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ON THE PROBLEM OF THE COVARIANT DEFINITION OF THE SPIN PSEUDO-VECTOR

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As is well known, the spin operator is defined by an antisymmetric tensor of the third rank, i.e., it is a pseudovector: $\sigma_\mu = (\boldsymbol{\sigma}, i\rho_1)$. In previous papers it has been shown[1,2] (cf. also Sections 17-20 in reference 3) that the longitudinal polarization of free Dirac particles can be characterized by the operator $(\boldsymbol{\sigma}\cdot\mathbf{k})/k$. This operator is an integral of the motion with eigenvalue s. We shall try to relate to the quantity s not only the longitudinal polarization, but also the transverse and time components of the spin vector.

The wave function for positive energy and with inclusion of the spin states has the form (cf. references 1-3)

$$\psi = L^{-3/2}\sum_s C_s b_s e^{-icKt+i\mathbf{k}\cdot\mathbf{r}}. \tag{1}$$

Here

$$b_s = \frac{1}{\sqrt{2}}\begin{Bmatrix} sf(K)\cos\theta_s \\ sf(K)\sin\theta_s e^{i\varphi} \\ f(-K)\cos\theta_s \\ f(-K)\sin\theta_s e^{i\varphi}\end{Bmatrix}$$

$$f(K) = \sqrt{1+k_0/K},\quad \theta_s = \theta/2 - (\pi/4)(1-s),$$
$$K = \sqrt{k^2+k_0^2},\quad \beta_1 = k/K. \tag{2}$$

The amplitude C_s describes the state with longitudinal spin component $s = \pm 1$, and θ and φ are the spherical angles of the vector $\mathbf{k}$.

The transverse and time components are not integrals of the motion, and therefore they can be characterized only by the average values

$$\zeta_\mu = K\int \psi^+\sigma_\mu\psi d^3x, \tag{3}$$

where the factor $K = k_0(1-\beta_1^2)^{-1/2}$ is introduced in order to preserve for the average values ζ_μ the same relativistic covariance as possessed by the expression $\psi^+\sigma_\mu\psi$.

Let us introduce an auxiliary coordinate system in which the z axis is directed along the momentum $\mathbf{k}$. Then, using the fact that for this system $\theta = \varphi = 0$, we find

$$\zeta_3 = K(C_1^+C_1 - C_{-1}^+C_{-1}) = Ks$$

(longitudinal component); for $C_1 \neq 0$ and $C_{-1} \neq 0$ the quantity $|s|$ will be smaller than unity:

$$\zeta_1 = k_0(C_1^+C_{-1} + C_{-1}^+C_1) = k_0\sqrt{1-s^2}\cos\delta,$$

$$\zeta_2 = ik_0(C_{-1}^+C_1 - C_1^+C_{-1}) = k_0\sqrt{1-s^2}\sin\delta$$

(transverse components); δ is the phase difference between the complex amplitudes C_1 and C_{-1}. Finally, $\zeta_4 = ik(C_1^+C_1 - C_{-1}^+C_{-1}) = iks$ is the time component.*

For an unpolarized beam of electrons $s = 0$ and the phase δ is a rapidly changing quantity, so that on the average $\cos\delta$ and $\sin\delta$ go to zero.

Partial polarization is also possible: for example, $0 < |s| < 1$, and the angle δ is again a rapidly changing quantity. For complete polarization the quantities s and δ are fixed constants. In this case one can make one of the transverse components zero by a rotation around the axis $\mathbf{k}$, and then we shall have $\zeta_3 = Ks$, $\zeta_1 = k_0(1-s^2)^{1/2}$, $\zeta_2 = 0$, and $\zeta_4 = iks$, i.e., the quantity s will determine all the components of the spin vector.

Let us assume that in some coordinate system the momentum vector $k_\mu(\mathbf{k}, iK)$ is parallel to the spin vector, $s = 1$, $\zeta_\mu(K\mathbf{k}/k, ik)$, i.e., the two vectors make the same angle $\theta_k = \theta_s = \theta$ with the z axis. Then in a new coordinate system moving relative to the first with the velocity $c\beta$ directed along the z axis these angles are already different:†

$$\cos\theta_k' = (\beta_1\cos\theta - \beta)/\sqrt{(1-\beta\beta_1\cos\theta)^2 - (1-\beta^2)(1-\beta_1^2)},$$

$$\cos\theta_s'$$

$$= (\cos\theta - \beta\beta_1)/\sqrt{(\beta_1 - \beta\cos\theta)^2 + (1-\beta^2)(1-\beta_1^2)} \neq \cos\theta'_k,$$

owing to which the quantity s' is smaller and is given by

$$s' = (\beta_1 - \beta\cos\theta)/\sqrt{(1-\beta\beta_1\cos\theta)^2 - (1-\beta^2)(1-\beta_1^2)} < 1.$$

Here $c\beta_1 = ck/K$ is the speed of the motion of the particle in the original coordinate system.

As is well known, in the decay of a stationary π meson into a μ meson and a neutrino the spin of the μ meson must be directed exactly along the momentum of the μ meson ($s = 1$). But in the laboratory system, relative to which the π meson may be in motion, we have $s' < 1$, and therefore there must be a transverse component of the spin of the μ meson, lying in the plane of the momenta of the π and μ mesons.

We note that for particles with zero rest mass ($k_0 = 0$) the speed is always given by $\beta_1 = 1$. In this case $\theta'_s = \theta'_k$, and therefore the quantity s' is unchanged ($s' = s = 1$). This fact was used by one of us, following the ideas of Lee and Yang, in constructing a theory of the neutrino with oriented spin,[6,7] in which neutrino and antineutrino are characterized by different values of s ($s = 1$ and $s = -1$).

*Analogous expressions for the average value of the spin vector with only positive energies taken into account can be obtained from the work of Tolhoek,[4] and also from that of Alikhanov et al.[5] But the expression for σ_μ^A introduced in reference 5 (see note on page 789),

$$\sigma^A = (K\rho_3\sigma + \rho_2[\mathbf{k}\times\sigma])/k_0 = \sigma + i\rho_2\mathbf{k}/k_0,$$

$$\sigma_4^A = i\rho_3(\sigma\cdot\mathbf{k})/k_0 = \sigma_4 - \rho_2K/k_0$$

will satisfy neither the law of conservation of total angular momentum nor the law of conservation of the longitudinal component of the spin. For $K > 0$ the results agree, since the average value of ρ_2 is zero.

†This is due to the fact that the momentum four-vector is timelike: $k_\mu k_\mu = -k_0^2$, and the spin is spacelike: $\zeta_\mu\zeta_\mu = k_0^2$, with the two vectors orthogonal to each other: $\zeta_\mu k_\mu = 0$.

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RANDOM FREE PRECESSION OF MAGNETIC MOMENTS OF ATOMIC NUCLEI

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THE free precession of magnetic moments of atomic nuclei in an external magnetic field H_0 is usually observed by measuring the voltage and the frequency $\omega_0 = \gamma H_0$ at the terminals of a receiving coil which contains a sample initially magnetized perpendicularly to H_0. In a sufficiently homogeneous field H_0 the phenomenon is observed as long as the signal which decays with the time constant $T_{||}$ exceeds the noise in the system.

After the signal has decayed the voltage at the terminals of the receiver coil is of a fluctuating nature and is determined on the one hand by the thermal noise in the receiver circuit, and on the other hand by the fluctuations in the magnetization of the sample.

The spectral density of the mean square of the voltage across the load resistor R_0 of the receiver circuit due to thermal noise in the receiver system is given by Nyquist's formula, which in the case under consideration is of the form:[1]

$$(V_T^2)_\omega = \frac{\hbar\omega}{2\pi}\coth\frac{\hbar\omega}{2kT}\frac{RR_0^2}{|Z(\omega)|^2}, \quad (1)$$

where $Z(\omega)$ is the impedance, while R is the effective resistance of the whole receiver circuit. The presence in the receiver coil of a sample containing atomic nuclei with a magnetic moment different from zero and situated in an external field H_0, leads to the appearance of an additional voltage due to the fluctuations of the component of the magnetization of the sample in the direction perpendicular to H_0. The spectral density of the mean square of the component of the magnetization in this direction is given by the expression

$$(M^2)_\omega = (\hbar\chi''/2\pi)\coth(\hbar\omega/2kT), \quad (2)$$

where

$$\chi'' = \frac{1}{2}\chi_0\left(\frac{\omega T_\perp}{1+(\gamma H_0+\omega)^2T_\perp^2} + \frac{\omega T_\perp}{1+(\gamma H_0-\omega)^2T_\perp^2}\right) \quad (3)$$

is the imaginary part of the complex nucleus susceptibility, while $T_\perp$ is the transverse relaxation time.

The spectral density of the mean square of the