U^{238}, and Cf^{252}). Secondly, the gamma quanta energies depends little on the excitation energy of the compound nucleus prior to fission.

The authors express their gratitude to Yu. I. Belyanin for insuring operation of the accelerated tube in the performance of this experiment.

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ZEMPLEN'S THEOREM IN RELATIVISTIC HYDRODYNAMICS

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KHALATNIKOV[1] has shown that for a relativistic shock wave of low intensity the theorem of Zemplen and the conditions of mechanical stability, $v_1 > c_1$, $v_2 < c_2$, are applicable provided only that the following inequality holds;

$$\left(\frac{\partial^2 (w/n)}{\partial p^2}\right)_s > 0 \qquad (1)$$

(where w is the heat function per particle, s the entropy per particle, n the density of particles measured in the rest system of the particles, and p the pressure.)

These results are also applicable for relativistic shock waves of any intensity. The proof can be done in a similar way to Landau and Lifshitz, (reference 2, paragraph 84,) for the case when the shock adiabate lies in the plane (p, w/n.) In this case, formula (84,6) will correspond to

$$w_2 T_2 ds_2 = {}^1/_2 (w_1/n_1 - w_2/n_2)^2 d(j^2),$$

and the expression

$$1 - \frac{v_2^2}{c_2^2} = (V_1 - V_2)\left[1 - \frac{j^2 (V_1 - V_2)}{2T_2}\left(\frac{\partial V_2}{\partial s_2}\right)_{p_2}\right]\frac{d(j^2)}{dp_2}$$

is replaced by

$$1 - \frac{u_2^2}{a_2^2}$$

$$= \left(\frac{w_1}{n_1} - \frac{w_2}{n_2}\right)\left[1 - \frac{j^2 (w_1/n_1 - w_2/n_2)}{2w_2 T_2}\left(\frac{\partial (w_2/n_2)}{\partial s_2}\right)_{p_2}\right]\frac{d(j^2)}{dp_2},$$

$$j = nu, \quad u = v/\sqrt{1 - v^2}, \quad a = c/\sqrt{1 - c^2},$$

(where c is the velocity of sound, and the velocity of light is taken as unity.) It follows from this that the quantity n/w, as well as the pressure and the density, are increased on the shock wave.

The inequality (1), for the nonrelativistic case, reduces to the well known conditions, $(\partial^2(1/n)\partial p^2)_s > 0$. For a relativistic ideal gas we have

$$\left(\frac{\partial^2 (w/n)}{\partial p^2}\right)_s = \frac{2(2-\gamma)}{\gamma(\gamma-1)}\frac{1}{pn^2}.$$

The last expression is always positive, since the quantity γ is within the interval[3] $1 < \gamma \leq 5/3$.

It should be noted that for an ultra-relativistic ideal gas,[2] $\gamma = 4/3$.

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ON ELECTROMAGNETIC SHOCK WAVES IN FERRITES

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WE investigate the propagation of a uniform plane electromagnetic wave in a medium with nonlinear dependence of the induction **B** on the magnetic field **H**.* We assume to begin with that the medium is isotropic, i.e., that a relation exists between the magnitudes of the fields ($B = B(H)$; $\mu(H) = \partial B/\partial H$). It is easy to show that Maxwell's equations for this case have solutions given by the relations

$$H_y = f\left(z \mp ct/\sqrt{\varepsilon\mu(H_y)}\right),$$

$$\sqrt{\varepsilon}\, E_x = \pm \int_0^{H_y} \sqrt{\mu(H)}\, dH\,, \quad (1)$$

where $f(\xi)$ is an arbitrary smooth function and ϵ the dielectric constant.

The solutions (1) describe simple waves in a nonlinear medium (waves of this type in gas dynamics are described, e.g., in references 1 and 2). The deformation of a simple wave can be most easily investigated by looking at the propagation of a single pulse. It can be easily seen that if $\mu(H)$ is a monotonically decreasing function then in the course of time the leading edge will become steeper and the trailing edge more sloping.† Beginning from a particular moment t^* there will appear in the function $H_y(t^*, z)$ an infinite derivative. This points to the appearance of an electromagnetic shock wave.

The boundary conditions which connect the fields on both sides of the discontinuity with its velocity of propagation, v, can be obtained by integrating Maxwell's equations over an infinitesimal interval Δz, containing the discontinuity. Thus we obtain

$$\{H_y\} = \frac{v}{c}\varepsilon\{E_x\}, \quad \{E_x\} = \frac{v}{c}\{B_y\},$$

$$\text{hence } v^2/c^2 = \{H_y\}/\varepsilon\{B_y\}, \quad (2)$$

where the brackets { } denote the jump in the values of the respective quantities across the discontinuity.

Immediately after the appearance the shock wave is weak ($\{\mu\} \ll \mu$) and the structure of the field is in this approximation the same as in a simple wave. Equation (2) together with (1) thus allow the description of the development of simple waves and of weak shock waves.

To investigate the structure of the shock front one needs further information on the characteristics of the medium. We shall investigate here the simplest example, of plane uniform waves in a ferrite saturated by a dc field parallel to the direction of propagation of the wave by means of a uniform field H_0. The connection between the magnetization **M** ($\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$; $M = \text{const}$) and the field $\mathbf{H}(z, t)$ in this case is given by the following equation[3]‡

$$\partial\mathbf{M}/\partial t = \gamma\mathbf{M}\times(\mathbf{H}+\mathbf{H}_0) - \lambda M^{-2}\mathbf{M}\times[\mathbf{M}\times(\mathbf{H}+\mathbf{H}_0)], \quad (3)$$

where γ is the gyromagnetic ratio for the electron spin and $1/\lambda = \tau_0$ is the relaxation time.

In particular, it follows from (3) that for sufficiently slow processes (with a characteristic time $T \gg \tau_0$) $\mathbf{M} \parallel \mathbf{H} + \mathbf{H}_0$. The connection between $H_\perp = H_y$ and $M_\perp = M_y$ is then given by

$$H_y = M_y\left(H_0/\sqrt{M^2 - M_y^2} - 4\pi\right). \quad (4)$$

By means of (4) it is easy to find the functions $B_y(H_y)$ and $\mu = 1 + 4\pi dM_y/dH_y = \mu(H_y)$ which enter Eqs. (1) and (2), for the case of simple and shock waves.

It turns out that it is not possible to obtain a general solution of Maxwell's equations taking into account the relation (3). We therefore limit ourselves to the case of a stationary plane shock wave.** The equations can in that case be easily integrated. It turns out[4] that in a stationary wave the vector **M** rotates around the direction of propagation of the wave, z (precession angle, φ), while the angle between M and the z-axis, θ, changes according to

$$\ln\frac{(\cos\theta - \cos\theta')^2}{(1+\cos\theta)^{1-\cos\theta'}(1-\cos\theta)^{1+\cos\theta'}}$$

$$= \left(\frac{z}{v} - t\right)\frac{2\lambda}{M}(H_y' + 4\pi M\sin\theta')\sin\theta', \quad (5)$$

The precession frequency $\omega = \partial\varphi/\partial t$ is given by

$$\omega = \gamma\{H_y'\cos\theta/\sin\theta' - (H_0 - 4\pi M\cos\theta)\}, \quad (6)$$

Here θ' and H_y' are the values of the respective magnitudes for $z \to -\infty$ (far behind the wave front); the wave velocity v depends on the amplitude of the transverse part of the magnetic field***

$$v^2/c^2 = H_y/(H_y + 4\pi M\sin\theta)$$

$$= H_y'/(H_y' + 4\pi M\sin\theta') = \text{const}, \quad (7)$$

while the angle θ' is connected with the field amplitude of the shock wave H_y' by the equation $H_y' \cot\theta' = H_0 - 4\pi M\cos\theta'$.

It follows from (5) that the time width of the wave front, τ, is given by

$$\tau = M/2\lambda(H_y' + 4\pi M\sin\theta')\sin\theta' \quad (8)$$

and it depends on the relaxation time of the ferrite, $\tau_0 = 1/\lambda$ and on the amplitude H_y'. In a very strong wave ($H_y' \gg H_0, M$) obtains

$$v \approx c, \quad \tau \approx \tau_0 M/2H_y', \quad \omega_{\max} \approx \gamma H_y'. \quad (9)$$

In a weak shock wave ($H_y' \ll H_0 - 4\pi M$) the passage time of the front is considerably longer and the maximum frequency component is less.

$$\omega_{\max} \approx {}^1\!/_2\,\gamma H_0[H_y'/(H_0 - 4\pi M)]^2, \quad \tau\omega_{\max} \approx {}^1\!/_4\,\gamma M\tau_0. \quad (10)$$

Thus for sufficiently large $\tau\omega_{max}$ the wave front of an electromagnetic shock wave consists of a circularly polarized oscillation with a variable frequency.

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*The case of a nonlinear relation between the electric displacement D and field E can be treated similarly, as well as the case of nonlinearity with respect to both the electric and the magnetic fields.

†This circumstance (for electromagnetic waves) was first pointed out and utilized by I. G. Kataev.

‡The anisotropy field will not be considered. In the following it will be assumed that $H_{z0} = H_0 - 4\pi M > 0$ since only this leads to stability in the initial conditions in the medium.

**In a stationary wave the field components (which, in general, are not transverse) have the form $f(z - vt)$ where the velocity $v = \text{const}$.

***We note that the value for the velocity of the shock wave determined from (2) and (4) coincides with that from (7).

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ON THE HEAT CONDUCTIVITY AND ATTENUATION OF SOUND IN SUPERCONDUCTORS

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We have previously calculated the electronic heat conductivity[1] κ_e, of superconductors and the phonon conductivity,[2] κ_p, determined by the scattering of phonons by electrons. It will be shown here that from the theoretical temperature dependence of κ_e and κ_p found, we can explain, to a considerable extent, all the relationships in the existing experimental data on the heat conductivity of superconductors.

According to our earlier paper[2] κ_p can be expressed as:*

$$\varkappa_p^s = \varkappa_p^n F(T)/F(T_k),$$

$$F(T) = -8(b^4 + b^3)(e^b - 1)^{-1}$$
$$+ 6\zeta(3)(e^b + 1) - 3(e^b + 1)\sum_s s^{-3}\exp(-2bs)$$
$$\times (4b^2s^2 + 4bs + 2) + 6\zeta(4)(e^b - 1)$$
$$-(e^b - 1)\sum_s s^{-4}\exp(-2bs)(8b^3s^3$$
$$+ 12b^2s^2 + 12bs + 6) + 32b^3(e^{2b} - 1)^{-1}$$
$$- a^4\sum_s\{s\exp(-2bs)\,\mathrm{Ei}[-s(2b-a)]\} + 6\sum_s s^{-3}\exp(-2bs),$$

$$a = 2b - 0{,}16, \quad \zeta(s) = \sum_{n=1}^{\infty} n^{-s}. \qquad (1)$$

In the normal state $\kappa_p^n = \text{const} \cdot T^2$; $b = \Delta(T)/kT$, where $\Delta(T)$ is the energy gap, and κ_s/κ_n depends only on T and T/T_k. For comparison with experiment one must use a specimen with sufficient impurity concentration for κ_e to be small. In Fig. 1 the theoretical curve is drawn according to Eq. (1) and the experimental points are for an In-Tl alloy measured by Sladek.[3]

If $(T_k - T)/T_k$ is not very small, κ_e is not appreciably affected by the electron-phonon interaction.

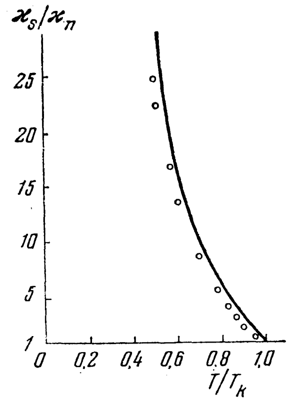

FIG. 1. Points – experimental data[3] for Tl concentration 38%. Solid curve – theoretical

As can be seen from Fig. 1, the conductivity κ_p increases exponentially as $T \to 0$, owing to the increase in phonon mean free path with decreasing scattering by electrons. At sufficiently low temperatures the lattice thermal resistance due to electron scattering, $1/\kappa_{pe}$, becomes less than the resistance due to scattering by lattice defects and crystal boundaries, $1/\kappa_{pd}$ (κ_{pd} is the same as κ_{pd} in a normal metal). Since the resulting lattice conductivity is $\kappa_p = \kappa_{pe}\kappa_{pd}/(\kappa_{pe} + \kappa_{pd})$, we get $\kappa_p \approx \kappa_{pd}$ at still lower temperatures. κ_{pd} usually decreases according to a power law[4] ($\sim T^3$) at low temperatures. For temperatures