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[1] Leipunskiĭ, Morozov, Makarov, and Yampol'skiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 393 (1957), Soviet Phys. JETP **5**, 305 (1957).

[2] A. W. Schardt, Bull. Am. Phys. Soc. Ser. II, **1**, 85 (1956).

[3] S. H. Vegors and P. Axel, Bull. Am. Phys. Soc. **30**, No. 7, 11 (1955).

[4] S. H. Vegors and P. Axel, Phys. Rev. **101**, 1067 (1956).

[5] A. W. Schardt, Phys. Rev. **108**, 398 (1957).

[6] Leipunskiĭ, Miller, Morozov, and Yampol'skiĭ, Dokl. Akad. Nauk SSSR **109**, 935 (1956), Soviet Phys. "Doklady" **1**, 505 (1956).

[7] O. B. Likin, Приборы и техника эксперимента (Instruments and Measurement Engg.) **2**, 000 (1958).

[8] Yu. V. Makarov, Report, Inst. of Chem. Phys., Acad. Sci. U.S.S.R., 1957.

[9] Glagolev, Kovrizhnykh, Makarov and Yampol'skiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1046 (1959), Soviet Phys. JETP **9**, in press.

[10] B. L. Cohen and E. Newman, Phys. Rev. **99**, 718 (1955).

[11] B. G. Dzhelepov and L. K. Peker, Схемы распада радиоактивных изотопов (Decay Schemes of Radioactive Isotopes), Acad. Sci. Press (1957).

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ON A RELATION IN QUANTUM STATISTICS

E. S. FRADKIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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As is well known, the density matrix ρ of a canonical ensemble has the form

$$\rho = e^{-\beta H}, \quad H = H_0 + H_1 = \int [H_0(x) + H_1(x)]\, d^3x, \quad (1)$$

where $\beta = 1/kT$; H is the Hamiltonian of the system; H_0 is the "free" Hamiltonian (in general it may also partially take into account the interaction between the particles, for example in the Hartree-Fock approximation); H_1 is the interaction Hamiltonian.

Let us take instead of H the Hamiltonian $H_\lambda = H_0 + \lambda H_1$. Then

$$-\partial\rho/\partial\beta = (H_0 + \lambda H_1)\rho. \quad (2)$$

Following the general methods of the S matrix (cf. e.g., reference 1), we write down the formal operator solution of Eq. (2):*

$$\hat{\rho} = e^{-\beta H_0}\, T\left\{\exp\left(-\lambda \int H_1(xt)\, dt d^3x\right)\right\}, \quad (3)$$

where T calls for arrangement of the operators from right to left in the order of increasing t, and any operator f(x, t) is connected with f(x) by the relation

$$f(xt) = e^{tH_0} f(x)\, e^{-tH_0}. \quad (4)$$

To determine all the thermodynamic quantities it is sufficient to know the function

$$Z = \ln \mathrm{Sp}\, e^{\alpha N - \beta H}, \quad (5)$$

where the averaging (Sp) is taken over a complete orthogonal system of eigenfunctions of the Hamiltonian H or of H_0; N is the operator for the total number of particles and commutes with the total Hamiltonian; $\alpha = \beta\mu$, where μ is the chemical potential. Using Eqs. (5) and (3), one can easily verify that

$$\frac{\partial Z}{\partial\lambda} = -\mathrm{Sp}\Big[\exp(\alpha N - \beta H_\lambda) \times \int \widetilde{H}_1(xt)\, d^3x dt\Big] \Big/ \mathrm{Sp}\,[\exp(\alpha N - \beta H_\lambda)], \quad (6)$$

where $\widetilde{H}_1(xt) = e^{H_\lambda t} H_1(x)\, e^{-H_\lambda t}$ (i.e., the operator in the Heisenberg representation).

From Eq. (6) it follows that

$$Z = Z_{\lambda=1} = Z_0 - \int_0^1 \frac{\mathrm{Sp}\,[\exp(\alpha N - \beta H_\alpha) \int \widetilde{H}_1(xt)\, d^3x dt]}{\mathrm{Sp}\,[\exp(\alpha N - \beta H_\lambda)]}\, d\lambda, \quad (7)$$

where Z_0 is the known expression for Z when $H = H_0$. It can be shown that the expression in the integrand of Eq. (7) is

$$\frac{1}{\lambda}\int (M(xt, x't')\, G(x't', xt)\, d^3x dt d^3x' dt',$$

where M is the mass operator for the "one-particle" Green's function[1] G(xt, x't') (integration with respect to t, t' from 0 to β).

In the case in which H_0 does not contain the charge g, we can take the charge g as the parameter λ, and Z takes the form

$$Z = Z_0 - \int_0^g \frac{dg'}{g'} \int M(xt, x't')\, G(x't', xt)\, d^3x dt dt' d^3x'. \quad (8)$$

Thus all the statistical characteristics of the system are determined by the mass operator of the "one-particle" Green's function. This fact makes the application of field-theory methods to statistics very fruitful. It is not hard to show from Eq. (8) that for the case $\beta = \infty$ (T = 0) one gets the following well known relation for the energy of the ground state

$$\overline{E} = E(g=0) + \int_0^g \frac{dg'}{g'} \int M(x\beta, x't')\, G(x't', x\beta)\, d^3x d^3x' dt'. \quad (9)$$

One can verify this by choosing as the complete orthogonal set of functions for the summation the eigenfunctions of the total Hamiltonian and noting that for $\beta = \infty$ the ground-state term is the only one contributing to the sum.

*The integration with respect to t is everywhere taken from 0 to β.

[1] T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955).

Translated by W. H. Furry
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POLARIZATION OF THE NUCLEUS BY CAPTURE OF POLARIZED NEGATIVE MUONS INTO THE MESONIC K SHELL

I. M. SHMUSHKEVICH

Leningrad Physico-Technical Institute, Academy of Sciences, U.S.S.R.

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THE formation of μ-mesonic atoms with transition of μ^- mesons to the K shell is accompanied by considerable depolarization of the originally polarized μ mesons.[1-3] If the nucleus has a spin I a polarization of the nucleus is also produced, owing to the magnetic interaction of the spins.

The hyperfine splitting in the ground state of the mesonic atom is much larger than $\hbar/\tau$, where τ is the lifetime of the μ^- meson. Therefore the states with $F = I + \frac{1}{2}$ and $F = I - \frac{1}{2}$ form an incoherent mixture.[4] Accordingly the spin state of the mesonic atom is best described by the corresponding density matrix ρ. Since the nucleus is supposed unpolarized before the capture, the only direction singled out if that of the original polarization of the μ^- mesons. Let **j** be the unit vector in that direction. For a given F, from the requirements of invariance and the Hermitian character of ρ and its being linear in **j**, we have

$$\rho_F = \frac{1}{2F+1}\left(1 + \frac{3\lambda_F}{F+1}\mathbf{jF}\right)P_F, \quad (1)$$

P_F is the projection operator onto states with the given value of F. From the condition $\overline{\mathbf{jF}} = \mathrm{Sp}(\rho_F \mathbf{jF}) \le F$ it follow that $|\lambda_F| \le 1$.

$$\text{For } F = I + {}^1/_2 \quad P_F = \frac{I + 1 + 2\mathbf{Is}}{2I+1};$$

$$\text{for } F = I - {}^1/_2 \quad P_F = \frac{I - 2\mathbf{Is}}{2I+1},$$

s and **I** are the spin operators of the μ^- meson and the nucleus. Substituting Eq. (2) in Eq. (1), we get:

for $F = I + \frac{1}{2}$:

$$\rho_+ = \frac{1}{2(2I+1)}\left\{1 + \frac{2\mathbf{Is}}{I+1} + \lambda_+\mathbf{j}\left(\frac{3\mathbf{I}}{I+1} + 2\mathbf{s}\right)\right.$$
$$\left. + \frac{3\lambda_+}{(I+1)(I+{}^3/_2)}\left[(\mathbf{jI})(\mathbf{sI}) + (\mathbf{sI})(\mathbf{jI}) - \frac{2I(I+1)}{3}\mathbf{js}\right]\right\}, \quad (3)$$

from which we have

$$\frac{\bar{\mathbf{s}}}{s} = \frac{1}{s}\mathrm{Sp}(\rho_+\mathbf{s}) = \lambda_+\mathbf{j}, \quad \frac{\bar{\mathbf{I}}}{I} = \frac{1}{I}\mathrm{Sp}(\rho_+\mathbf{I}) = \lambda_+\mathbf{j}; \quad (4)$$

for $F = I - \frac{1}{2}$:

$$\rho_- = \frac{1}{2(2I+1)}\left\{1 - \frac{2\mathbf{Is}}{I} + \lambda_-\frac{I-{}^1/_2}{I+{}^1/_2}\mathbf{j}\left(\frac{3\mathbf{I}}{I} - 2\mathbf{s}\right)\right.$$
$$\left. - \frac{3\lambda_-}{I(I+{}^1/_2)}\left[(\mathbf{jI})(\mathbf{sI}) + (\mathbf{sI})(\mathbf{jI}) - \frac{2I(I+1)}{3}\mathbf{js}\right]\right\}; \quad (5)$$

$$\frac{\bar{\mathbf{s}}}{s} = \frac{1}{s}\mathrm{Sp}(\rho_-\mathbf{s}) = -\lambda_-\frac{I-{}^1/_2}{I+{}^1/_2}\mathbf{j},$$

$$\frac{\bar{\mathbf{I}}}{I} = \frac{1}{I}\mathrm{Sp}(\rho_-\mathbf{I}) = \frac{(I-{}^1/_2)(I+1)}{(I+{}^1/_2)I}\lambda_-\mathbf{j}. \quad (6)$$

For nuclei with $I = \frac{1}{2}$, for example for μ-mesonic hydrogen,

$$\rho_+ = {}^1/_4\left[1 + \lambda_+\mathbf{j}(\sigma_p + \sigma_\mu) + {}^1/_3\,\sigma_p\sigma_\mu\right],$$

$$\rho_- = {}^1/_4(1 - \sigma_p\sigma_\mu). \quad (7)$$

One must take this effect (polarization of the nucleus) into account in the analysis of experiments relating to the capture of μ^- mesons by nuclei, in particular in the determination of the angular distribution of the neutrons produced. Formulas for the asymmetry parameter of the angular distribution of the neutrons produced in the reaction $\mu^- + p \to n + \nu$ have been obtained in references 5 — 7. In these papers it was assumed that before the capture the proton is unpolarized, despite the fact that the μ^- meson located in the K shell is polarized. It thus appears that the results in question are unreliable.