DEVELOPMENT OF THE NUCLEAR-ACTIVE COMPONENT OF EXTENSIVE AIR SHOWERS

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The spectrum of the nuclear-active particles in extensive atmospheric showers and the particle and absorption ranges are computed, and the rate of structure bursts is estimated, on the basis of certain simple assumptions regarding the nature of the elementary act. It is shown that the extensive atmospheric shower has certain characteristics that depend weakly on the nature of the elementary act, and certain characteristics that are sensitive to the latter.

THE Guzhavin's and Zatsepin[1] calculated the altitude variation of nuclear-active (n.a.) high-energy particles and of the number of high-energy muons at sea level. They also calculated the altitude variation of the nuclear-active and soft components of extensive atmospheric showers (e.a.s.). The elementary act for super-high energies was chosen in one of the variants of calculations after Landau.[2] In another variant it was assumed, in accordance with Vernov's hypothesis[3] that the nucleon always retains approximately 70% of the energy of the incident particle. The effective cross section for the collision was assumed constant for all energies and corresponding to a free path in air $\lambda_0 = 65 - 70$ g/cm^2.

It was found that the results of the calculations depended greatly on λ_0. We note that there is no known experimental value of λ_0 at energies $\gtrsim 10^{10}$ ev. We therefore undertook a calculation of various characteristics of nuclear-active component of extensive atmospheric showers, in which the magnitude of λ_0 is determined by the type of elementary act and by the experimental value of the absorption range of nuclear-active high-energy particles ($\approx 10^{12}$ ev). By making simple assumptions concerning the character of the elementary act, we calculated the spectrum of the n.a. particles in the e.a.s., the absorption ranges of n.a. particles, and the energy fluxes in the showers. We also estimated the probability of observation of one or two n.a. high-energy particles at a given level. The principal purpose of the calculation is to determine the e.a.s. characteristics that are sensitive to the character of interaction, by varying different parameters of the elementary act and the magnitude of λ_0.

Let us assume that upon collision between n.a. particles of energy ϵ' with an air nucleus, the probability of observing secondary n.a. particles of energies $\epsilon_1, \ldots, \epsilon_n$ is a function of only the ratios ϵ_i/ϵ', and can be written

$$\Phi(\varepsilon', \varepsilon_1, \ldots, \varepsilon_n) = \delta(\alpha_1 - \varepsilon_1/\varepsilon') \ldots \delta(\alpha_n - \varepsilon_n/\varepsilon'), \quad (1)$$

where α_i = const.

We assume that the energy carried away by the secondary n.a. particles amounts to $\sim 70 - 80\%$ of the energy of the incident particle. The assumed fraction of the energy retained by the secondary n.a. particles takes into account the transformation of a portion of the energy into π^0 mesons, and also considers to some extent the decay of the $\pi^\pm$ mesons. If we assume that the principal portion of the energy of the primary particle is carried away by π mesons and nucleons, their energy should be greater than $\sim 60\%$ of ϵ'. Analogous conclusions concerning the fraction of the energy remaining in the nuclear-active component were reached by Rozental' and Gerasimova in reference 5.

Taking this into account, we limit our analysis to the following three variants: 1) the collision produces one particle with energy $0.7\,\epsilon'$ ($\alpha_1 = 0.7$); 2) two particles of essentially different energies, $0.7\,\epsilon'$ and $0.1\,\epsilon'$ ($\alpha_1 = 0.7$, $\alpha_2 = 0.1$), are produced; 3) seven particles are produced, each with energy $0.1\,\epsilon'$ ($\alpha_1 = \ldots = \alpha_7 = 0.1$).

We shall calculate the spectrum of the n.a. particles in the shower for each of the three variants. For this purpose it is necessary to know the range for the interaction, λ_0. Assuming that the range for the interaction is independent of the energy, we determine its magnitude, using data on the absorption of high-energy n.a. particles in the atmosphere. It is known that if the spectrum of the secondary particles is written in form (1), and the spectrum of the primary cosmic radiation is in the form of a power law, the following relation holds[6]

$$\lambda_0 = \lambda\left(1 - \sum_{i=1}^{n} \alpha_i^{\gamma}\right), \qquad (2)$$

where λ is the range for absorption of n.a. high-energy particles in the atmosphere, and γ is the exponent of the integral spectrum of the primary cosmic radiation.

By taking $\lambda = 120$ g/cm^2 (reference 4) and $\gamma = 1.6$ (reference 7), we obtain for the first case $\lambda_0 = 52.7$ g/cm^2, for the second case $\lambda_0 = 50.2$ g/cm^2, and for the third case $\lambda_0 = 99.0$ g/cm^2.

Knowing λ_0, we calculate the average number of n.a. particles, $N(\epsilon, x)$, in an e.a.s. with energy $\geq \epsilon$, at a depth x (ϵ is the energy of the n.a. particles as a fraction of the energy E_0 of the particle producing the shower). The depth of the atmosphere x is measured in units of λ_0.

The average number of particles with energy ϵ at a depth x can be written in the following form (see reference 8)

$$N(\varepsilon, x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} e^{-x} F_k(\varepsilon),$$

where in the absence of decay

$$F_k(\varepsilon) = \int_{\varepsilon}^{1} F_{k-1}(\varepsilon') W(\varepsilon', \varepsilon) d\varepsilon'.$$

$W(\epsilon, \epsilon')$ is the probability of observing secondary particles with energies ϵ upon collision of a particle with energy ϵ' with a nucleus; this probability is determined as follows:

$$W(\varepsilon', \varepsilon) d\varepsilon = \int_0^{\varepsilon'} \ldots \int_0^{\varepsilon'} \Phi(\varepsilon', \varepsilon, \varepsilon_2, \ldots, \varepsilon_n)(\varepsilon')^{-n} d\varepsilon d\varepsilon_2 \ldots d\varepsilon_n$$
$$+ \int_0^{\varepsilon'} \ldots \int_0^{\varepsilon'} \Phi(\varepsilon', \varepsilon_1, \varepsilon, \varepsilon_3, \ldots, \varepsilon_n)(\varepsilon')^{-n} d\varepsilon_1 d\varepsilon d\varepsilon_3 \ldots d\varepsilon_n + \ldots$$
$$+ \int_0^{\varepsilon'} \ldots \int_0^{\varepsilon'} \Phi(\varepsilon', \varepsilon_1, \ldots, \varepsilon_{n-1}, \varepsilon)(\varepsilon')^{-n} d\varepsilon_1 \ldots d\varepsilon_{n-1} d\varepsilon$$
$$= \sum_{i=1}^{n} \delta\left(\alpha_i - \frac{\varepsilon}{\varepsilon'}\right) \frac{d\varepsilon}{\varepsilon'}. \qquad (3)$$

By calculating the e.a.s. produced by the primary particle of energy E_0, we obtain $F_0(\epsilon) = \delta(1-\epsilon)$. Then in our case

$$F_1(\varepsilon) = \int_{\varepsilon}^{1} d\varepsilon' \delta(1-\varepsilon') \sum_{i_1=1}^{n} \delta\left(\alpha_{i_1} - \frac{\varepsilon}{\varepsilon'}\right) \frac{1}{\varepsilon'} = \sum_{i_1=1}^{n} \delta(\alpha_{i_1} - \varepsilon),$$
$$F_2(\varepsilon) = \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \delta(\alpha_{i_1}\alpha_{i_2} - \varepsilon), \ldots, F_k(\varepsilon)$$
$$= \sum_{i_1=1}^{n} \ldots \sum_{i_k=1}^{n} \delta(\alpha_{i_1} \ldots \alpha_{i_k} - \varepsilon).$$

After calculating $F_k(\epsilon)$ we can determine the integral energy spectrum of n.a. particles at a given depth, since

$$N(\geqslant \varepsilon, x) = \int_{\varepsilon}^{1} N(\varepsilon', x) d\varepsilon' = \sum_{i=0}^{\infty} \frac{x^i}{i!} e^{-x} F_i(\geqslant \varepsilon), \qquad (4)$$

where

$$F_i(\geqslant \varepsilon) = \int_0^1 F_i(\varepsilon') d\varepsilon'.$$

The energy dependence of the average number of n.a. particles with energies $\geq \epsilon$ in e.a.s. at sea level, calculated from formula (1) for the three variants of the elementary act, is shown in Fig. 1.

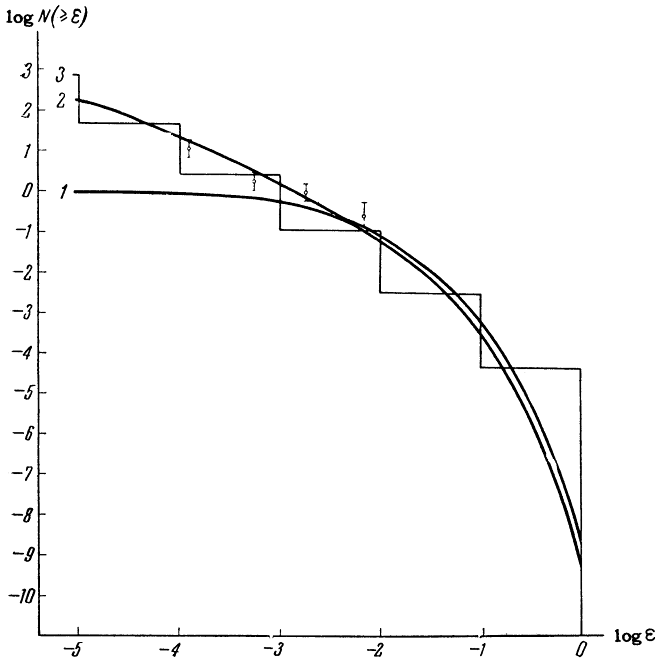

FIG. 1. Energy dependence of the average number of the nuclear-active particles with energy $\geq \varepsilon$ in e. a. s. at sea level. The curves are calculated from formula (4) for three respective variants of the elementary act. The points indicate the experimental results taken from reference 9.

To compare the resultant spectra with experiments, we used the data obtained in reference 9, shown as dots in Fig. 1. The authors of reference 9 measured the average number of n.a. particles with energies $\geq 10^{12}$ ev in showers with different numbers of particles, N_s, at sea level.

We assume that E_0 can be determined with sufficient accuracy from the number of shower particles.* We can then find E_0 with the aid of the relation $E_0 = kN_s$, derived by Cocconi[1] from an analysis of the altitude variation of the showers. At sea level we have $k = 10^{10}$ ev/particle.

*Estimates as well as calculations carried out by Grigorov and Shestoperov[10] show that this assumption is satisfactory when N_s is large ($\gtrsim 10^3$ particles).

As can be seen from Fig. 1, the variation of $N(\geq \epsilon)$, calculated according to the first variant, contradicts the experimental data, and will not be discussed further. $N(\epsilon)$, calculated with the aid of the second and third variants, agree with experiment. Our calculations show that variants intermediate between the second and third (for example, $\alpha_1 = 0.5$, $\alpha_2 = 0.12$, $\alpha_3 = 0.06$ and $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, $\alpha_3 = 0.1$) also give good agreement with experiment. We note that spectra of n.a. particles at $10^{-5} \leq \epsilon \leq 1$ depend little on the presence of secondary low-energy particles during the collision. Thus, for example, if ten more particles are produced in the third variant, each with energy 0.01 of the energy of the incident particle, then $N(\geq 10^{-2})$ increases by the factor of 1.2, and $N(\geq 10^{-4})$ increases by 1.02 times. If still another 100 particles are formed with energies of 0.001 each, then $N(\geq 10^{-3})$ increases by 1.3 times.

The interaction of n.a. particles need not always occur in the same manner; for example, it may occur with a certain degree of probability in accordance with the second variant or according to the third. It is then to be expected that, with a suitable choice of λ_0, the calculated and experimental values will agree.

We note also that the function $N(\geq \epsilon, x)$ depends on the form of the function $W(\epsilon', \epsilon)$. Since the function $W(\epsilon', \epsilon)$ is obtained from Φ by formula (3) it may have the same form even if the function Φ has different forms. This will occur, for example, for

$$\Phi_1 = \delta(0.1 - \varepsilon_1/\varepsilon') \ldots \delta(0.1 - \varepsilon_7/\varepsilon')(\varepsilon')^{-7}$$

and

$$\Phi_2 = {}^1/_2\delta(0.1 - \varepsilon_1/\varepsilon') \ldots \delta(0.1 - \varepsilon_6/\varepsilon')(\varepsilon')^{-6} + {}^1/_2\delta(0.1 - \varepsilon_1/\varepsilon') \ldots \delta(0.1 - \varepsilon_8/\varepsilon')(\varepsilon')^{-8},$$

although in the first case seven particles are always produced in a nuclear collision, and in the second case six or eight particles may be produced, with probabilities of $\frac{1}{2}$.

Thus, an analysis of the spectrum of the n.a. particles in a shower for one level (if the value of λ_0 is not known from experiment and is determined from the absorption range of n.a. particles of energy $E \gtrsim 10^{12}$ ev) does not make it possible to determine, even approximately, the characteristics of the elementary act, if there is no possibility of observing the nuclear-active particles in the interval $0.1 \geq \epsilon \geq 1$. Therefore, in addition to calculating $N(\geq \epsilon)$ in e.a.s. for sea level, we performed analogous calculations for two other depths, 650 and 1250 g/cm^2.

On the basis of these calculations we obtained values of ranges for the absorption of n.a. particles, $\lambda_N(\geq \epsilon)$ with energies $\geq \epsilon$ and for the absorption of the energy $\lambda_E(\geq \epsilon)$ carried away by these particles (Fig. 2). As can be seen from the diagram, the quantities $\lambda_N(\geq \epsilon)$ and $\lambda_E(\geq \epsilon)$, calculated for the second and third variants, do not agree. Therefore an experimental measurement of these absorption ranges may prove important for the determination of the type of elementary act at super-high energies.

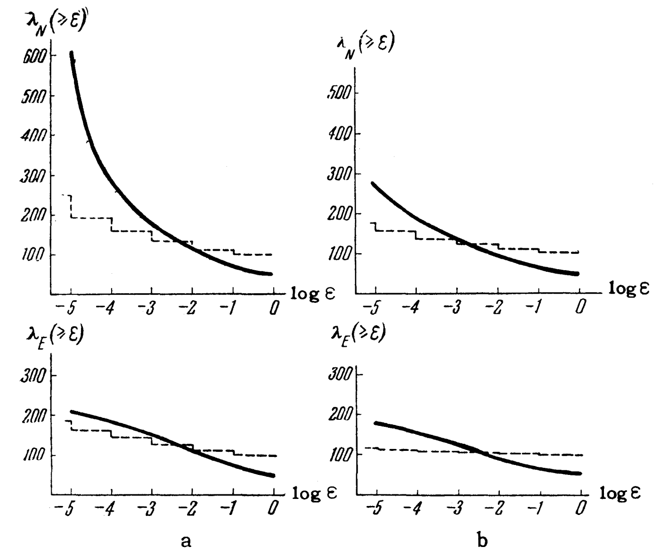

FIG. 2. Absorption ranges of nuclear-active particles $\lambda_N(\geq\varepsilon)$ with energy $\geq\varepsilon$, and absorption range $\lambda_E(\geq\varepsilon)$ for the energy carried by these particles: a – for the depth interval 650-1000 g/cm^2 for the second (solid) and third (dotted) interaction variants; b – the same for 1000 – 1250 g/cm^2.

Let us calculate, for the second and third variants, the probability of observing at a depth x_0 in a shower one particle $P(1, x_0)$ or two particles $P(2, x_0)$ of energy $\epsilon \geq 0.1$. For the second variant, the probability $P(1, x_0)$ can be written

$$P(1,x_0) = \int_0^{x_0} Q_1(x)\{Q_1(x_0 - x)\,Q_2(x_0 - x) + Q_3(x_0 - x)\,Q_4(x_0 - x)\}\,dx. \qquad (5')$$

Here $Q_1(x)\,dx = e^{-x}dx$ determines the probability of the primary particle interacting at a depth $x, x + dx$. As a result of this interaction there are formed at depth x two particles with energies $\epsilon = 0.7$ and $\epsilon = 0.1$. $Q_1(x_0 - x)$ determines the probability that a particle with energy $\epsilon = 0.1$ will traverse a distance $x_0 - x$ without interaction;

$$Q_2(x_0 - x) = 1 - (1 + \ldots + (x_0 - x)^6/5!)\exp\{-(x_0 - x)\}$$

determines the probability of a particle with energy $\epsilon = 0.7$ interacting more than five times along a path $x_0 - x$ and reaching the observation level with energy $\epsilon < 0.1$. $Q_3(x_0 - x) = 1 - Q_1(x_0 - x)$ determines the probability of a particle with energy $\epsilon = 0.1$ interacting at least once and reaching the observation level with $\epsilon < 0.1$. $Q_4(x_0 - x) = 1 - Q_2(x_0 - x)$ is the probability of a particle with $\epsilon = 0.7$ interacting not more than five times along the path $x_0 - x$ and reaching the observation level with an energy $\epsilon > 0.1$.

Analogously, we obtain for $P(2, x_0)$ the expression

$$P(2, x_0) = \int_0^{x_0} Q_1(x) Q_1(x_0 - x) Q_4(x_0 - x)\, dx. \quad (6')$$

Calculating the integrals in (5′) and (6′), we get

$$P(1, x_0) = (2x_0 + x_0^2/2 + x_0^3/6 + x_0^4/24 + x_0^5/120 + x_0^6/720 - 12)\exp(-x_0) + (10x_0 + 4x_0^2 + x_0^3 + x_0^4/6 + x_0^5/60 + 12)\exp(-2x_0), \quad (5)$$

$$P(2, x_0) = 6\exp(-x_0) - (5x_0 + 2x_0^2 + x_0^3/2 + x_0^4/12 + x_0^5/120 + 6)\exp(-2x_0). \quad (6)$$

For the third variant, $P(1, x_0)$ can be written

$$P(1, x_0) = C_7^1 \int_0^{x_0} Q_1(x) Q_1(x_0 - x)[1 - Q_1(x_0 - x)]^6\, dx; \quad (7')$$

Since the interaction of the primary particle at the depth x, $x + dx$ results in the production of seven particles with energy $\epsilon = 0.1$, the probability of at least one of the seven particles traversing a distance $x_0 - x$ without interaction is $C_7^1 Q_1(x_0 - x)$, and the probability that the remaining six particles will interact at least once is $[1 - Q_1(x_0 - x)]^6$. Analogously

$$P(2, x_0) = C_7^2 \int_0^{x_0} Q_1(x) Q_1^2(x_0 - x)(1 - Q_1(x_0 - x))^5\, dx. \quad (8')$$

Here C_7^1 and C_7^2 is the number of combinations of 7 elements one and two at a time.

After calculating the integrals contained in (7′) and (8′), we get

$$P(1, x_0) = (7x_0 - 17.15)\exp(-x_0) \text{ for } \exp(-x_0) \ll 1 \quad (7)$$

$$P(2, x_0) = 3.5\exp(-x_0) \text{ for } \exp(-x_0) \ll 1/6. \quad (8)$$

The value of $P(2, x_0)/[P(1, x_0) + P(2, x_0)]$, calculated at sea level, is found to be 5×10^{-5} and 0.06 for the second and third variants respectively. Thus, this quantity depends more strongly on the type of the elementary act than the ratio of the numbers of nuclear-active particles with energies greater than the given value.*

More detailed calculations of e.a.s. may disclose, apparently, other characteristics that depend strongly on the choice of the elementary act. In particular, various characteristics of the μ-meson component of an extensive atmospheric shower may be disclosed. Thus, the extensive atmospheric shower contains, along with nuclear-active component characteristics that depend little on the type of the elementary act, also characteristics that are quite sensitive to the character of the interaction. A detailed experimental study of these characteristics may yield important data on the elementary act.

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*Reference 10 contains the so called "association curves." The association curves are the results of the spectrum of nuclear-active particles in an extensive atmospheric shower and the spectrum of primary particles, and they are therefore not discussed separately in our paper. Let us note that it follows from our calculations that there is no need for introducing two types of elementary acts to explain the association curves.