NUCLEAR EXCITATION BY HIGH ENERGY PARTICLES

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Submitted to JETP editor September 2, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 922-924 (March, 1959)

THE quantities which, in the main, characterize the first stage of interaction of fast particles with nuclei are the mean number of nucleons $\overline{N}$ knocked out and the mean excitation energy $\overline{U}$ of the remaining nucleus.[1] Knowledge of these quantities is also important in the description of the evaporation stage. Experimental determination of them is very difficult;[2,3] therefore, the well-known Monte-Carlo method[2] is used to provide estimates. Use of it also involves a lot of work. Thus, it would be of interest to have another method of calculation, which would make it possible to obtain $\overline{N}$ and $\overline{U}$ analytically as functions of the energy E_0 and mass number A.

We consider a case, averaged over neutrons and protons ($A \approx 2Z$), and will assume that all cascade nucleons travel mainly in the forward direction.[2,3] Then, the weak dependence of the effective cross section for collision with the nucleons inside the nucleus makes it possible to use results of the theory of slowing down of neutrons in hydrogen. (The mean effective cross section is $\sigma_{eff} \approx \frac{1}{2}(\sigma_{pp} + \sigma_{pn})(1 - 7E_f/5E_c)$; $\sigma_{pp} \approx \sigma_{nn}$, $\sigma_{pn} = \sigma_{np}$ are the total cross sections for elastic collisions of free nucleons; E_f = 22 to 25 Mev is the Fermi energy; $E_c = E_0 + V_0$ is the kinetic energy of the nucleons in the nucleus, and V_0 is the depth of the potential well.) In this case, the number of cascade nucleons at distance 2Rx (x is dimensionless) and with energy in the interval dE is equal to

$$dN = \left\{\delta(E_c - E)\,e^{-\beta x} + \sum_{n=1}^{\infty} 2^n W_n\right\} dE, \qquad (1)$$

where the summation is carried out over all the collisions and W_n is of the form[4]

$$W_n = \frac{(\beta x)^n}{n!}\, e^{-\beta x}\, \frac{\ln^{n-1}(E_c/E)}{E_c(n-1)!}\,. \qquad (2)$$

The excitation energy of the remaining nucleus, averaged over the geometrical cross section, is equal to

$$\overline{U}(E_0, \beta) = E_0 + |\varepsilon| - (V_0 + B/2)\overline{F} + (V_0 - |\varepsilon|)\overline{N}, \qquad (3)$$

where the following quantities have been introduced

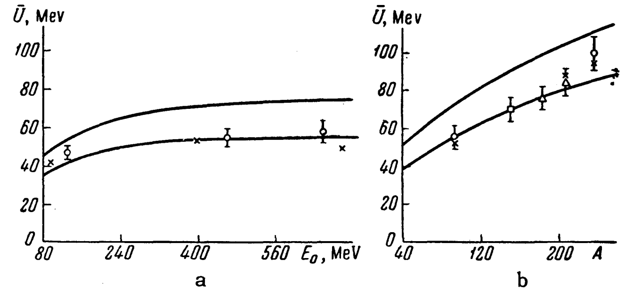
$$\overline{N} = 2\beta^{-2}\int_0^{\beta} y e^{-y}\Psi(\alpha, 2y)\,dy; \quad \overline{F} = 2\beta^{-2}e^{\alpha}\int_0^{\beta} y e^{-y}\Psi(2\alpha, y)\,dy,$$

$$\Psi(x,z) = 1 + \int_0^{\sqrt{4xz}} \cdot \exp\left\{-\frac{t^2}{4z}\right\} I_1(t)\,dt;$$

$$\alpha = \ln\frac{E_0 + V_0}{V_0 + B/2}; \quad \beta = \frac{3A\bar{\sigma}_{эф}}{2\pi R^2}; \qquad (4)$$

$I_1(t)$ is the Bessel function of imaginary argument, ϵ is the mean binding energy of nucleons in the nucleus and B is the Coulomb barrier for protons.

On the figure we show values of $\overline{U}$ calculated from these formulae with the initial data: $V_0 \approx$ 31 Mev, $R = 1.41 \cdot 10^{-13} A^{1/3}$ cm (σ_{pp} and σ_{pn} are taken from reference 5, with mean error ±10%, corresponding to the two curves on the figure, the upper for +10% and the lower for −10%), as well as several experimental data and results of Monte Carlo calculations. Although most of these data are only claimed to be estimates,[2,6] the agreement is quite satisfactory.



Dependence of the mean excitation energy: a – on proton energy for AgBr nuclei; b – on the mass number, for $E_0 \approx 460$ kev. o, □ and Δ are experimental data from references 3 and 6, × – are Monte Carlo calculations;[2,7] the solid curves are from the present work.

The main feature of the proposed method is its simplicity. Using it, it is possible to evaluate cross sections for the reactions (p, xn); (p, pxn) etc. at high energies, and to show simply the influence of errors in the initial data, e.g., in the nuclear radius, or in the cross sections σ_{pp}, σ_{pn}. For the latter purpose, it is useful to employ approximate expressions for $\overline{F}$ and $\overline{N}$

$$\overline{F} \approx {}^1\!/_2 e^{\alpha}\{1 + \Phi(\sqrt{2\alpha} - \sqrt{y_0})\};$$

$$\overline{N} \approx {}^1\!/_2 e^{y_0}\{1 + \Phi(\sqrt{\alpha} - \sqrt{2y_0})\}, \qquad (5)$$

where $y_0 = A\sigma_{eff}/\pi R^2$; $\Phi(z)$ is the error function.

Bombardment of nuclei by fast particles of mass a and energy E_a is equivalent to bombardment of them by a nucleons, each of mean energy E_a/a. Therefore, we have the following expressions for $\overline{U}_a$:

$$\overline{U}_a(E_a, \beta) \approx |\varepsilon_a| - |\varepsilon| + a\overline{U}(E_a/a, \beta). \quad (6)$$

Here ϵ_a is the binding energy of particles of the same type as the incident ones in the nucleus and $\overline{U}(E_a/a, \beta)$ can be obtained from Eqs. (3) and (4). From this formula and the preceding ones it follows that bombardment of a nucleus by complex particles is advantageous with respect to obtaining high excitation energies of the remaining nucleus. From analysis of the graph in the figure, we conclude that there is little chance of a cascade for $E_a/a \lesssim 30$ to 50 Mev and that nuclear reactions here go through formation of the compound nucleus.

In conclusion, I would like to thank M. M. Agrest and I. M. Rozman for advice and discussion, and N. V. Khaikhyan for calculational help.

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Translated by G. E. Brown
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A METHOD OF EVALUATING ELECTRICAL CONDUCTIVITY

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Submitted to JETP editor September 15, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 924-926 (March, 1959)

It is well known that in the electron theory of solids the standard transport equation method meets with a number of difficulties (see, e.g., references 1 – 3). A number of authors have in this connection in recent years made attempts to construct an accurate theory of the transport coefficients using relaxation functions.[4-7] In the present note we give a new method to evaluate the reaction of a system of particles to an external field, using quantum-mechanical Green functions. To be specific, we shall deal with the electrical conductivity, but it will be clear from the results that the method could be equally well used to evaluate any parameter characterizing the reaction of the system. We note also that to apply the general results to the Bose case one needs only change the sign in some of the intermediate equations. It is obvious that if one wants to evaluate the average current produced in the system by the action of an external electrical field it is sufficient to find the change in the "one-particle" density matrix $R_1(\mathbf{x}, \mathbf{y};\ t)$. It was shown in reference 8 that this matrix is connected with the one-fermion Green function through the equation

$$R_2(\mathrm{x}, \mathrm{y}; t) = i \lim G(x, y) \quad \text{as} \quad x_0 \to y_0 = t, \quad x_0 < y_0 \quad (1)$$

(x, y, are points in four-dimensional space, the spin indices are omitted for the sake of simplicity). Let the field be characterized by the four-potential A_i ($i = 0, 1, 2, 3$, $A_0 = -\varphi$, φ is the scalar potential). If we are interested only in effects which are linear in the field we have for the change in the Green function

$$\Delta G(x, y) = (e/\hbar)\int dz dz' dz'' G_0(x_1 z') \times \Gamma_i(z', z''; z)\, G_0(z'', y)\, A_i(z), \quad (2)$$

where $\Gamma_i = -\hbar G^{-1}(z', z'')/\delta e A_i(z)$ is the "electromagnetic" vertex part and G_0 the Green function for $A_i = 0$.

For a number of problems it is sufficient to restrict the discussion to the case of spatial uniformity, when ($\hbar = 1$)

$$G_0(x, y) = \int dp G_0(p)\, e^{-i(p, x-y)},$$

$$\Gamma_i(x, y; z) = (-2\pi)^{-8}\int dq' dq'' \Gamma_i(q', q'')\, e^{-i(q', x-z)+i(q'', y-z)} \quad (3)$$

($px = p_0 x_0 - \mathbf{p}\cdot\mathbf{x}$). We shall also assume $A_i = A_{mi} e^{-ikz - s|z_0|}$ (s is a small positive number; if damping is present, e.g., when the electrical conductivity is finite, we can at once put $s = 0$). Equation (2) then gives

$$\Delta G(x, y)\Big|_{s=0} = eA_i(y)(2\pi)^4 \times \int dp G_0(p)\, G_0(p-k)\, \Gamma_i(p, p-k)\, e^{-i(p, x-y)}. \quad (4)$$

The electrical conductivity can be evaluated from this in a trivial manner.