ON RADIATIVE TRANSITIONS BETWEEN ROTATIONAL LEVELS IN SPIN 1/2 NUCLEI

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The relative intensities of electric and magnetic transitions between rotational levels in spin $\frac{1}{2}$ nuclei are considered. The calculation is based on the coupling scheme previously proposed by the author. As an example the Tm^{169} nucleus is considered. It is shown that the observed intensity ratio does not contradict the proposed coupling scheme.

To calculate the probabilities for the magnetic dipole and electric quadrupole transitions between the rotational levels of deformed nuclei with spin $\frac{1}{2}$, we shall assume:

(1) These nuclei are in $\Sigma^+_{g,u}$ states, i.e., in the one-nucleon states described in Hund's coupling scheme b.[1] The indices g and u refer to the states $\frac{1}{2}^+$ and $\frac{1}{2}^-$, respectively.

(2) Correspondingly, we have, besides the trivial ones, the following integrals of the motion: the nucleon spin **s**, the total rotational momentum $\boldsymbol{\kappa} = \mathbf{l} + \mathbf{R}$ (**R** is the moment vector of the collective rotation), the projection of the orbital momentum of the nucleon $\mathbf{l}$ on the nuclear axis $\Lambda = 0$, and the projection of the vector $\boldsymbol{\kappa}$ on the nuclear axis $\kappa_{z'} = 0$. The total angular momentum is $\mathbf{J} = \boldsymbol{\kappa} + \mathbf{s}$.

The wave function for these states can be written in the form

$$\Psi^{JM}_{\varkappa,s} = \varphi_0 \sum_{m,m_s} (\varkappa {}^1\!/_2\, m m_s \,|\, \varkappa {}^1\!/_2\, JM)\, Y_{\varkappa,m}\, \chi_{1/2, m_s}, \quad (1)$$

where φ_0 is the wave function describing the motion of the nucleon relative to the nuclear axes, and χ. is the spin function of the extra nucleon in the laboratory system. The rotational quantum number runs through the values 0, 2, 4, ... for the states Σ^+_g, and through 1, 3, 5, ... for the states Σ^+_u.

The wave function (1) is conveniently written in a system of coordinates connected with the nuclear axes. For this purpose we note that

$$Y_{\varkappa,m}(\theta,\varphi) = \sqrt{(2\varkappa+1)/8\pi^2}\, D^{\varkappa}_{m,0}(\theta,\varphi,\psi), \quad (2)$$

where (θ, φ, ψ) are the Eulerian angles defining the orientation of the nuclear axes in space. The function $D^{K}_{m,\nu}(\theta, \varphi, \psi)$ describes the unitary transformation from the fixed system to the system of coordinates connected with the nucleus.

Noting that

$$\chi_{1/2,m_s} = \sum_{m'_s} D^{1/2}_{m_s m'_s}\, \chi_{1/2, m'_s}, \quad (3)$$

we can write the wave function (1) in the coordinates referred to the nuclear axes in the form

$$\Psi^{JM}_{\varkappa s} = \sqrt{(2\varkappa+1)/8\pi^2}\, \varphi_0 \sum_{m'_s} (\varkappa {}^1\!/_2\, 0 m'_s \,|\, \varkappa {}^1\!/_2\, J m'_s)\, D^{J}_{M,m'_s}\, \chi_{1/2, m'_s}. \quad (4)$$

The calculation of the probabilities for electric quadrupole transitions between the rotational levels of spin $\frac{1}{2}$ nuclei with the help of the wave function (3) presents no difficulties. The result is the same as in the paper of A. Bohr and B. Mottelson,[2] where these probabilities were calculated for a different coupling scheme. It is now of interest to calculate the probabilities for magnetic dipole transitions. The operator for magnetic dipole transitions can be written in the form[2]

$$\mathfrak{M}(1,m) = \sum_{\nu} D^{1}_{m,\nu}\, \mathfrak{M}'(1,\nu), \quad (5)$$

where

$$\mathfrak{M}'(1,\nu) = \nabla r Y_{1\nu} [(g_l - g_R)\mathbf{l} + (g_s - g_R)\mathbf{s} + g_R \mathbf{J}]$$

is the operator for a magnetic dipole transition in the coordinate system connected with the nuclear axes; g_l, g_s, and g_R are the gyromagnetic ratios for the single-nucleon and collective motions, respectively.

With the help of the wave function (4) one can without great difficulty compute the probability for a magnetic dipole transition between levels with the same rotational quantum number (transitions within a doublet). The expression for this probability is of the form

$$T(M1, J+1 \to J) = \frac{16\pi}{9\hbar}\left(\frac{\omega}{c}\right)^3 B_{\varkappa}(M1, J+1 \to J), \quad (6)$$

where

$$B_{\varkappa}(M1, J+1 \to J) = \left(\frac{e\hbar}{2mc}\right)^2 (g_s - g_R)^2 \frac{3}{16\pi}\frac{2J+1}{J+1}.$$

Another possible case is the transtion in which κ changes by two units. In this case the magnetic transition probability is zero, if the transition operator (5) is used. This difficulty can be avoided in the case of nuclei with an extra proton by adding to the operator (5) a term arising from the spin-orbit coupling:[3]

$$\mathfrak{M}_{sl}(1, m) = \frac{\lambda}{mc^2}\left[\nabla U(\mathbf{r})(\mathbf{sr}) - \mathbf{s}(\mathbf{r}\nabla U)\right]\nabla r Y_{1m}\frac{e\hbar}{2mc}. \quad (7)$$

Here $\mathbf{r}$ is the radius vector of the odd proton, λ is the spin-orbit coupling constant, and $U(\mathbf{r})$ is the self-consistent potential of the nucleus.

Using the operator (7), relation (6), and the wave function (4), we can obtain the abovementioned probability for the magnetic dipole transition between states whose κ value differ by two units (and $\Delta J = 1$):

$$B(M1, J+1 \to J)$$

$$= \frac{\lambda}{mc^2}\left\langle\left|(n_{z'}^2 - n_{x'}^2)\, r\,\frac{\partial U}{\partial r}\right|\right\rangle \frac{3}{4\pi}\left(\frac{e\hbar}{2mc}\right)^2 \frac{2J+1}{2^4(J+1)}. \quad (8)$$

The diagonal matrix element appearing in this expression is taken between the wave functions φ_0 and can be determined from the magnitude of the doublet splitting in the rotational spectrum of the corresponding nucleus.[1,3] Writing the Hamiltonian related to the rotational terms in the form

$$H_{\mathrm{coll}} = \hbar^2\varkappa^2/2I + \gamma\varkappa\mathbf{s}, \quad (9)$$

where I is the moment of inertia of the nucleus, we obtain

$$\frac{\lambda}{mc^2}\left\langle\left|(n_{z'}^2 - n_{x'}^2)\, r\,\frac{\partial U}{\partial r}\right|\right\rangle = \frac{2\gamma}{3\sqrt{5/4\pi}\,\beta\hbar^2/I}$$

with β the deformation parameter.

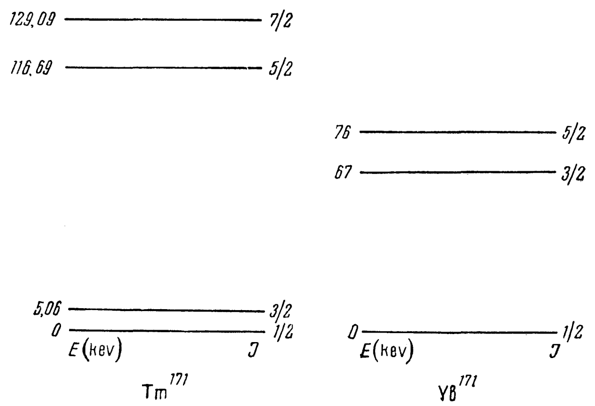
It is known from experiment that the rotational levels of spin $\frac{1}{2}$ nuclei have doublet character. The transitions between the components of different doublets with $\Delta J = 1$ are essentially of the magnetic dipole type. The electric quadrupole admixture apparently does not exceed 10 to 20%.

To determine the possible order of magnitude of the probability for magnetic transitions, we calculate the ratio $T(M1)/T(E2)$ for a transition with an energy of 110 kev in the rotational spectrum of Tm^{169}.[4] For this nucleus $\beta \approx 0.3$ (reference 5) and $\gamma/(\hbar^2/I) = 0.24$ (reference 4). Using (8) and (9), we then obtain for the ratio $T(M1)/T(E2) \approx 4$.

It follows from this estimate that the spin-orbit effect can account for the observed ratio of the probabilities in these transitions with an odd proton. In the case of an extra neutron one must introduce an effective charge arising from the interaction of the neutron with the nuclear core.[6]

In any case, it should be noted that the magnetic moment,[3] the character of the rotational spectrum, and the electric transition probabilities for the nucleus Tm^{169} do not stand in contradiction to our proposed classification.

It is also interesting to note that data on the Coulomb excitation of the Yb^{171} nucleus have recently appeared.[7] This nucleus is a daughter of Tm^{171} in β decay, whose ground state has spin $\frac{1}{2}$. The Coulomb excitation gives rise to two levels of Yb^{171}. Their energies and spins are given in the figure. In the same figure we also give the energies and spins of the rotational levels of Tm^{171}, for comparison.[8]



If our classification is correct, it follows necessarily from the form of the rotational spectra that the ground state of Tm^{171} corresponds to the Σ_u^+ one-nucleon term ($\frac{1}{2}^-$), and the ground state of Yb^{171} to the Σ_g^+ term ($\frac{1}{2}^+$). This corresponds to the fact that Tm^{171} changes its parity during β decay.

This classification in general does not contradict the available data on the rotational levels of spin $\frac{1}{2}$ nuclei. There is, however, some disagreement in the parities of the ground states of several nuclei, as given by our classification and that of Nilsson.[9] For example, we ascribe negative parity to the ground state of the Tm^{169} nucleus. while in the Nilsson scheme this nucleus has positive parity.[5]

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