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NEW SHORT-LIVED ISOMERS As^{75m} *AND* Ga^{70m} *EXCITED BY FAST PROTONS*

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In earlier work we observed short-lived isomeric activity in germanium bombarded with fast protons (E_γ = 0.31 Mev, $T_{1/2}$ = 17.5 ± 2.0 msec).[1] Through comparison with the results obtained by other investigators[2,3] this was identified as belonging to As^{75m} from the reaction Ge^{76}(p, 2n)As^{75m}. This identification has been confirmed by Vegors and Axel[4] and by Schardt.[5] In this connection it is of interest to look for short-lived proton-induced isomeric activity in arsenic.

The 19.6-Mev linear accelerator of the Ukrainian Academy of Sciences Physico-Technical Institute with a repetition rate of 2 pulses per second served as the proton source. The measuring technique was described earlier in references 1 and 6. Gamma-ray energy and half-life was measured by means of a gray-wedge pulse-height analyzer[7] and multichannel time-delay analyzer,[8] respectively. We used an external proton beam extracted through a 40 μ aluminum window, which was monitored by a liminescent thin willemite screen placed in front of the target.

The observed radiation was compared with the result obtained by bombarding a blank carbon target, which revealed no activity.

In the present work we obtained more accurate values for the properties of As^{75m} than those obtained previously through fast-proton bombardment of germanium: E_γ = 0.30 ± 0.01 Mev, $T_{1/2}$ = 16 ± 1 millisec. Through Ge^{76} enrichment from 7.67 to 70% it was also established that Ge^{76} is the isotope involved in the reaction that produces the short-lived isomer.

A metallic arsenic target irradiated by fast protons revealed very intense short-lived emission with E_γ = 0.29 ± 0.01 Mev and $T_{1/2}$ = 16 ± 1 millisec, and with the reaction threshold ≈ 13 Mev for a thick target. These results are in good agreement with the improved values for As^{75m} (E_γ = 0.305 Mev, $T_{1/2}$ = 17 millisec) given by Schardt in reference 5, where As^{75m} is produced by an E2 transition from the 402-kev level which arises from electron capture in Se^{75}. Thus fast-proton bombardment of arsenic also produces As^{75m} through the reaction As^{75}(p, p′)As^{75m}.

In our earlier work[1] we also detected short-lived isomeric gamma rays (of a few milliseconds) from gallium reacting with fast protons. The present work established E_γ = 0.19 ± 0.01 Mev and $T_{1/2}$ = 19 ± 1 millisec for this radiation.

A short-lived isomer with E_γ = 0.17 ± 0.01 Mev and $T_{1/2}$ = 16 ± 1 millisec was discovered in our laboratory during 1957[9] through neutron bombardment of germanium. The good agreement of these values with those obtained in the present work suggests that the same isomeric level is involved in both instances.

There are two stable gallium isotopes (69, 71) and five stable germanium isotopes (70, 72, 73, 74, 76). An analysis of the possible reactions between protons and gallium nuclei and between neutrons and germanium nuclei which would result in the same isomeric level shows that the excited nuclei of Ga^{70}, Ge^{69}, Ge^{70}, or Ge^{71} could be the final reaction products. Ge^{70}: which is an even-even nucleus, is excluded. Thus only the following proton-gallium reactions can produce these excited nuclei: Ga^{71}(p, pn)Ga^{70}, Ga^{69}(p, n)Ge^{69} and Ga^{71}(p, n)Ge^{71}. The experimental thresholds for the production of these nuclei in their ground states are 9, 4.1, and 1.0 Mev, respectively.[10]

By using enriched gallium isotopes (with Ga^{69} enriched from 60.2 to 97.6% in one instance and depleted to 1.3% in another instance) we determined that Ga^{71} is involved in a reaction where a short-lived level is formed. The experimental threshold of this reaction for a thick target is about 9 Mev. We can thus infer that the aforementioned level E_γ = 0.19 Mev belongs to Ga^{70m} produced in the reaction Ga^{71}(p, pn)Ga^{70m}. Glagolev et al.[9] obtained this level through the reaction Ge^{70}(n, p)Ga^{70m}. The half-life for an E3 transition is estimated to be of the order 10^{-2} sec, which agrees with experimental results.

The ground-state spin of Ga^{70} is 1^+.[11] If the spin of the excited Ga^{70} state is assumed to be 4^-, which is likely for a number of reasons, the observed short-lived isomeric transition is of the E3 type.

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ON A RELATION IN QUANTUM STATISTICS

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As is well known, the density matrix ρ of a canonical ensemble has the form

$$\rho = e^{-\beta H}, \quad H = H_0 + H_1 = \int [H_0(x) + H_1(x)]\, d^3x, \tag{1}$$

where $\beta = 1/kT$; H is the Hamiltonian of the system; H_0 is the "free" Hamiltonian (in general it may also partially take into account the interaction between the particles, for example in the Hartree-Fock approximation); H_1 is the interaction Hamiltonian.

Let us take instead of H the Hamiltonian $H_\lambda = H_0 + \lambda H_1$. Then

$$-\partial\rho / \partial\beta = (H_0 + \lambda H_1)\rho. \tag{2}$$

Following the general methods of the S matrix (cf. e.g., reference 1), we write down the formal operator solution of Eq. (2):*

$$\hat{\rho} = e^{-\beta H_0} T\left\{\exp\left(-\lambda \int H_1(xt)\, dt\, d^3x\right)\right\}, \tag{3}$$

where T calls for arrangement of the operators from right to left in the order of increasing t, and any operator f(x, t) is connected with f(x) by the relation

$$f(xt) = e^{tH_0} f(x)\, e^{-tH_0}. \tag{4}$$

To determine all the thermodynamic quantities it is sufficient to know the function

$$Z = \ln \mathrm{Sp}\, e^{\alpha N - \beta H}, \tag{5}$$

where the averaging (Sp) is taken over a complete orthogonal system of eigenfunctions of the Hamiltonian H or of H_0; N is the operator for the total number of particles and commutes with the total Hamiltonian; $\alpha = \beta\mu$, where μ is the chemical potential. Using Eqs. (5) and (3), one can easily verify that

$$\frac{\partial Z}{\partial \lambda} = -\mathrm{Sp}\Big[\exp(\alpha N - \beta H_\lambda) \times \int \tilde{H}_1(xt)\, d^3x dt\Big] \Big/ \mathrm{Sp}\,[\exp(\alpha N - \beta H_\lambda)], \tag{6}$$

where $\tilde{H}_1(xt) = e^{H_\lambda t} H_1(x)\, e^{-H_\lambda t}$ (i.e., the operator in the Heisenberg representation).

From Eq. (6) it follows that

$$Z = Z_{\lambda=1} = Z_0 - \int_0^1 \frac{\mathrm{Sp}\left[\exp(\alpha N - \beta H_\alpha) \int \tilde{H}_1(xt)\, d^3x dt\right]}{\mathrm{Sp}\,[\exp(\alpha N - \beta H_\lambda)]}\, d\lambda, \tag{7}$$

where Z_0 is the known expression for Z when $H = H_0$. It can be shown that the expression in the integrand of Eq. (7) is

$$\frac{1}{\lambda} \int (M(xt, x't')\, G(x't', xt)\, d^3x dt d^3x' dt',$$

where M is the mass operator for the "one-particle" Green's function[1] G(xt, x't') (integration with respect to t, t′ from 0 to β).

In the case in which H_0 does not contain the charge g, we can take the charge g as the parameter λ, and Z takes the form

$$Z = Z_0 - \int_0^g \frac{dg'}{g'} \int M(xt, x't')\, G(x't', xt)\, d^3x dt dt' d^3x'. \tag{8}$$