GEODESICS IN FRIEDMAN-LOBACHEVSKY SPACE

I. G. FIKHTENGOL' TS

Leningrad Institute of Precision Mechanics and Optics

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WE consider the equations for the geodesics in the space in which the square of the line element ds has the form

$$ds^2 = H^2(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2), \quad (1)$$

where H is some function of the variables x_0 and $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

Taking the time coordinate x_0 as the independent parameter, the equations for the geodesics are then written in the form (see, e.g., reference 1)

$$\ddot{x}_i - \dot{x}_i \Gamma^0_{\alpha\beta}\dot{x}_\alpha \dot{x}_\beta + \Gamma^i_{\alpha\beta}\dot{x}_\alpha\dot{x}_\beta = 0 \qquad (i = 1, 2, 3). \quad (2)$$

Here $\Gamma^\nu_{\alpha\beta}$ is the Christoffel symbol of the second kind; the dot denotes the derivative with respect to the variable x_0; greek indices run through the values 0, 1, 2, 3, and identical indices are understood to be summed from 0 to 3.

Starting with formula (1), we obtain the following expressions for the Christoffel symbol of the second kind:

$$\Gamma^0_{00} = \frac{1}{H}\frac{\partial H}{\partial x_0}, \quad \Gamma^i_{00} = \Gamma^0_{0i} = \frac{1}{rH}\frac{\partial H}{\partial r} x_i, \quad \Gamma^i_{0k} = \Gamma^0_{ik} = \frac{1}{H}\frac{\partial H}{\partial x_0}\delta_{ik},$$

$$\Gamma^l_{ik} = \frac{1}{rH}\frac{\partial H}{\partial r}(x_i\delta_{kl} + x_k\delta_{il} - x_l\delta_{ik}). \quad (3)$$

In these expressions $\delta_{ik} = 1$ for $i = k$, and $\delta_{ik} = 0$ for $i \neq k$; the Latin indices i, k, l run through the values 1, 2, 3.

With the expressions (3), the equations in (2) now take the form

$$\ddot{x}_i + \frac{1 - \dot{r}^2}{H}\left(\frac{1}{r}\frac{\partial H}{\partial r} x_i + \frac{\partial H}{\partial x_0}\dot{x}_i\right) = 0. \quad (4)$$

If we now set[1] $H = H(S)$, where $S = \sqrt{x_0^2 - r^2}$, then we finally obtain, according to (4),

$$\ddot{x}_i + \frac{\dot{r}^2 - 1}{S}\frac{H'}{H}(x_i - \dot{x}_i x_0) = 0, \quad (5)$$

where the prime denotes the derivative with respect to the variable S.

It is seen immediately that the relations

$$x_i = \dot{x}_i x_0 \quad (6)$$

yield $\dot{x}_i = \text{const}$; the functions x_i defined by them are therefore particular solutions of (5).

The relations (6) are used in a well-known way for the explanation of the phenomenon of the "recession of the galaxies," by regarding the quantities $\dot{x}_i$ as the coordinates of the corresponding mass in the accompanying system of coordinates.

[1] V. A. Fock, Теория пространства, времени и тяготения (The Theory of Space, Time, and Gravitation), GTTI, 1955.

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ON THE PRODUCTION OF PION AND MUON PAIRS BY THE ANNIHILATION OF HIGH-ENERGY POSITRONS

A. I. NIKISHOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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THE study of the processes $e^+ + e^- \rightleftarrows \mu^+ + \mu^-$ and $e^+ + e^- \rightleftarrows \pi^+ + \pi^-$ is of interest in connection with the possibility of detecting deviations from local theory at distances $\sim 10^{-13}$ cm. If we describe the deviation by a factor $F(q^2)$ in the expression for the transition current, we get the cross-sections for these processes from the corresponding expressions of local theory by simply multiplying by the squares of the form factors $F(q^2)$ for the particles in question. Since in the annihilation of two particles $q^2 = -4E^2$ (in the center-of-mass system), the introduction of the form factors does not change the angular distributions for these processes. The values of the form factors for $q^2 < 0$ (annihilation of particles) cannot be obtained from the values of $F(q^2)$ for $q^2 > 0$ (scattering).*

The absolute squares of the matrix elements in the center-of-mass system, averaged over the initial spin states and summed over the final (for those particles having spin), are as follows:†

$$|M|^2(e^+ + e^- \rightleftarrows \mu^+ + \mu^-) = \frac{1}{16E^4}\left\{1 + \left(\frac{\mu}{E}\right)^2 + \frac{p_\mu^2}{E^2}\cos^2\vartheta\right\},$$

$$|M|^2(e^+ + e^- \rightleftarrows \pi^+ + \pi^-) = \frac{p_\pi^2}{32E^4}\sin^2\vartheta.$$

We note that the matrix elements for the processes involving π mesons are small for nonrelativistic values of the velocities v_π. The maximum of the angular distribution for this process lies in the plane perpendicular to the line of impact.

The corresponding differential cross-sections in the center-of-mass system are:

$$d\sigma(e^+ + e^- \to \mu^+ + \mu^-) = F_\mu^2 F_e^2 \frac{r_0^2}{16\gamma^2}\left\{1 + \left(\frac{\mu}{E}\right)^2 + \frac{p_\mu^2}{E^2}\cos^2\vartheta\right\} d\Omega,$$

$$d\sigma(e^+ + e^- \to \pi^+ + \pi^-) = F_\pi^2 F_e^2 \frac{r_0^2}{32\gamma^2}\frac{p_\pi^3}{E^3}\sin^2\vartheta\, d\Omega,$$

$$d\sigma(\mu^+ + \mu^- \to e^+ + e^-)$$

$$= F_\mu^2 F_e^2 \frac{r_0^2}{8\gamma^2}\frac{1}{v_{\rm rel}}\left\{1 + \left(\frac{\mu}{E}\right)^2 + \frac{p_\mu^2}{E^2}\cos^2\vartheta\right\} d\Omega,$$

$$d\sigma(\pi^+ + \pi^- \to e^+ + e^-) = F_\pi^2 F_e^2 \frac{r_0^2}{\gamma^2}\frac{v_\pi}{32}\sin^2\vartheta\, d\Omega,$$

$r_0 = 2.8 \times 10^{-13}$ cm; $\gamma = E/m$; μ and m are the masses of meson and electron; E is the energy of a particle; $v_{\rm rel} = 2v_\mu$ is the relative velocity of the mesons in the beam; v_π, p_π are the velocity and momentum of a π meson; ϑ is the angle between the colliding and emerging particles; $q^2 = -4E^2$; $\hbar = c = 1$.

In the limit v_π, $v_\mu \approx c$ we get for the cross sections integrated over the angles

$$\sigma(e^+ + e^- \to \mu^+ + \mu^-)/\sigma(e^+ + e^- \to \pi^+ + \pi^-) = 4F_\mu^2/F_\pi^2.$$

We note that the probability for decay of the bound system $\mu^+\mu^-$ into e^+e^- is given by

$$w = |\psi(0)|^2 (v_{\rm rel}\sigma)_{v_{\rm rel}=0} = 4\cdot 10^{11}\ \text{sec}^{-1} \approx w_{\mu^+ + \mu^- \to 2\gamma}.$$

Because of the small velocities v_π the corresponding probability $w(\pi^+ + \pi^- \to e^+ + e^-)$ is vanishingly small.

If for an estimate we set F = 1 for all particles, the largest values of the total cross-sections are of the order $10^{-30} - 10^{-31}$ cm^2.

Finally we note that if in the process $\pi + N \to N + e^+ + e^-$ the angular characteristics of the pair do not differ strongly from those for the process $\pi^+ + \pi^- \to e^+ + e^-$, then it may be possible to distinguish it experimentally, in spite of the very large background of pairs from the decay $\pi^0 \to e^+ + e^- + \gamma$.

In conclusion I express my gratitude to I. Ya. Pomeranchuk, I. L. Rozental', and E. L. Feĭnberg for fruitful discussions.

*It is possible that this bears a relation to the fact that the average multiplicity of the mesons from the annihilation of antinucleons is somewhat larger than the value given by the statistical theory with $R = 1.4 \times 10^{-13}$ cm.[1]

†The production of meson pairs by the annihilation of positrons was first discussed by I. Ya. Pomeranchuk and V. B. Beresterskiĭ.[2] In their paper a factor 4 is omitted from the expression for $\sigma(e^+ + e^- \to \mu^+ + \mu^-)$.

[1] Belen'kiĭ, Maksimenko, Nikishov, and Rozental', Usp. Fiz. Nauk **62**, No. 2, 1 (1957).

[2] V. B. Berestetskiĭ and I. Ya. Pomeranchuk, J. Exptl. Theoret. Phys. **29**, 864 (1955), Soviet Phys. JETP **2**, 580 (1956).

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ON PAIR PRODUCTION BY THE COLLISION OF TWO CIRCULARLY POLARIZED GAMMA-RAY QUANTA

F. S. SADYKHOV and B. K. KERIMOV

Moscow State University

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THE present note presents the results of a calculation of the electron-positron pair production in the collision of two circularly polarized γ-ray quanta, with account taken of the longitudinal polarization of the pair particles. An examination of this problem is of definite interest, since beams of γ-rays of high energy are now available ($E_\gamma \sim 0.5 - 1$ Bev).[1,2] The circularly polarized γ-rays are produced in the deceleration radiation of longitudinally polarized high-energy electrons,[3] and also in nuclear β-decay processes.[4]

The equation that describes the process $\gamma + \gamma' \to e^- + e^+$ is of the form

$$D\psi_2 = \{U(\varkappa)D^{-1}U(\varkappa') + U(\varkappa')D^{-1}U(\varkappa)\}\psi_0, \qquad (1)$$

where ψ_0 is the wave function of the initial state and ψ_2 that of the final state, D is the Dirac operator, and $U(\kappa)$ and $U(\kappa')$ are the operators for the interaction of electrons with the quanta having the momenta $\hbar\boldsymbol{\kappa}$ and $\hbar\boldsymbol{\kappa}'$. The polarization vectors $\mathbf{a}_l \equiv \mathbf{a}_l(\boldsymbol{\kappa})$ and $\mathbf{a}'_{l'} \equiv \mathbf{a}_{l'}(\boldsymbol{\kappa}')$ of the quanta are taken in the form[5,6]

$$\mathbf{a}_l = (\boldsymbol{\beta} + il[\boldsymbol{\varkappa}^0\boldsymbol{\beta}])/\sqrt{2}, \quad \mathbf{a}'_{l'} = (\boldsymbol{\beta} + il'[\boldsymbol{\varkappa}'^0\boldsymbol{\beta}])/\sqrt{2}. \qquad (2)$$

Here $\boldsymbol{\beta}$ is a unit vector perpendicular to the momenta of the γ-ray quanta, $\boldsymbol{\kappa}^0 = \boldsymbol{\kappa}/\kappa$, and $\boldsymbol{\kappa}'^0 = \boldsymbol{\kappa}'/\kappa'$. In the case $l = l' = 1$ we have quanta with right-handed polarization (the spins of the quanta are in the direction of motion), and for $l = l' = -1$ we have left-handed polarization (spin opposite to motion). Using Eqs. (21) and (15) of reference 5 for the total cross sections for electron-positron