Mev is lower than in the first excited state and even than in the ground state. This level has positive parity (the two first ones have negative parity) and is distinguished by relatively great stability. Apparently it corresponds to the breaking up of the s shell. If a neutron goes over from the s shell into the p shell then $r_0 = 1.79 \times 10^{-13}$ cm, but if a proton is excited (which, apparently, is more probable since we then obtain the triton + deuteron system, which explains the great stability), then $r_0 = 1.73 \times 10^{-13}$ cm.

The radius of the nuclei He^6-Li^6-Be^6 increases very sharply with energy; these nuclei are similar to the deuteron.

The radius of the nuclei Li^7-Be^7 does not depend strongly on the excitation energy.

The radius of nuclei with A = 8 depends very weakly on excitation energy. The level of Li^8 with E = 3.3 Mev and the level corresponding to it in Be^8 of 19.9 Mev have not been accurately identified, the following values are given $J = (1^+, 2^+)$. The graph shows both points, the upper one corresponding to $J = 2^+$, the lower one corresponding to $J = 1^+$.

The level of Li^8 with E = 2.28 Mev in the paper by Ajzenberg and Lauritsen[3] is compared with the level in Be^8 of 19.2 Mev. The corresponding point does not appear on the graph. Apparently the analogue to this Li^8 level in the case of Be^8 is not the 19.2-Mev level, but the 19.0 or the 18.9-Mev level. The graph shows all three points: the top one corresponds to E = 18.9 Mev, the middle one to 19.0, and the lowest one to 19.2 Mev.

4. CONCLUSIONS

On the basis of the simple method described above one can draw conclusions with respect to nuclear size and its increase, the "swelling" of nuclei in going to excited levels. It turns out that the electromagnetic nuclear radii increase monotonically with increasing excitation energy with the exception of one level of He^5-Li^5 which is distinguished by changed parity and by great stability. Moreover, the strong dependence of the radius on energy is observed only in the case of nuclei with A = 6 which are analogous to the deuteron.

Apparently it is possible by this method to make more precise the classification of levels of certain nuclei, since the Coulomb energy depends on the spatial correlation of nucleons in the nucleus. The possibility of obtaining more precise values for the radii of light nuclei in their different states is also of importance for theoretical calculations referring to levels of light nuclei utilizing the two nucleon interaction Hamiltonian by methods of the type used in references 7 and 8.

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[1] Balashov, Tumanov, and Shirokov, Труды конференции по ядерным реакциям при малых и средних энергиях (Proceedings of the Conference on Nuclear Reactions at Low and Medium Energies, Moscow State University and Physics Institute Academy of Sciences, U.S.S.R.) 1957.

[2] A. H. Wapstra, Physica **21**, 367 (1955).

[3] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[4] B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954).

[5] R. Hofstadter, Revs Modern Phys. **28**, 214 (1956).

[6] R. Hofstadter, Report at the Paris Conference on Nuclear Reactions, 1958 (preprint).

[7] Balashov, Dorofeev, Kalitkin, Kaminskii, Smirnov, Tumanov, and Shirokov, Труды конференции по ядерным реакциям при малых и средних энергиях (Proceedings of the Conference on Nuclear Reactions at Low and Medium Energies, Moscow State University and Physics Institute Academy of Sciences, U.S.S.R.) 1957.

[8] Balashov, Tumanov, and Shirokov, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 167 (1957), Soviet Phys. JETP **5**, 136 (1957).

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