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EMPIRICAL REGULARITIES IN THE NUCLEON PAIR PRODUCTION ENERGIES IN NUCLEI

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The variation of the pairing energies of nucleons and of the energy of the residual n-p interaction of odd nucleons is investigated on the basis of the most recent experimental data. It is found that the pairing energy depends not only on the total angular momentum of the nucleons but also on the position of the pair in the nuclear shell and on the deformation of the nucleus. The decrease in pairing energy with mass number A is slower than predicted by the theory. The pairing energy remains almost constant if two nucleons of different types and added to the nucleus. The energy of the residual n-p interaction of the odd nucleons is not zero, decreases with A, and is smaller than the pairing energy.

MODERN theory of nuclear structure starts out, in the zero approximation, from the concept of a separate nucleon moving independently of all remaining nucleons in the averaged field of the nucleus. Mayer[1,2] was able to explain the existence of nuclear shells for a spherically symmetrical nuclear field, assuming the existence of a strong spin-orbit coupling. However, many experimental data indicate that the single-particle approximation is inadequate. The total nuclear angular momentum j, which is the sum of the orbital momentum l and the spin s, equals zero in all even-even nuclei. This experimental fact can be attributed to the presence of a residual interaction between the nucleons. Identical nucleons with equal but opposite j form pairs with zero angular momentum. The presence of this residual interaction and the production of pairs is confirmed by the fact that the joining energy of, say, an even neutron is always greater than the nucleon binding energy of the preceding odd neutron.

The difference in the nucleon binding energies between the even neutron and the preceding odd neutron is the energy of production of a pair of neutrons in the nucleus, and is called the pairing energy P_n of the neutrons. The proton pairing energy P_p is analogously defined.

Considering the residual interaction between identical nucleons with equal but opposite directions of j, Mayer and Jensen[1,2] calculated the pairing energy P. They used delta forces as the interaction forces. This yielded for the pairing energy

$$P = -C(2j+1)/A, \qquad (1)$$

where A is the mass number of the nucleus to which the pair is joined and C is a constant that depends on the type of potential. Expression (1) should be valid only near filled nuclear shells, since it considers the residual interaction of only two nucleons.

Birbrair and Sliv[3] have shown that the presence of spherical symmetry in even-even nuclei that are not very close to nuclei with filled shells is explained by the existence of pairing energies. They have also shown that in deformed nuclei the pairing energies are lower than in spherically-symmetrical nuclei.

The author of this paper[4] has attempted to ascertain the dependence of the pairing energy and the total angular momentum j. The experimental data of 1952 did not disclose this dependence. Quisenberry, Scolman, and Nier,[5] Johnson and Nier,[6] and Giese and Benson[7] also failed to find a dependence of the pairing energies on j in new, more accurate experimental data.

Nomoto[8] checked the experimental values of the pairing energy against the Mayer formula (1) for nuclei with mass numbers less than 115. Although the constant C was indeed calculated in reference 8, the curves show that the agreement between the experimental data and formula (1) is not good enough.

The purpose of this paper is an attempt to establish the principal regularities in the variation of pairing energies, using the latest experiments. It is proposed here not only to compare the principal theoretical laws of Mayer and of Birbrair and Sliv with experiment, but also to point to new regularities in regions where the existing theories do not give satisfactory results.

1. EXPERIMENTAL DATA AND CALCULATION OF PAIRING ENERGIES

The initial experimental data for the calculation of the pairing energies are the masses of the atoms and binding energies of the nuclei calculated from them. For isotopes in the interval from hydrogen to zinc $(1 \leq Z \leq 30)$ and from xenon to europium $(54 \leq z \leq 62)$, use was made of masses and binding energies calculated from the values of the mass-spectrometric doublets by Quisenberry, Scolman, Nier, Giese, Benson, and Johnson[5-7,9-11] and from the reaction and decay energies. These masses and binding energies are listed in a review by the author.[12] The masses of the atoms and the binding energies from gallium to xenon $(31 \leq Z \leq 54)$ are taken from the Wapstra mass tables.[13]

The nucleon binding energies of the last neutrons in the nuclei of the isotopes from gadolinium to lead $(64 \leq Z \leq 82)$ are taken from the measurements of Johnson and Bhanot.[14,12] The atomic masses and the binding energies of the nuclei from thallium to uranium $(81 \leq Z \leq 92)$ are taken from the tables of Huizenga.[15] The nucleon binding energy of the last neutrons and protons in the nuclei of the isotopes from uranium to fermium $(92 \leq Z \leq 100)$ are taken from the survey of Hyde and Seaborg.[16]

The pairing energy of the N-th and the $(N-1)$-th neutrons in a nucleus with protons is

$$P_n(Z, N) = B_n(Z, N) - B_n(Z, N-1), \qquad (2)$$

where N is an even number and $B_n(Z, N)$ is the nucleon binding energy of the last N-th neutron to the nucleus with Z protons.

Analogously, the pairing energy for the Z-th and $(Z-1)$-th protons in a nucleus with N neutrons is

$$P_p(Z, N) = B_p(Z, N) - B_p(Z-1, N), \qquad (3)$$

where Z is an even number and $B_p(Z, N)$ is the energy of the binding of the last Z-th proton to the nucleus with N neutrons.

For a description of the total binding energy of the nuclei, $E(Z, N)$, use is made of a geometrical representation, namely the energy surface $E = f(Z, N)$ in the three-dimensional space E, Z, N. This surface has four branches: even-even (even Z and even N), even-odd (even Z and odd N), odd-even (odd Z and even N), and odd-odd (Z and N both odd).

After determining the joining energy of the last neutrons, we can express the pairing energy in the terms of the total binding energy of the nuclei $E(Z, N)$ as follows:

$$P_n(Z, N) = 2\{{}^1/_2[E(Z, N) + E(Z, N-2)] - E(Z, N-1)\},$$

$$P_p(Z, N) = 2\{{}^1/_2[E(Z, N) + E(Z-2, N)] - E(Z-1, N)\}. \qquad (4)$$

For even Z and N it is easy to verify that $P_n(Z, N)$ is twice the distance along the E axis between the even-even and even-odd energy surfaces. $P_p(Z, N)$ is the distance along the E axis between the even-even and odd-even energy surfaces. From the well-known Weizzäcker-Fermi semi-empirical formula for the binding energies,[17] we can express the pairing energy from (4) as follows

$$P = -2\delta A^{-3/4}. \qquad (5)$$

In the present paper we consider in addition to the pairing energy also the residual reaction between the odd neutrons and protons. To determine this interaction, we have calculated from the experimental data the value of R_{np}, the difference between the energy of simultaneous binding of a neutron and proton to an even-even nucleus (Z, N) and the sum of the energies of the binding of the neutron and proton separately to the same nucleus

$$R_{np}(Z+1, N+1) = B_{np}(Z+1, N+1) - B_p(Z+1, N) - B_n(Z, N+1),$$

where

$$B_{np}(Z+1, N+1) = E(Z+1, N+1) - E(Z, N). \qquad (6)$$

Here Z and N are even numbers.

We shall call the quantity R_{np} the residual n-p interaction. To calculate R_{np} it is more convenient to use other formulas:

$$R_{np}(Z+1, N+1) = B_p(Z+1, N+1) - B_p(Z+1, N) = B_n(Z+1, N+1) - B_n(Z, N+1). \qquad (7)$$

If B_n and B_p are considered negative, all values of the residual n-p interaction energy are always negative, within the limits of experimental errors.

Certain other authors[5-7] consider $P_n(Z, N)$ with odd Z and $P_p(Z, N)$ with odd N, and call these quantities pairing energies, too. In view of the presence of an odd nucleon of another type in the nucleus, these quantities depend also on the residual n-p interaction and therefore should not be called pairing energies; they are not considered in the present paper.

2. VARIATIONS OF PAIRING ENERGIES AND OF THE RESIDUAL n-p INTERACTION

To investigate the variation of the pairing energies we plotted the graphs shown in Figs. 1 — 5. The first three show the dependence of the pairing energy of the neutrons P_n on the number of neutrons N in the nucleus. All the points representing the pairing energies in the nuclei of isotopes of the same element are connected by straight lines. The broken lines thus formed are tagged with the symbol of the element. The fraction near each point represents the total angular momentum of the preceding odd neutron, known from experiment or assumed from the decay data; the values of j are taken from the tables of Dzhelepov and Peker[18] and from the paper by Peker.[19]

Figures 4 and 5 show the depencence of the pairing energy of protons P_p on the number of the protons Z in the nucleus. As in Figs. 1 — 3, the points representing the pairing energy in nuclei with equal number of neutrons are joined by a broken line, tagged with the number of neutrons in these nuclei. The fraction at each point indicates the value of j of the preceding odd proton, taken from references 18 and 19.

Everywhere in the intervals $4 \le N \le 40$ (Fig. 1) and $4 \le Z \le 30$ (Fig. 4), the errors in the pairing energies are less than the circles representing the points, with the exception of the cases represented by vertical segments. In the intervals $40 \le N \le 56$ (Fig. 1) and $32 \le Z \le 42$ (Fig. 4), the errors in the pairing energies increase approximately from ±0.30 Mev to ±0.60 Mev. In Fig. 2, in the interval $78 \le N \le 90$, the error in P_n fluctuates about ±0.15 Mev, and in the interval $92 \le N \le 126$ the error is ±0.03 Mev almost everywhere in the region of the heavy nuclei. In Figs. 3 and 5, the pairing energies have on the average errors from ±0.10 to ±0.30 Mev.

In the regions missing from the diagrams, the mass-measurement accuracy is insufficient for the calculation of the pairing-energy values suitable for the investigation of the regularities.

A study of the course of variation of the pairing energies with the number of nucleons in the nuclei and with their total angular momentum j shows that the pairing energies frequently depend on j. We see in Figs. 1, 3, and 4 that the pairing energy of nucleons with $j = 1/2$ is as a rule lower than that of nucleons with larger j (for example, in Fig. 1 for N = 8 and 16, in Fig. 3 for N = 126, in Fig. 4 for Z = 8, 16, and 40, and in other cases). There are exceptions, however; for example, in Fig. 1 at N = 40 the pairing energy for nucleons with $j = 1/2$ is higher than at N = 34 with $j = 3/2$, with the values of P_n for zinc being shown to a high degree of accuracy.

It is seen from the curves that most frequently the pairing energy increases with increasing j, but the increase in the pairing energy is apparently slower than $(2j + 1)$.

From an examination of the intervals $20 \le N \le 28$ $(j = 7/2)$ and $38 \le N \le 50$ $(j = 9/2)$ in Fig. 1 and of the intervals $20 \le Z \le 28$ $(j = 7/2)$ and $30 \le Z \le 40$ $(j = 3/2)$ in Fig. 4 it can be seen that for equal j, the pairing energy depends on the position of the pair in the shell. As the shell becomes filled, the pairing energy decreases. The pairing energy reaches a maximum value either at the middle of the shell, or at the beginning.

An examination of the curves for P_n in Figs. 1 — 3 shows that the neutron pairing energy P_n

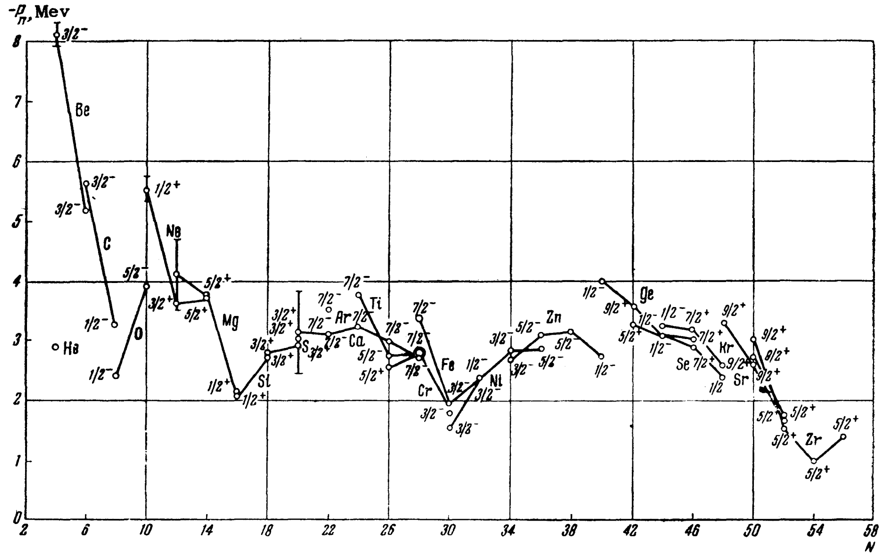

FIG. 1. Pairing energies of neutrons in nuclei with even atomic number Z, for the 4th to the 56th neutrons. Located near each point is the value of the spin of the preceding odd neutron, known from experiment. The broken lines are tagged with the symbol of the element in whose nucleus the pair is produced.

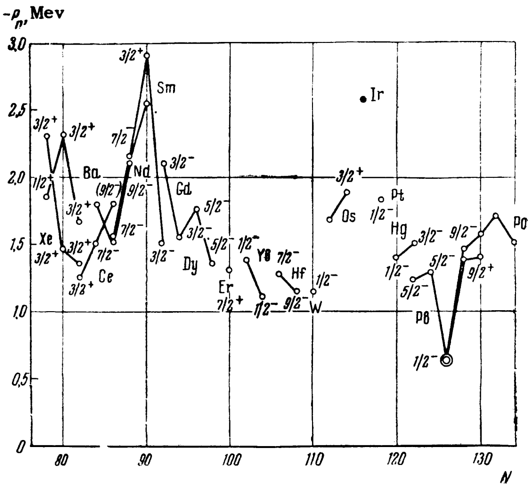

FIG. 2. Pairing energies of neutrons in nuclei with even atomic number Z for the 78th to 132d neutrons. All other data are as in Fig. 1: o – points for even-even nuclei, • – point for the 116th neutron, which produces a pair in the odd-even nucleus of iridium.

depends little on the number of even protons in the nucleus, Z, since most frequently the spread of the points at given values of N barely exceeds the errors. As can be seen from Figs. 4 – 5, the pairing energy of the protons P_p also depends little on the number of even neutrons N in the nucleus, although in many cases the spread in the points, at the given Z, is somewhat greater than in the case of P_p. This experimental fact shows that the residual energy of the n-p interaction is small for the nucleons that produce the pairs.

In many cases, as indicated by Mayer,[1] when levels with large j are filled, the last odd nucleon may occupy the nearest level with small j,

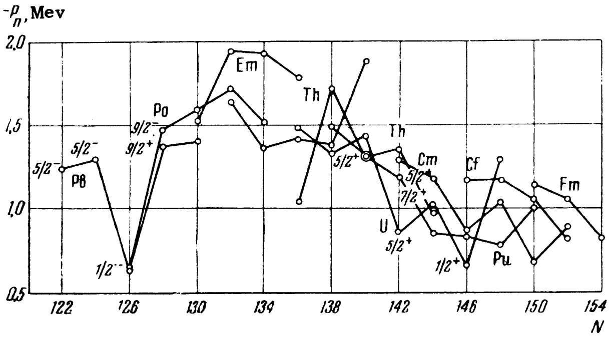

FIG. 3. Pairing energies of neutrons in nuclei with even atomic number Z for the 122d to 154th neutrons. The rest is as in Fig. 1.

this being explained by the presence of a large pairing energy at large j. When the next even nucleon appears, the odd nucleon goes to a level with large j and a pair with large j in the opposite direction is produced. For example, in Fig. 1 in the interval $42 \leq N \leq 50$, according to the Mayer scheme, the level with $j = 9/2$ should be filled, but along with this j we find also $j = 1/2$. Here the $j = 1/2$ appear at the middle and at the start of the level, where P_n is large, and upon approaching the end of the shell, when P_n becomes smaller, the large $j = 9/2$ appear more frequently. This apparently confirms the Mayer hypothesis concerning the pairing energy and the appearance of individual nucleons at levels with small j.

The increase in pairing energy halfway between shells can explain to some extent the increased value of the pairing energy P_n at $N = 40$ (Fig. 1), for here we deal exactly with the middle of the shell, between $N = 28$ and $N = 50$.

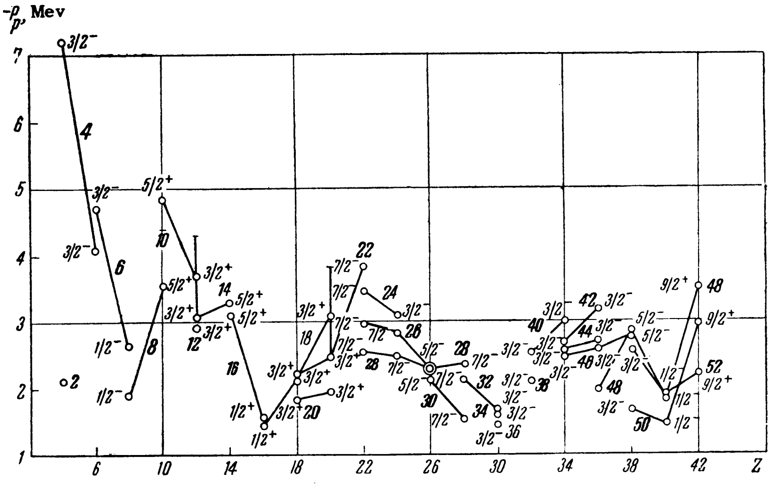

FIG. 4. Pairing energies of protons in nuclei with even number of neutrons, for the 4th to 42d protons. Each broken line is tagged with the number of neutrons in the nucleus in which the pair is produced. Near each point is the experimentally-known value of the spin of the preceding odd proton.

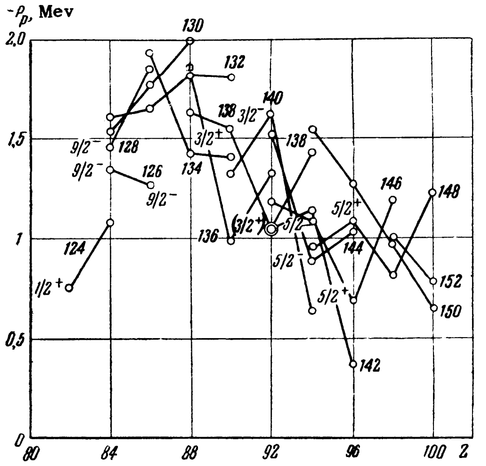

FIG. 5. Pairing energies of protons in nuclei with even number N, for the 82d to the 100th protons. The rest is as in Fig. 4.

The new mass values given in references 5 — 7 and 9 — 11 no longer give rise to the subshells mentioned by the author[20] for the case of $N = 34, 40$, etc. The same is confirmed by Zeldes,[21] who, using the new values of the masses, sought but could not find the same subshells. The decrease in P_p at $Z = 40$ is possibly due to the fact that the neutron shell $N = 50$ is filled in this region. This confirms the theoretical premise that the Mayer theory, including formula (1) for the pairing energy, is suitable only in the vicinity of nuclei with closed shells.

Figure 3 shows graphs for the variation of pairing energies in the region of deformed nuclei of rare-earth isotopes, the interval $90 \leq N \leq 114$. The neutron pairing energy rises sharply and reaches a maximum on the boundary of the region of deformed nuclei at $N = 90$. Within the region of deformed nuclei the neutron pairing energy drops toward the middle of the shell and reaches a minimum at $N = 110$ in tungsten. This is followed by a rise with a maximum at $N = 116$ in iridium ($Z = 77$). The point $N = 116$ was taken for an odd-even nucleus, since P_n was not measured for even Z for $N = 116$. On the basis of the experimental law indicated by Quisenberry et al.,[5] the residual interaction energy for even-even nuclei is always less than the pairing energy. Consequently, the point at $N = 116$ for the true pairing energy of even-even nuclei should lie above the point shown for iridium in the curve.

In the region of light deformed nuclei near magnesium on Fig. 1 at $10 \leq N \leq 14$ and on Fig. 4 at $10 \leq Z \leq 14$, it is also possible to observe a reduction in the pairing energy for deformed nuclei. For heavy deformed nuclei at $Z \geq 88$ in Figs. 3 and 5, we also note a drop in the pairing energies. The drops in the pairing energies in the region of the deformed light and heavy nuclei are not as clearly pronounced as in the rare-earth region. This reduction in the pairing energies for deformed nuclei was theoretically explained by Birbrair and Sliv.[3]

Figures 6 and 7 show the dependence of the residual n-p interaction (R_{np}) on the number of neutrons and on the number of protons for light and heavy nuclei. The ordinates represent R_{np} and the abscissas the number of neutrons in the nucleus N; the points pertaining to nuclei with the same Z are joined by straight lines. It is seen from the curves that R_{np} is everywhere negative, with two exceptions that do not exceed the errors. R_{np} diminishes with mass number A in the nucleus. If R_{np} is compared with the pairing energies we see that on the average, in nuclei with equal mass numbers, $|R_{np}| < |P|$, i.e., the energy of the residual n-p interaction is less in absolute magnitude than the pairing energy.

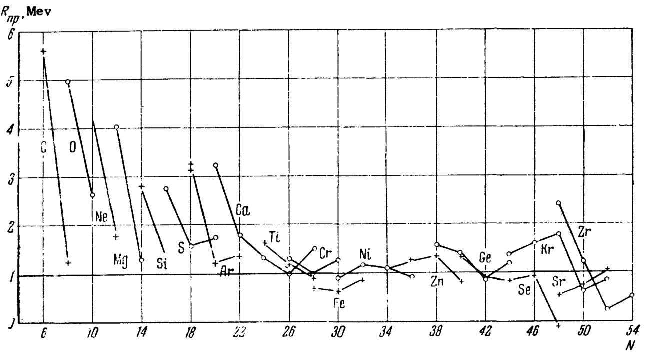

FIG. 6. Energy of residual n–p interaction of odd nucleons in nuclei of isotopes from carbon to zirconium, as a function of the number of neutrons N in the nucleus to which the nucleons are joined. Each broken line is tagged with the symbol of the element to whose nucleus the nucleons are joined.

Nomoto[8] compared the Mayer formula (1) for the pairing energy with the experimental data for nuclei with $10 \leq A \leq 120$. It is clearly seen from Nomoto's curves that the experimental values diminish more slowly than $1/A$, as would follow from (1). Since the dependence of the pairing energy on j is also subject to doubt, the dependence on A was investigated for only one fixed j. A total momentum $j = 1/2$ was chosen, since almost all nucleons with $j = 1/2$ are found in nuclei close to nuclei with filled shells, and these nuclei fit the Mayer model better. The P_n at $N = 40$ were omitted in the investigation, since a nucleus with these values of P_n is relatively far away from the nuclei with filled shells. We considered the pairing energies of all spherical nuclei with $j = 1/2$, including heavy nuclei, but omitting $P_n(40)$. We calculated the products PA, $PA^{3/4}$, and

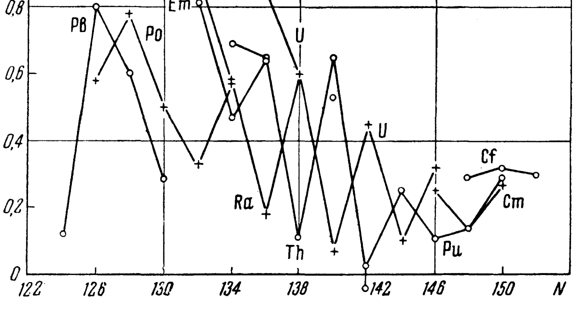

FIG. 7. Energy of residual n–p interaction of odd nucleons in nuclei of the isotopes from mercury to curium. The rest is the same as in Fig. 6.

$PA^{1/2}$. The smallest spread is in the product $PA^{1/2}$. Thus, the pairing energies of protons and neutrons with $j = 1/2$, located near the end or the beginning of the nuclear shells, diminish in inverse proportion to $\sqrt{A}$.

3. CONCLUSIONS

A study of the experimentally-obtained pairing energies of nucleons (mass-spectroscopic measurements, measurements of the reaction and decay energies) parmits the following conclusions:

1. The pairing energy is usually greater for nucleons with large total angular momenta j. The increase in pairing energy with j is slower than (2j+1). A deviation from this rule is found only for nuclei with a number of nucleons that differ substantially from magic.
2. The pairing energy increases as the number of pairing nucleons deviates from magic.
3. The pairing energy of the neutrons depends little on the number of paired protons in the nucleus. The analogous rule for protons is less clearly pronounced.
4. The course of variation of the pairing energies and of j confirm the Mayer hypothesis that in even-odd nuclei the odd nucleons may temporarily occupy near-lying levels with small j, and that in even-even nuclei they again pass to the level with larger j.
5. The pairing energies have a large maximum at the beginning of the region of deformed nuclei, then drop to a minimum in the middle of the region of deformed nuclei, rising again towards the end of the region.
6. Residual n-p interaction energies different from zero exist for odd nucleons. The residual n-p interaction diminishes with increasing mass number A and is always less than the pairing energy.
7. The decrease in pairing energies with increasing mass number A is slower than 1/A. In nuclei close to those with filled shells, the pairing energy of nucleons with $j = 1/2$ diminishes in inverse proportion to $A^{1/2}$.

For further refinement of the course of variation of pairing energies it is desirable to obtain more accurate values for the masses of atoms heavier than zinc. Of particular importance are measurements of stable isotopes of molybdenum and ruthenium, and of the masses of the isotopes from gadolinium to lead.

In conclusion, I consider it my pleasant duty to thank B. L. Birbrair and L. A. Sliv for discussing the results of this investigation.

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