EFFECT OF SPECIMEN SHAPE ON FERROMAGNETIC RESONANCE IN A STRONG RADIO-FREQUENCY FIELD

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We analyze the exact solutions of the Landau-Lifshitz equations for nonspherical ferromagnetic specimens in a radio-frequency field of arbitrary amplitude. An expression is derived for the threshold field h_c, above which instability in the motion of the magnetization vector begins. The slow decrease of the magnetization component and the shift of the resonance field strengths $h_0 > h_c$ are explained. It is shown that for $h_0 > h_c$ the height of the absorption peak decreases and its width increases.

1. Experiments by Damon[1] and by Bloembergen and Wang[2] have shown that the magnetization component M_z in the direction of a constant magnetizing field H_0 decreases with increasing microwave power. This decrease is, however, much slower than expected from a theory based on the approximate solution of the equation of motion for the magnetization.

It has been found, furthermore, that for a radio-frequency field with amplitude greater than a certain threshold field h_0 (considerably smaller than the width H of the absorption line), the contour of χ'' begins to broaden, and the position of the resonant absorption maximum shifts while its magnitude decreases. If $h_0 > h_c$, a broad additional absorption peak appears in fields H_0 that are somewhat less than the resonant field H_r.

A successful attempt at explaining the principal results of these experiments was made by Suhl.[3,4] Calling attention to the nonlinear character of the magnetization equation of motion

$$\dot{\mathbf{m}} + \gamma[\mathbf{m}\times\mathbf{H}^{ef}] + \alpha[\dot{\mathbf{m}}\times\mathbf{m}] = 0,$$

$$\mathbf{m} = \mathbf{M}/M_s, \quad \alpha > 0, \quad \gamma > 0, \qquad (1)$$

where in the case of a thin ferromagnetic disk

$$H_x^{ef} = h_x - N_\perp M_s m_x,$$

$$H_y^{ef} = h_y - N_\perp M_s m_y, \quad H_z^{ef} = H_0 - N_\parallel M_s m_z, \qquad (2)$$

Suhl shows that starting with a certain critical field value

$$h_c = \Delta H\,(3.08\,\Delta H/4\pi M_s)^{1/2}, \qquad (3)$$

the motion of the vector $\mathbf{m}$ becomes unstable close to ferromagnetic resonance.

Assuming further that the homogeneity of magnetization is disturbed in ferromagnetic specimens of spheroidal shape by thermal excitation, Suhl improved on Eq. (3).

The theory developed by Suhl is based to a considerable extent on the assumption that the magnetization vector deviates little from the equilibrium position ($m_z \approx 1$); as will be shown later, this theory is not suitable when $h_c \approx \Delta H$.

It is easy to dispense with the foregoing assumption by using the exact solutions of Eq. (1), as done in reference 5. It now becomes possible to find for the threshold field h_c an expression that is valid over the entire range of values of the parameters that characterize the system.

2. In a coordinate system rotating with a frequency ω about the direction of the constant field $H_0 = H_z$, Eq. (1) for steady motion becomes

$$[\mathbf{m}\times\boldsymbol{\xi}] + \mathbf{m}\times[\mathbf{m}\times\boldsymbol{\Omega}] = 0. \qquad (4)$$

where

$$\boldsymbol{\xi} = (\gamma\mathbf{H}^{ef} - \boldsymbol{\omega})/\alpha\omega, \quad \boldsymbol{\Omega} = \boldsymbol{\omega}/\omega. \qquad (5)$$

By choosing the axes of the rotating coordinate system such that the radio-frequency $\mathbf{h} = (h_0 \cos t, h_0 \sin t, 0)$ has only a component along the abscissa axis, and solving (4) for m_x and m_y, we obtain

$$m_x = a(\xi_0 - \xi_N m_z)m_z/[(\xi_0 - \xi_N m_z)^2 + m_z^2], \qquad (6)$$

$$m_y = -a m_z^2/[(\xi_0 - \xi_N m_z)^2 + m_z^2], \qquad (7)$$

where

$$\xi_0 = \frac{\gamma H_0 - \omega}{\alpha\omega}, \quad a = \frac{\gamma h_0}{\alpha\omega}, \quad \xi_N = \gamma\frac{N_\parallel - N_\perp}{\alpha\omega}M_s. \qquad (8)$$

It is now easy to determine χ' and χ'', the real and imaginary parts of the complex susceptibility

$$\chi' = \chi_0(\xi_0 - \xi_N m_z)m_z/[(\xi_0 - \xi_N m_z)^2 + m_z^2], \qquad (9)$$

$$\chi'' = \chi_0 m_z^2 / [(\xi_0 - \xi_N m_z)^2 + m_z^2, \tag{10}$$

where $\chi_0 = \gamma M_S / \alpha\omega$ is the value of χ'' at resonance in rf fields with $h_0 \leq h_c$, when

$$\gamma H_r = \omega + \gamma M_s (N_\parallel - H_\perp). \tag{11}$$

It is readily seen that the half-width ΔH of the absorption line equals in this case $\alpha\omega m_z/\gamma$, where $m_z = (1 + \sqrt{1 - a^2})/2$. Thus, in weak fields, when $a^2 \ll 1$, the half-width is $\Delta H_0 = \alpha\omega/\gamma$. When $h_0 \leq h_c$ is increased, the half-width decreases to a value

$$\Delta H_c = {}^1/_2 \Delta H_0 \{[1 - (h_c / \Delta H_0)^2]^{1/2} + 1\}. \tag{12}$$

This decrease in width is usually small, since $h_c < \Delta H_0$.

Using (6) and (7) together with the condition $m^2 = 1$ we obtain the equation

$$\xi_0 = m_z \left[\xi_N \pm \left(\frac{a^2}{1 - m_z^2} - 1 \right)^{1/2} \right], \tag{13}$$

in which we can analyze and plot the dependence of m_z on ξ_0 for various values of the parameter a (Fig. 1). For homogeneous strictly spherical specimens, $\xi_N = 0$ and the biquadratic equation (13) is readily solved[5] with respect to m_z. As the field h_0 (the parameter a) increases, the minima of the curves drop and shift along the dotted line $\xi_0 = \xi_N m_z$. The projection of the component of the magnetization vector on the line $\xi_0 - \xi_N m_z$ is $m_x = 0$ ($\chi' = 0$).

Equation (13) is correct for all values of ξ_N and $m_z \leq 1$, to the extent that the initial equation (1) is correct.

The approximate equation analogous to (13), used by Suhl, has the following form in our symbols

$$\xi_0 = m_z \xi_N \pm \left(\frac{a^2}{1 - m_z^2} - 1 \right)^{1/2} \tag{14}$$

and differs from (13) in that it does not contain the factor m_z in front of the parentheses.

3. As can be seen from the curves of Fig. 1, within a fixed range of values of ξ_0, $m_z(\xi_0)$ is not single valued after a certain value $a = a_c$ is reached. The critical value of the parameter a_c can be determined from the condition $d\xi_0/dm_z = 0$, which, according to (13), leads to the equation

$$a^4 - (1 - m_z^2)^2 [2 + \xi_N^2 (1 - m_z^2)] a^2 + (1 + \xi_N^2)(1 - m_z^2)^4 = 0,$$

the solutions of which are real if

$$1 - m_z^2 \geqslant 2 [\sqrt{1 + \xi_N^2} - 1] \xi_N^{-2}.$$

Since here $a^2 > 1 - m_z^2$, we have

$$a_c^2 = 2 [\sqrt{1 + \xi_N^2} - 1] \xi_N^{-2}. \tag{15}$$

If $\xi_N \gg 1$, then

$$a_c = (2 / \xi_N)^{1/2} \ll 1, \tag{16}$$

which is the same expression found by Suhl.

4. When $a \leq a_c$, the motion of the magnetization vector is stable everywhere. When $a > a_c$, the motion is unstable on the sections of the curve of Fig. 1 marked 2—3—4. The variation of m_z with the external field H_0 (the parameter ξ_0) follows therefore the curve 1—2—4—5. In this case the minimum of m_z and the maximum of χ'' occur not at the point 3, but at the point 4, where $\chi' \neq 0$. Therefore the resonant field, determined from the position of the maximum of χ'', will correspond not to condition (11), as would appear from an analysis of (10), but to a value of ξ_0 other than $\xi_0 m_z$. The geometric locus of the points were χ'' has a maximum is shown in Fig. 1 by a dotted line passing through the discontinuities of m_z.

Thus, the discontinuity in the motion of the magnetization vector should cause a slower reduction in m_z at resonant with increasing amplitude of

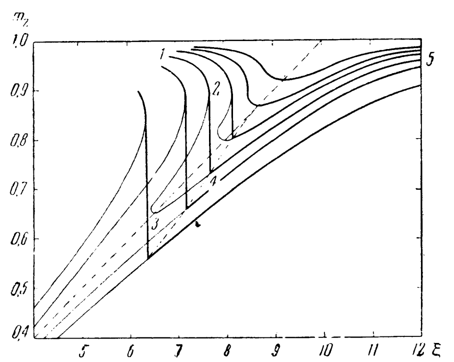

FIG. 1. Dependence of m_z on ξ for various values of the parameter a, at $\xi_N = 10$; the values of a (where $a^2 = 0.15$, 0.25, 0.36, 0.58, 1.0, and 2.0) are given in order of increasing gaps on the curves.

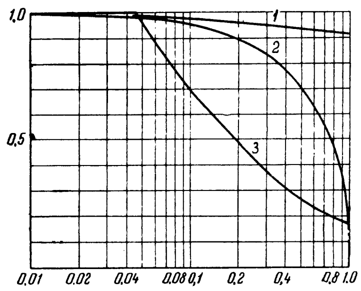

FIG. 2. Effect of non-sphericity of specimen on the character of the dependence of m_z on a^2 (curve 1) and of χ'' on a^2 (curve 3), for $\xi_N = 100$. The $m_z(a^2)$ curve for a homogeneously magnetized spherical specimen (curve 2) is shown for comparison.

the radio-frequency field h_0 than expected from condition (11) (Curve 1, Fig. 2).

In sufficiently weak fields, $a \leq a_c$, resonance occurs when condition (11) is satisfied. In this case $\chi'' = \chi_0$, i.e., constant. To ascertain the dependence of χ'' on a^2 when $a_c < a < 1$, we use the expression

$$\chi'' = \chi_0 (1 - m_z^2) / a^2, \tag{17}$$

which is readily obtained from (10) and (13). According to the above, $1 - m_z^2$ increases more slowly than a^2. Therefore $\chi''(a^2)$ diminishes with increasing microwave power (a^2) when $h_0 > h_c$. A more rigorous analysis shows that the drop in $\chi''(a^2)$ begins with a value somewhat greater than h_c (Curve 3, Fig. 2). The greater $|\xi_N|$, the slower the drop in m_z and the faster the decrease in $\chi''(a^2)$.

It follows from (17) that the minimum of $m_z(\xi_0)$ for a given a corresponds to a maximum of $\chi''(\xi_0)$. Therefore, as the microwave power increases, the resonant frequency should shift. For $a \leq a_c$ this shift is determined by condition (11). When $a > a_c$ the shift can be determined graphically (dotted curve of Fig. 1).

By using (17) and (13) it is easy to plot curves of $\chi''(\xi_0)$ for various values of the parameter a. The width of the absorption line is found to increase with increasing $a > a_c$.

Thus, by accounting for the demagnetizing field in the specimen, it is possible to describe qualitatively the experimentally-observed peculiarities of ferromagnetic resonance in strong radio-frequency fields without making any assumptions whatever regarding the magnitude of M_z. However, certain details of the phenomenon are elusive and cannot be explained here. Thus, it does not follow directly from the analysis that an additional absorption maximum exists in fields where $a > a_c$, nor is it clear why the threshold field is small for specimens of nearly spherical shape.

Actually, for spherical specimens $N_{||} = N_{\perp}$ and $\xi_N = 0$, and therefore $a_c = 1$. If the specimen is assumed spheroidal, even with a semi-axis ratio of 1.3, then according to Osborne[6] we have for nickel ferrite[3] ($\Delta H_0 = 50$ oe, $4\pi M_s / \Delta H_0 = 65$) $\xi_N = 6.6$ and $a_c = 0.51$, while the experimentally observed value is $a_c = 0.18$, corresponding to $\xi_N = 62$, which according to (8) can occur only for a thin disk. Thus, the assumption regarding the nonsphericity of the specimen cannot explain fully the small value of critical field observed in this case.

The presence of pores and of internal stresses, and the block-like nature of the specimen structure, may reduce substantially the value of the critical field. However, this problem has not been investigated experimentally.

To explain the smallness of the critical field in spheroidal specimens and the appearance of an additional absorption maximum it is necessary, according to Suhl, to take into account the presence of thermal magnetization fluctuations in the specimens. For this purpose it is necessary to include in the effective field (2) the fluctuation field, which disturbs the homogeneous precession. But in this case the peculiarities in the behavior of ferromagnets in a strong radio-frequency field can be described without the assumption that $m_z \approx 1$.

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