We note that the theory can be tested for both $\Gamma_f \gg \Gamma_n$ and $\Gamma_f \ll \Gamma_n$ by studying $\sigma_f(\vartheta)\ \sigma_f(\pi/2)$. For $\vartheta \neq 0$ this ratio approaches a constant limit as $\alpha^2 j^2_{\max}$ increases. Thus $\sigma_f(\pi/4)/\sigma_f(\pi/2) \to \sqrt{2}$ as $\alpha^2 j^2_{\max} \to \infty$ and is independent of the nuclear parameters.

In conclusion I wish to thank D. P. Grechukhin for a discussion and I. Halpern for his kindness in making experimental data available.

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[1] G. A. Pik-Pichak, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 341 (1958), Soviet Phys. JETP **7**, 238 (1958).

[2] V. M. Strutinskiĭ, Атомная энергия (Atomic Energy) **2**, 508 (1957). (Translation, Consultants Bureau, Inc., p. 621).

[3] C. T. Coffin and I. Halpern, Phys. Rev. **112**, 536 (1958).

Note added in proof (January 27, 1959). $\sigma(0°)/\sigma(90°)$ was also calculated for Ra, assuming $\xi = 1$ but with a more exact value of the angular momentum of the compound nucleus (See I. Halpern and V. M. Strutinskiĭ, Report P/1315 at the Second Geneva Conference on the Peaceful Uses of Atomic Energy, 1958). It was assumed that $\tau = \sqrt{a(U - E_f)}$, where E_f is the fission threshold and the constant a was determined from the anisotropy for Pu. The initial temperature of the nucleus Pu + α particle at the saddle point is 1.3 to 1.5 Mev. The anisotropy for Ra is calculated to be 2.0. For Bi, taking into account the dependence of fissionability on excitation energy, we obtain $\sigma(0°)/\sigma(90°) = 1.9\,(\sigma(0°)/\sigma(90°)_{\text{exp}} = 2.02$; $\Gamma_f^{(0)}/\Gamma_n \sim \exp[(E_n - E_f)/\tau]$, where $E_f - E_n \approx 8$ Mev and $E_f \approx 15$ Mev. Agreement of the calculated and experimental anisotropy for Ra shows that we have no reason to assume the absence of correspondence between the nuclear moment of inertia and that of a rigid body, including the cases of excitations below 10 or 12 Mev, as was suggested by Halpern and Strutinskiĭ on the basis of neutron experiments.

The results given here were obtained in collaboration with V. M. Strutinskiĭ.

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ON THE THERMODYNAMICS OF HELIUM

M. P. MOKHNATKIN

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In the paper by Esel'son, Kaganov, and Lifshitz[1] it is shown that the lambda transition in a solution of helium isotopes is a phase transition of the second order. Using the condition for equilibrium between phases and the equations for the chemical potentials of the components in the gaseous phase, the authors obtain the equation

$$kT\ln P_3 + \chi(T) = \varphi + (1 - x_{\text{liq}})\,\partial\varphi/\partial x_{\text{liq}},$$

$$kT\ln P_4 + \chi(T) = \varphi - x_{\text{liq}}\,\partial\varphi/\partial x_{\text{liq}} \qquad (2)$$

(the equation numbers are those of reference 1). If we take the total derivative of both sides of Eq. (2), we get, according to the authors, the expression

$$-S_3 + kT\left(\frac{d}{dT}\ln P_3\right) = \frac{\partial\varphi}{\partial T} + (1 - x_{\text{liq}})\frac{\partial^2\varphi}{\partial x_{\text{liq}}\partial T} \qquad (3)$$

and so on. It is further asserted that at the temperature T_λ one obtains

$$kT_\lambda\Delta\left(\frac{d}{dT}\ln P_3\right) = (1 - x_{\text{liq}})\,\Delta\left(\frac{\partial^2\varphi}{\partial T\partial x_{\text{liq}}}\right) \qquad (4)$$

and so on, since the first derivative with respect to the thermodynamic potential has a break and the second derivative a jump at a phase transition of the second order. Substituting, according to Eqs. (6) and (9), for the quantities which occur here the authors obtain the following equation for the total pressure

$$kT_\lambda\Delta\left(\frac{d}{dT}\ln P\right) = (x_{\text{vap}} - x_{\text{liq}})\frac{\Delta C_p}{T_\lambda}\,\frac{\partial T_\lambda}{\partial x_{\text{liq}}}\,. \qquad (10)$$

Since, as is shown in reference 1, $\partial T_\lambda/\partial x_{\text{liq}} < 0$, $\Delta C_p > 0$, $x_{\text{vap}} > x_{\text{liq}}$, we have $\Delta\{d\ln p/dT\} < 0$ which is in accordance with experimental data.[2]

The authors conclude from this that the lambda transition is a phase transition of the second order.

We must draw attention to an error which has crept in in the process of this proof. If we take into account that along the equilibrium curve the total derivative with respect to the temperature is given by the expression $d\mu/dT = \partial\mu/\partial T + (\partial\mu/\partial P)\,dP/dT$ one must write the basic equations (3) and (4) in the form

$$\frac{d}{dT}\left[kT\ln P_3 + \chi(T)\right] = \frac{\partial\varphi}{\partial T} + (1 - x_{\text{liq}})\frac{\partial^2\varphi}{\partial T\partial x_{\text{liq}}}$$

$$+\left[(1 - x_{\text{liq}})\frac{\partial^2\varphi}{\partial P\partial x_{\text{liq}}} + \frac{\partial\varphi}{\partial P}\right]\frac{dP}{dT}\,, \qquad (3')$$

$$kT_\lambda\Delta\left(\frac{d}{dT}\ln P_3\right)=(1-x_{\text{liq}})\,\Delta\frac{\partial^2\varphi}{\partial T\partial x_{\text{liq}}}$$

$$+\left\{\frac{\partial\varphi}{\partial P}\Delta\left(\frac{dP}{dT}\right)+(1-x_{\text{liq}})\left[\Delta\frac{\partial^2\varphi}{\partial P\partial x_{\text{liq}}}\frac{dP}{dT}\right]\right\} \tag{4$'$}$$

and so on. The authors of reference 1 have incorrectly discarded the additional terms within the curly brackets in Eq. (4′) and thus obtained some agreement of the theory with experiment. However, an analysis of the complete Eq. (4′) for phase transitions does not confirm those conclusions which were made by the authors on the basis of the incorrect Eqs. (3) and (4). In that case any assertions about the character of the lambda transition in solutions of helium isotopes remain unproved assumptions.

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[1] Esel'son, Kaganov, and Lifshitz, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 936 (1957), Soviet Phys. JETP **6**, 719 (1958).

[2] Esel'son, Bereznyak, and Kaganov, Dokl. Akad. Nauk SSSR **111**, 568 (1956), Soviet Phys. "Doklady" **1**, 683 (1956).

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REPLY TO THE LETTER BY M. P. MOKHNATKIN

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As was shown at the beginning of our paper, the character of the lambda transition in solutions of helium isotopes follows from a large number of experimental data, in particular from the fact that the elasticity of the vapor over the solution depends essentially on the lighter isotope concentration over a large range of temperatures (absence of a plateau in the P-x diagram).

The proof of the fact that the lambda transition is of the second order was therefore not the object of our paper. The object of the paper was the analysis of different thermodynamic consequences from the fact that a lambda transition (a transition of the second order) existed in solutions and a comparison of these consequences with experimental data. In particular the character of the singularity in the P-T curve at the lambda point was elucidated in the paper. The conclusions of that consideration were in accordance with experiment and corroborated the conclusion about the second order transition.

M. P. Mokhnatkin asserts that this conclusion is illegitimate. An error was discovered in the fact that we neglected in a number of formulae derivatives of the pressure with respect to the chemical potential of the condensed phase. This neglect was, to be sure, made deliberately and was justified by the smallness of the specific volume of the liquid (v_{liq}) as compared with the specific volume of the vapor (v_{vap}). We remark that in those cases where this was necessary we took the derivative of the pressure with respect to the chemical potential of the liquid into account [see Eq. (19) and following].

Even if we retain the terms which M. P. Mokhnatkin writes down [see his Eqs. (3′) and (4′)] which, of course, leads to needlessly complicated equations, the conclusions obtained on the basis of Eq. (10) remain completely the same. Indeed, when the derivatives of the pressure with respect to the chemical potential of the liquid are taken into account, the equation analogous to Eq. (10) has the form

$$\frac{kT_\lambda}{P}\left\{1-\frac{v_{\text{liq}}+(x_{\text{liq}}-x_{\text{vap}})(\partial v_{\text{liq}}/\partial x_{\text{liq}})_\lambda}{v_{\text{vap}}}\right\}\Delta\frac{dP}{dT},$$

$$=(x_{\text{vap}}-x_{\text{liq}})\frac{\Delta C_p}{T_\lambda}\frac{\partial T_\lambda}{\partial x_{\text{liq}}}\left\{1-\left(\frac{\partial P}{\partial T}\right)_\lambda\left(\frac{\partial T_\lambda}{\partial P}\right)\right\}.$$

We have used here the notation of our paper and the index λ at the derivatives indicates that one must take half of the sum of the values for the two phases at $T = T_\lambda$. The expressions within curly brackets differ quite insignificantly from unity: on the left hand side the difference is about 10^{-2} and on the right hand side about 10^{-3}, which also confirms the legitimacy of the omissions made.

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