ALPHA DECAY OF Th^{227} *IN THE COLLECTIVE MODEL AND THE SPIN OF* Ra^{223} *IN ITS LOWEST STATE*

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FRILLEY et al.[1] have shown that the relative intensities of the α-groups of Th^{227}, which arise during its decay from the ground state to the single-particle levels of Ra^{223} (0.59, 286 kev), are approximately the same and amount to 17 — 21%. Such levels must therefore be considered as the ground states for the three rotational series, one of which was considered by the author elsewhere.[2]

In this communication we establish the spin of the lower level of Ra^{223} with the aid of the Ter-Martirosyan formula[3] for the probability of the α decay of a parent nucleus of spin I_0.

$$W_I^{I_0} = C \sum_{l=I-I'}^{I+I'} \left| C_{II'lm}^{I_0 I_0} \right|^2 (2l+1) \exp\{-\alpha E_r - \beta l(l+1)\}. \quad (1)$$

In our calculations the projection m of the orbital moment of the α particle l on the symmetry axis of the nucleus is assumed to differ from zero. Here $m = I_0 - I'$, where I' is an analogous projection of the spin I of the corresponding level of the daughter nucleus. The semi-empiral coefficients α and β are taken from reference 2 and the values of the excitation energy E_r are taken from the level scheme given in reference 1. It was also established in reference 1 that 59 — 29 — 0 kev level transitions are electric quadrupole, and consequently the following assumptions can be made concerning the spin of the lowest level: (a) $I = I' = I_0 - 2$; (b) $I = I' = I_0 - 1$; (c) $I = I' = I_0$. The 29-kev level, in view of the relatively low intensity of the α particles, (5%) should logically be considered the first rotational sublevel of the lower level. In the case (c) we could not obtain an empirical relation for the intensities of the α groups for the 29 and 0 kev levels, leaving therefore only cases (a) and (b). Upon calculating the Clebsch-Gordan coefficients that enter into (1), and considering that by the parity rule l can be only even, we obtain for case (a) the following ratios for the intensities of the groups for the 29 and 0 kev levels

$$\frac{J_{29}}{J_0} = \frac{2}{I_0+1} e^{-14.2 \cdot 0.029} \quad (2)$$

(Here $l = 2$ and $m = 2$, and the terms with $l > 2$ in the sum of Eq. (1) can be neglected). Relation (2) assumes the empirically-established value of 5% : 19%, either when the spin of the ground state of the daughter nucleus (Th^{227}) is $I_0 = 7/2$, or when $I_0 = 9/2$. We therefore conclude that the spin of the lower level of Ra^{223} is either $3/2$ or $5/2$. Calculations made for case (b) lead to $I_0 = 3/2$, which, however, from considerations given in reference 2, is excluded.

The same method of calculations can also be applied to the calculation of the spin of the 286-kev ground state of the third rotational series. The 307-kev level must, for the same reason, be taken as the first sublevel of this series. Since a magnetic dipole 286 — 0 kev transition and an electric dipole 286 — 238 kev transition were empirically established in reference 1, the only assumption that can be made is $I = I' = I_0 - 1$, for the 286-kev level and $I = I_0$ and $I' = I_0 - 1$ for the first sublevel (307 kev). In both cases we have $l = 2$ and $m = 1$. The calculated ratio of the intensities of the α groups for the 307 and 286 kev levels is

$$\frac{J_{307}}{J_{286}} = \frac{3(2I_0-1)}{8(I_0-1)(2I_0+3)} e^{-14.2 \cdot 0.02} \quad (3)$$

and when $I_0 = 7/2$ and $9/2$ it is close to the empirical ratio, 1% : 17%. The best agreement occurs in this case for the second value, corresponding to a spin of $5/2$ for the lower level. The calculated ratio of the intensities for the α groups of the 59-kev and 0 levels is ≈ 1.3; the empirical value is 0.9.

Sliv and Peker[4] computed the effect of the nonsphericity of the nucleus on the transparency coefficient of the nuclear barrier for an α particle with velocity $\beta = v/c$. This effect is introduced into the probability of the α decay by the factor

$$P = \exp\left\{-\frac{8Z}{137\beta}\frac{4}{5}\sqrt{x(1-x)}\,\zeta\right\}, \quad (4)$$

where $x = E_p/V_0$, with E_p being the decay energy and V_0 the maximum height of the potential barrier; ζ is the relative deformation of the nucleus. The calculated ratio of the foregoing intensities agrees with the empirical values at a difference of 0.1 in the relative deformations (elongations) at the 59-kev and 0 level. For the 238-kev and 0 levels, the difference becomes 0.15. Both values are fully acceptible and readily explain the empirically established fact that the rotation constants B for the first and third rotational series (4.5 and 4 kev) coincide approximately with the rotation constant (5 kev) of the second rotational band, considered in reference 2.

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170

ON THE PROBLEM OF THE COVARIANT DEFINITION OF THE SPIN PSEUDOVECTOR

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As is well known, the spin operator is defined by an antisymmetric tensor of the third rank, i.e., it is a pseudovector: $\sigma_\mu = (\boldsymbol{\sigma}, i\rho_1)$. In previous papers it has been shown[1,2] (cf. also Sections 17-20 in reference 3) that the longitudinal polarization of free Dirac particles can be characterized by the operator $(\boldsymbol{\sigma}\cdot\mathbf{k})/k$. This operator is an integral of the motion with eigenvalue s. We shall try to relate to the quantity s not only the longitudinal polarization, but also the transverse and time components of the spin vector.

The wave function for positive energy and with inclusion of the spin states has the form (cf. references 1-3)

$$\psi = L^{-3/2}\sum_s C_s b_s e^{-icKt+i\mathbf{k}\cdot\mathbf{r}}. \tag{1}$$

Here

$$b_s = \frac{1}{\sqrt{2}}\begin{Bmatrix} sf(K)\cos\theta_s \\ sf(K)\sin\theta_s e^{i\varphi} \\ f(-K)\cos\theta_s \\ f(-K)\sin\theta_s e^{i\varphi} \end{Bmatrix}$$

$$f(K) = \sqrt{1+k_0/K},\quad \theta_s = \theta/2 - (\pi/4)(1-s),$$
$$K = \sqrt{k^2+k_0^2},\quad \beta_1 = k/K. \tag{2}$$

The amplitude C_S describes the state with longitudinal spin component $s = \pm 1$, and θ and φ are the spherical angles of the vector $\mathbf{k}$.

The transverse and time components are not integrals of the motion, and therefore they can be characterized only by the average values

$$\zeta_\mu = K\int \psi^+\sigma_\mu\psi d^3x, \tag{3}$$

where the factor $K = k_0(1-\beta_1^2)^{-1/2}$ is introduced in order to preserve for the average values ζ_μ the same relativistic covariance as possessed by the expression $\psi^+\sigma_\mu\psi$.

Let us introduce an auxiliary coordinate system in which the z axis is directed along the momentum $\mathbf{k}$. Then, using the fact that for this system $\theta = \varphi = 0$, we find

$$\zeta_3 = K(C_1^+C_1 - C_{-1}^+C_{-1}) = Ks$$

(longitudinal component); for $C_1 \neq 0$ and $C_{-1} \neq 0$ the quantity $|s|$ will be smaller than unity:

$$\zeta_1 = k_0(C_1^+C_{-1} + C_{-1}^+C_1) = k_0\sqrt{1-s^2}\cos\delta,$$

$$\zeta_2 = ik_0(C_{-1}^+C_1 - C_1^+C_{-1}) = k_0\sqrt{1-s^2}\sin\delta$$

(transverse components); δ is the phase difference between the complex amplitudes C_1 and C_{-1}. Finally, $\zeta_4 = ik(C_1^+C_1 - C_{-1}^+C_{-1}) = iks$ is the time component.*

For an unpolarized beam of electrons $s = 0$ and the phase δ is a rapidly changing quantity, so that on the average $\cos\delta$ and $\sin\delta$ go to zero.

Partial polarization is also possible: for example, $0 < |s| < 1$, and the angle δ is again a rapidly changing quantity. For complete polarization the quantities s and δ are fixed constants. In this case one can make one of the transverse components zero by a rotation around the axis $\mathbf{k}$, and then we shall have $\zeta_3 = Ks$, $\zeta_1 = k_0(1-s^2)^{1/2}$, $\zeta_2 = 0$, and $\zeta_4 = iks$, i.e., the quantity s will determine all the components of the spin vector.

Let us assume that in some coordinate system the momentum vector $k_\mu(\mathbf{k}, iK)$ is parallel to the spin vector, $s = 1$, $\zeta_\mu(Kk/k, i\mathbf{k})$, i.e., the two vectors make the same angle $\theta_k = \theta_S = \theta$ with the z axis. Then in a new coordinate system moving relative to the first with the velocity $c\beta$ directed along the z axis these angles are already different:†

$$\cos\theta'_k = (\beta_1\cos\theta - \beta)/\sqrt{(1-\beta\beta_1\cos\theta)^2 - (1-\beta^2)(1-\beta_1^2)},$$

$$\cos\theta'_s = (\cos\theta - \beta\beta_1)/\sqrt{(\beta_1 - \beta\cos\theta)^2 + (1-\beta^2)(1-\beta_1^2)} \neq \cos\theta'_k,$$

owing to which the quantity s′ is smaller and is given by