# *ANGULAR DISTRIBUTION AND ANGULAR CORRELATION OF THE RADIATIONS FROM NUCLEI WITH ORIENTED ELECTRON SHELLS*

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Submitted to JETP editor October 16, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1240-1245 (April, 1959)

The effect of an oriented electron shell on the angular correlation of nuclear radiations is investigated. The angular distribution due to this effect is obtained.

If the lifetime of the intermediate nuclear level is not small as compared to the precession period of the nuclear moment in the field of the electron shell, the interaction with the electron shell leads to a redistribution of the m-sublevels of the nucleus; then a "perturbed" correlation of the nuclear radiations is observed.[1]

Alder[2] obtained a formula which takes account of this effect for the case of an electron shell which remains in a stationary state during the nuclear transitions. Later Coester[3] investigated the deviations from the Alder formula for the case when the stationary condition is not satisfied.

Owing to the large magnetic moment the electon shells orient themselves more easily than the nucleus. It is therefore meaningful to consider the effect of the oriented electron shell on the radiation of the nucleus.

In the present paper we investigate the angular correlation of two successive radiations of the nucleus; we study the correlation of the directions as well as the polarization effects with respect to the $\alpha$, $\beta$, and $\gamma$ rays and the conversion electrons[4] coming from an oriented electron shell. The basic formula for the correlation function is different from that used in the papers of Goertzel[1] and Alder,[2] since the correlation will also depend significantly on the hyperfine structure of the initial level of the nucleus in the presence of an orientation "of the k-th order."

## THE CORRELATION FUNCTION

**1.** We consider the radiation from nuclei whose electron shells are oriented. In general, the correlation function will then depend not only on the directions of the radiations and on their polarizations, but also on the rotational symmetry axis of the total angular momenta of the electron shells $\eta$.

The probability for the emission of two rays in the nuclear cascade decay $A \to B \to C$ is given by the expression

$$W = \sum_{\beta\beta'} \mathscr{E}^{(1)}(\beta\beta') E^{(2)}(\beta'\beta), \quad (1)$$

where

$$\mathscr{E}^{(1)}(\beta\beta') = \mathsf{S}_1 \sum_{\alpha\alpha'} a(m_{e\alpha}) \frac{(\beta|H_1|\alpha)(\alpha|\alpha')(\alpha'|H_1|\beta')}{1+(\omega_{\alpha\alpha'}\tau_A)^2}, \quad (1a)$$

$$E^{(2)}(\beta'\beta) = \mathsf{S}_2 \sum_{\gamma} \frac{(\beta'|H_2|\gamma)(\gamma|H_2|\beta)}{1+(\omega_{\beta\beta'}\tau_B)^2}. \quad (1b)$$

$\mathsf{S}_i$ implies summation over all unchanged properties of the i-th radiation $(i = 1, 2)$; $\omega_{\alpha\alpha'}$ and $\tau_A$ ($\omega_{\beta\beta'}$ and $\tau_B$) give the hyperfine structure and the lifetime of the nuclear level A (B); $a(m_{e\alpha})$ is the probability that the projection of the total angular momentum of the electron shell before the beginning of the decay is $m_{e\alpha}$.

The denominators in (1a) and (1b) can be obtained from the descriptive discussions of Abragam and Pound.[5]

**2.** We choose $\eta$ as the axis of quantization z. Then the matrix element $(\beta|H_2|\gamma)$ can be written in the form

$$(\beta|H_2|\gamma) \equiv (F_b m_b|H_2|F_c m_c\,\Omega_2\sigma_2)$$
$$= \sum_{L_2 M_2 \pi_2} (\Omega_2\sigma_2|L_2 M_2 \pi_2)(F_b m_b|H_2|F_c m_c L_2 M_2 \pi_2). \quad (2)$$

Here $F_i$ is the quantum number of the total angular momentum of the nucleus $(j_i)$ and the electron shell $(j_e)$; $m_i$ is the projection of $F_i$ on the z axis. $\Omega_2$ and $\sigma_2$ denote the direction and the polarization of the radiation; $(\Omega_2\sigma_2|L_2 M_2|\pi_2)$ is the wave function of the radiation with a given angular momentum $L_2$, projection $M_2$, and parity $\pi_2$.

We now go over to the coordinate system of the radiation:

$$(\Omega_2\sigma_2|L_2 M_2 \pi_2) = \sum_{\mu_2} D^{L_2}_{\mu_2 M_2}(R_2^{-1})(0\,\sigma_2|L_2\mu_2\pi_2). \quad (3)$$

$D^L_{\mu M}$ is the irreducible representation of the three dimensional rotation group of dimension $2L+1$; $R_2^{-1}$ implies a rotation from the direction of the radiation to the $\eta$ axis.

We recall that the matrix element

$$(F_b m_b | H_2 | F_c m_c L_2 M_2 \pi_2) = \sum_{m m_2 m_e m'_e} (-1)^{j-j_e+m_b} (2F_b+1)^{1/2} \times \begin{pmatrix} j & j_e & F_b \\ m & m_e & -m_b \end{pmatrix} (j m j_e m_e | H_2 | j_2 m_2 j_e m'_e L_2 M_2 \pi_2) (-1)^{j_2-j_e+m_c} \times (2F_c+1)^{1/2} \begin{pmatrix} j_2 & j_e & F_c \\ m_2 & m'_e & -m_c \end{pmatrix}, \quad (4)$$

(where $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$ is the 3j symbol of Wigner[6]), and assume that the electron shell is stationary during the nuclear transitions. In accordance with the Eckart theorem,[7] this leads to

$$(j m j_e m_e | H_2 | j_2 m_2 j_e m'_e L_2 M_2 \pi_2) = (-1)^{L_2-j_2+m} (2j+1)^{1/2} \begin{pmatrix} L_2 & j_2 & j \\ M_2 & m_2 & -m \end{pmatrix} \delta_{m_e m'_e} (j \| L_2 \| j_2). \quad (5)$$

Then expression (2) can be written as

$$(\beta | H_2 | \gamma) = \sum_{L_2 M_2 \pi_2 \mu_2} (-1)^{-j_2-j_e+L_2+m_b} (j \| L_2 \| j_2) (2j+1)^{1/2} (2F_b+1)^{1/2} \times (2F_c+1)^{1/2} \begin{Bmatrix} F_c & j_e & j_2 \\ j & L_2 & F_b \end{Bmatrix} \begin{pmatrix} F_c & L_2 & F_b \\ m_c & M_2 & -m_b \end{pmatrix} \times (0\sigma_2 | L_2 \mu_2 \pi_2) D^{L_2}_{\mu_2 M_2}(R_2^{-1}), \quad (6)$$

where the curly brackets stand for the 6j symbol of Wigner.[6] Formula (6) also determines

$$(\beta' | H_2 | \gamma) \equiv (F'_b m'_b | H_2 | F_c m_c \Omega'_2 \sigma'_2).$$

Using the properties of the matrix $D^L_{\mu M}$ (reference 7) and introducing the notation ($y$ specifies the type of radiation)

$$C_{\nu_2 \tau_2}(L_2 L'_2 \pi_2 y) \equiv S_2 \sum_{\mu_2 \mu'_2} (-1)^{L_2+\mu_2} (2\nu_2+1)^{1/2} \times \begin{pmatrix} L_2 & L'_2 & \nu_2 \\ \mu_2 & -\mu'_2 & -\tau_2 \end{pmatrix} (0\sigma_2 | L_2 \mu_2 \pi_2)^* (0\sigma'_2 | L'_2 \mu'_2 \pi'_2), \quad (7)$$

we now apply the formulas for the contraction of the 3j and 6j symbols of Wigner. In the end we obtain for the density matrix

$$E^{(2)}(\beta'\beta) = \sum_{\substack{L_2 L'_2 \pi_2 \\ \nu_2 \rho_2 \tau_2}} (-1)^{j_2-j_e+L_2+F_b+F'_b+m_b} (j \| L'_2 \| j_2) \times (j \| L_2 \| j_2)^* (2j+1)(2\nu_2+1)^{1/2} \frac{(2F_b+1)^{1/2}(2F'_b+1)^{1/2}}{1+(\omega_{F_b F'_b} \tau_B)^2} \quad (8)$$

$$\times \begin{Bmatrix} j & j & \nu_2 \\ L'_2 & L_2 & j_2 \end{Bmatrix} \begin{Bmatrix} j & j & \nu_2 \\ F'_b & F_b & j_e \end{Bmatrix} \begin{pmatrix} F'_b & F_b & \nu_2 \\ m'_b & -m_b & \rho_2 \end{pmatrix} C_{\nu_2 \tau_2}(L_2 L'_2 \pi_2 y) D^{\nu_2 *}_{\tau_2 \rho_2}(R_2^{-1}).$$

3. The density matrix for the first transition is given by expression (1a).

Repeating the calculations of Sec. 2, we find

$$(\alpha | H_1 | \beta) = \sum_{\substack{L_1 M_1 \pi_1 \\ \mu_1 m_1 m}} (-1)^{2j_e+j_1+L_1+m_b+m_a+m_1} (j_1 \| L_1 \| j) \times (2F_a+1)^{1/2} (2F_b+1)^{1/2} \begin{pmatrix} j_1 & j_e & F_a \\ m_1 & m_e & -m_a \end{pmatrix} \begin{pmatrix} L_1 & j & j_1 \\ M_1 & m & -m_1 \end{pmatrix} \times \begin{pmatrix} j & j_e & F_b \\ m & m_e & -m_b \end{pmatrix} (0\sigma_1 | L_1 \mu_1 \pi_1) D^{L_1}_{\mu_1 M_1}(R_1^{-1}), \quad (9)$$

where $R_1^{-1}$ implies the rotation from the direction of the first radiation to the $\eta$ axis, and

$$\mathscr{E}^{(1)}(\beta\beta') = \sum_{\substack{L_1 L'_1 \pi_1 \\ \nu_1 F_a F'_a}} (-1)^{2j_e+L'_1} (j_1 \| L'_1 \| j)(j_1 \| L_1 \| j) \times (2\nu_1+1)^{1/2} (2F_b+1)^{1/2} (2F'_b+1)^{1/2} \frac{(2F_a+1)(2F'_a+1)}{1+(\omega_{F_a F'_a} \tau_A)^2}$$

$$\times \sum (-1)^{m_1+m'_1+M_1+m_b+m'_b} a m''_e \begin{pmatrix} j_1 & j_e & F'_a \\ m''_1 & m''_e & -m_a \end{pmatrix} \begin{pmatrix} j_1 & j_e & F_a \\ m''_1 & m''_e & -m_a \end{pmatrix} \times \begin{pmatrix} j_1 & j_e & F'_a \\ m'_1 & m'_e & -m'_a \end{pmatrix} \begin{pmatrix} j_1 & j_e & F'_a \\ m_1 & m_e & -m'_a \end{pmatrix} \begin{pmatrix} j & j_e & F'_b \\ m' & m'_e & -m'_b \end{pmatrix} \begin{pmatrix} j & j_e & F_b \\ m & m_e & -m_b \end{pmatrix}$$

$$\times \begin{pmatrix} L'_1 & j & j_1 \\ M'_1 & m' & -m'_1 \end{pmatrix} \begin{pmatrix} L_1 & j & j_1 \\ M_1 & m & -m_1 \end{pmatrix} \begin{pmatrix} L_1 & L'_1 & \nu_1 \\ M_1 & -M'_1 & -\rho_1 \end{pmatrix} \times C_{\nu_1 \tau_1}(L_1 L'_1 \pi_1 x) D^{\nu_1 *}_{\tau_1 \rho_1}(R_1^{-1}). \quad (10)$$

The internal sum in (10) goes over the magnetic numbers $m$, $m_1$, $m_e$, $m'$, $m'_1$, $m'_e$, $m''_1$, $m''_e$, $M_1$, $M'_1$, $\rho_1$, $\tau_1$, $m_a$, and $m'_a$. The summation over the first six of these numbers is easily performed.

In order to apply Racah's formula for the contraction of the 3j symbols of Wigner in the sum

$$\sum_{M_1 M'_1} (-1)^{M_1} \begin{pmatrix} F_b & L_1 & F_a \\ m_b & M_1 & -m_a \end{pmatrix} \begin{pmatrix} F'_b & L'_1 & F'_a \\ m'_b & M'_1 & -m'_a \end{pmatrix} \begin{pmatrix} L_1 & L'_1 & \nu_1 \\ M_1 & -M'_1 & -\rho_1 \end{pmatrix},$$

which can also be written as

$$\sum_{M_1 M'_1 m''_b m'''_b \rho} (-1)^{M_1} \begin{pmatrix} F_b & L_1 & F_a \\ -m''_b & M_1 & -m_a \end{pmatrix} \begin{pmatrix} F'_b & L'_1 & F'_a \\ m'''_b & M'_1 & -m'_a \end{pmatrix} \times \begin{pmatrix} L_1 & L'_1 & \nu_1 \\ M_1 & -M'_1 & \rho \end{pmatrix} \delta_{-m_b m''_b} \delta_{m'_b m'''_b} \delta_{-\rho_1 \rho}, \quad (11)$$

we replace the Kronecker symbols by the expressions

$$\delta_{m'_b m'''_b} \delta_{-\rho_1 \rho} = \sum_{l x \lambda} \delta_{x\lambda} (2l+1) \begin{pmatrix} F'_b & \nu_1 & l \\ m'''_b & \rho & -\lambda \end{pmatrix} \begin{pmatrix} F'_b & \nu_1 & l \\ m'_b & -\rho_1 & -x \end{pmatrix};$$

$$\delta_{-m_b m''_b} \delta_{x\lambda} = \sum_{k\sigma} (2k+1) \begin{pmatrix} F_b & l & k \\ m''_b & \lambda & -\sigma \end{pmatrix} \begin{pmatrix} F_b & l & k \\ -m_b & x & -\sigma \end{pmatrix}. \quad (12)$$

**4.** We further introduce the 9j symbol of Wigner,[6]

$$\begin{Bmatrix} F_b & L_1 & F_a \\ F_b' & L_1' & F_a' \\ \nu_2 & \nu_1 & k \end{Bmatrix} = \sum_l (-1)^{2l} \begin{Bmatrix} \nu_2 & F_b & F_b' \\ l & \nu_1 & k \end{Bmatrix} \begin{Bmatrix} F_a' & L_1' & F_b' \\ \nu_1 & l & L_1 \end{Bmatrix} \begin{Bmatrix} F_a & F_b & L_1 \\ l & F_a' & k \end{Bmatrix}, \qquad (13)$$

and obtain for the correlation function, using the unitarity of the representation $D^{\nu}_{\tau\rho}$,

$$W = \sum (-1)^{j_1+j_2+L_2+F_a-F_b} (j_1 \| L_1 \| j)(j_1 \| L_1' \| j)(j \| L_2 \| j_2)(j \| L_2' \| j_2)$$
$$\times C_{\nu_1\tau_1}(L_1 L_1' \pi_1 x) C_{\nu_2\tau_2}(L_2 L_2' \pi_2 y)(2j+1)(2\nu_1+1)^{1/2}(2\nu_2+1)^{1/2}$$
$$\times (2k+1)^{1/2} \frac{(2F_a+1)(2F_a'+1)(2F_b+1)(2F_b'+1)}{[1+(\omega_{F_aF_a'}\tau_A)^2][1+(\omega_{F_bF_b'}\tau_B)^2]} \begin{Bmatrix} j & j & \nu_2 \\ L_2 & L_2' & j_2 \end{Bmatrix}$$
$$\times \begin{Bmatrix} j & j & \nu_2 \\ F_b & F_b' & j_e \end{Bmatrix} \begin{Bmatrix} L_1 & F_a & F_b \\ j_e & j & j_1 \end{Bmatrix} \begin{Bmatrix} L_1' & F_a' & F_b' \\ j_e & j & j_1 \end{Bmatrix} \begin{Bmatrix} k & F_a & F_a' \\ j_1 & j_e & j_e \end{Bmatrix}$$
$$\times \begin{Bmatrix} F_b & L_1 & E_a \\ F_b' & L_1' & F_a' \\ \nu_2 & \nu_1 & k \end{Bmatrix} \langle |(j_e j_e) k0\rangle \begin{pmatrix} \nu_1 & \nu_2 & k \\ \rho & -\rho & 0 \end{pmatrix} D^{\nu_1}_{\rho\tau_1}(R_1) D^{\nu_2}_{-\rho\tau_2}(R_2), \qquad (14)$$

where the summation goes over the corresponding quantum numbers; $\nu_1 + \nu_2 + k$ is an even number. $\langle |(j_e j_e) k0\rangle$ is the statistical tensor of Fano, which is proportional to

$$\langle |(j_e j_e) k0\rangle = \sum_{m_e} (-1)^{j_e - m_e} (2k+1)^{1/2} \begin{pmatrix} j_e & j_e & k \\ m_e & -m_e & 0 \end{pmatrix} a(m_e)$$
$$= \binom{2k}{k} j_e^k \left[ \frac{(2j_e+k+1)!}{(2k+1)(2j_e-k)!} \right]^{-1/2} j_k(j_e). \qquad (15)$$

The matrix elements in (14) are chosen to be real, which is always possible[4] in view of the invariance under time reversal. This leads to the following normalization for the correlation function (14):*

$$(8\pi^2)^{-2} \int W \, d\Omega_1 \, d\Omega_2 \, d\chi_1 \, d\chi_2 = \sum_{L_1 L_2} (j_1 \| L_1 \| j)^2 (j \| L_2 \| j_2)^2. \qquad (16)$$

The radiation parameters $C_{\nu\tau}(LL'\pi x)$ are given by Biedenharn and Rose.[4] Alder, Stech, and Winther[8] calculated these parameters for $\beta$ decay including the possibility of parity non-conservation.

For $(\omega_{F_aF_a'}\tau_A)^2 \ll 1$, we sum over $F_a$ and $F_a'$ in (14) and obtain

$$W = \sum (-1)^{j_1+j-j_2+L_1'+L_2+j_e+F_b+\nu_2} (j_1 \| L_1 \| j)(j_1 \| L_1' \| j)$$
$$\times (j \| L_2 \| j_2)(j \| L_2' \| j_2) C_{\nu_1\tau_1}(L_1 L_1' \pi_1 x) C_{\nu_2\tau_2}(L_2 L_2' \pi_2 y)(2j+1)$$
$$\times (2\nu_1+1)^{1/2}(2\nu_2+1)^{1/2}(2k+1)^{1/2} \begin{Bmatrix} j & j & \nu_1 \\ L_1 & L_1' & j_1 \end{Bmatrix} \begin{Bmatrix} j & j & \nu_2 \\ L_2 & L_2' & j_2 \end{Bmatrix}$$
$$\times \frac{(2F_b+1)(1F_b'+1)}{1+(\omega_{F_bF_b'}\tau_B)^2} \begin{Bmatrix} j & j & \nu_2 \\ F_b & F_b' & j_e \end{Bmatrix} \begin{Bmatrix} \nu_2 & k & \nu_1 \\ F_b & j_e & j \\ F_b' & j_e & j \end{Bmatrix} \langle |(j_e j_e) k0\rangle$$
$$\times \begin{pmatrix} \nu_1 & \nu_2 & k \\ \rho & -\rho & 0 \end{pmatrix} D^{\nu_1}_{\rho\tau_1}(R_1) D^{\nu_2}_{-\rho\tau_2}(R_2). \qquad (17)$$

If $(\omega_{F_bF_b'}\tau_B)^2 \ll 1$, expression (17) does not depend on the orientation of the electron shell, as was to be expected. It coincides in this case with the correlation function for the radiations from isolated nuclei:

$$W = \sum (-1)^{j_1-j_2+L_1'+L_2+\nu} (j_1 \| L_1 \| j)(j_1 \| L_1' \| j)(j \| L_2 \| j_2)(j \| L_2' \| j_2)$$
$$\times (2j+1) \begin{Bmatrix} j & j & \nu \\ L_1 & L_1' & j_1 \end{Bmatrix} \begin{Bmatrix} j & j & \nu \\ L_2 & L_2' & j_2 \end{Bmatrix} C_{\nu\tau_1}(L_1 L_1' \pi_1 x) C_{\nu\tau_2}$$
$$\times (L_2 L_2' \pi_2 y) D^{\nu}_{-\tau_2\tau_1}(R_2^{-1} R_1). \qquad (18)$$

If the total angular momenta of the electron shells are randomly distributed [i.e., $a(m_e) =$ const], (14) goes over into an expression which differs from (18) by the reducing factor

$$\sum_{F_bF_b'} \frac{(2F_b+1)(2F_b'+1)}{(2j_e+1)[1+(\omega_{F_bF_b'}\tau_B)^2]} \begin{Bmatrix} j & j & \nu \\ F_b & F_b & j_e \end{Bmatrix}^2$$

under the summation sign.*

**5.** It has been known that any interaction of the nucleus with the external field leads to a decrease in the anisotropy; in the limiting case $(\omega\tau)^2 \gg 1$ only the "hard core" remains.[9] The explanation for this has been that the external field leads to a redistribution of the m-sublevels in the intermediate state such that their population becomes more uniform.

It is seen from the example of the well-studied $\gamma$-$\gamma$ cascade $7/2\ (1, 2)\ 5/2\ (2)\ 1/2$ in $Cd^{111}$ that the orientation of the shell leads to a weakening of this effect.

To obtain maximal anisotropy, we place the axis of orientation in (17) in the direction of the first quantum. Substituting the value $j_e = 3/2$ for the total angular momentum of the electron shell, we obtain for the anisotropy:

$$A_{\text{unperturbed}} = -0.247; \quad A_{\text{unoriented}} = -0.103;$$
$$A_{\text{fully oriented}} = -0.149.$$

It turns out that in some cases the interaction between the nucleus and the oriented shell can lead to an increase of the anisotropy as compared to the case of an isolated nucleus. For example, using the same assumptions for the $\gamma$-$\gamma$ cascade $7/2\ (1)\ 5/2\ (2)\ 1/2$, we obtain

*The rotation is defined by the Eulerian angles, $\varphi$, $\theta$, and $\chi$; the element of solid angle in the direction of the radiation is $d\Omega = \sin\theta \, d\theta d\varphi$; $\chi$ defines its transverse polarization.

*This is Alder's result.[2]

$A_{\text{unperturbed}} = -0.1034;$ $A_{\text{unoriented}} = -0.0417;$ $A_{\text{fully oriented}} = -0.1557.$

This can be explained in the following fashion. In the intermediate state the system nucleus + shell tends toward equilibrium. If the sublevels of the electron shell $m_e$ are not uniformly populated, the non-uniformity of the population of the m-sublevels of the nucleus can be increased through the interaction.

## ANGULAR DISTRIBUTION

Integrating (14) over $d\Omega_2 d\chi_2$, we obtain the angular distribution of the nuclear radiation:

$$W = \sum (-1)^{-j_1 - j - L_1 + F_a - F'_a - k} (j_1 \| L_1 \| j)(j_1 \| L'_1 \| j)$$
$$\times C_{k\tau_1}(L_1 L'_1 \pi_1 x) \frac{(2F_a + 1)(2F'_a + 1)}{1 + (\omega_{F_a F'_a} \tau_A)^2} \begin{Bmatrix} L_1 & L'_1 & k \\ j_1 & j_1 & j \end{Bmatrix} \begin{Bmatrix} F_a & F'_a & k \\ j_e & j_e & j_1 \end{Bmatrix}$$
$$\times \begin{Bmatrix} F_a & F'_a & k \\ j_1 & j_1 & j_e \end{Bmatrix} \langle |(j_e j_e) k0 \rangle D^k_{0\tau_1}(R_1). \qquad (19)$$

Expression (19) is normalized according to the condition

$$\frac{1}{4\pi} \int W \sin\theta \, d\theta \, d\chi = \sum (j_1 \| L_1 \| j)^2. \qquad (20)$$

It is seen from (19) that the distribution becomes isotropic if either the lifetime of the initial state is much smaller than the precession period of the nuclear moment in the field of the electron shell $[(\omega\tau_A)^2 \ll 1]$, or the electron shell is not oriented $[f_k(j_e) = \delta_{k0})$.

If the transition under consideration is preceded by others, one must take into account the disorientation of the electron shell in these transitions. The corresponding expression for one such transition is obtained from (14) by integrating over $d\Omega_1 d\chi_1$.

In conclusion the author expresses his gratitude to K. A. Ter-Martirosyan for interest in this work.

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Translated by R. Lipperheide
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