RESONANCE SCATTERING OF LOW-ENERGY GAMMA RAYS ON NUCLEI

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The mechanism of resonance scattering of low-energy gamma rays is discussed. It is shown that this scattering is of the nature of nuclear resonance fluorescence and that excitation of the nucleus can be described by means of one-nucleon transitions. The gamma-ray scattering cross section is calculated on the basis of the shell theory. Excited level widths are estimated from the Fermi gas model and the results obtained by Signell and Marshak[1] in connection with the theory of nucleon scattering. The results are in satisfactory agreement with the experimental data.

THE scattering of gamma rays with energies $E_\gamma < 30$ Mev exhibits two maxima, one of which is in the giant resonance region and is accounted for by the dipolar vibrations which can be excited in all nuclear matter. The other maximum* is below the particle threshold; Fuller and Hayward[3] have obtained experimental evidence that gamma-ray scattering in this energy region depends essentially on nuclear structure. The scattering cross section at the giant-resonance maximum increases relatively smoothly with the atomic number A, whereas the "subthreshold" scattering maximum varies sharply depending on the nucleus. The variation is especially great in the case of nuclei with a closed shell structure. It has also been found that scattering in this region exhibits a few very sharp resonances.

It was the purpose of the present work to determine the mechanism of gamma-ray scattering in the subthreshold region. It was assumed that scattering occurs at separate one-nucleon levels and can be described as nuclear resonance fluorescence. In the dipole approximation the cross section for gamma-ray scattering on a nucleus is given by

$$\sigma(\gamma,\gamma)=\frac{2\pi}{3}\frac{e^4\omega^2\hbar^2}{c^4M^2}\left|\sum_{n'}\frac{f_{nn'}}{E_\gamma-(E_{n'}-E_n)+i\Gamma/2)}\right|^2. \qquad (1)$$

Here ω is the frequency of the scattered gamma ray; M is the nucleon mass; Γ is the total width of the intermediate (excited) level; $f_{nn'}$ is the oscillator strength for a transition from state n to state n′ and is defined (when the gamma ray is polarized along the Z axis of the nucleus) by

$$f_{nn'}=2M\hbar^{-2}(E_{n'}-E_n)\,|Z_{nn'}|^2.$$

We shall consider real transitions corresponding to resonances. In the given approximation and for $E < E_{thr}$ as a rule one transition corresponds to each fixed ground level. Therefore only one term in the sum on the right side of (1) is important.

Schröder's method[4] was used to calculate the ground and excited nucleon levels. Although the calculation took strongly complicating factors into account — the Coulomb interaction of protons within the nucleus, the spin-orbit interaction, the diffuse nuclear boundary — this method permits a relatively simple and clear solution of the problem. On the other hand, consideration of these factors results in a more accurate charge distribution within the nucleus and in potentials that are in agreement with the phenomenological model of Feshbach, Porter, and Weisskopf.[5]

In the assumed model the radial part of the wave equation for neutrons is given by

$$\left\{\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{l(l+1)}{r^2}+\frac{2M}{\hbar^2}[E_{n,l,j}-V]\right\}R_{n,l,j}(r)=0,$$

where

$$V=V_N-0.38\left(\frac{\hbar}{m_\pi c}\right)^2\frac{1}{r}\frac{d}{dr}V_N\frac{(\mathbf{lS})}{\hbar^2}. \qquad (2)$$

In (2) V_N is the potential for neutrons, while the second term represents the spin-orbit interaction. The specific character of V_N for Pb^{208} is given in reference 4.

A Coulomb interaction term must, of course, be added to the equation in order to determine the proton levels; in this procedure Schröder was followed exactly.

The total potential $V(r)+(\hbar^2/2M)\,l(l+1)\,r^{-2}$ was approximated by the parabola

*Bethe[2] has given a rough qualitative explanation of this resonance based on competition between reactions.

$$W = -u\left[1 - \left(\frac{r - r_1}{w_1}\right)^2\right].$$

The parameters u, r_1, w_1 are clearly functions of l and j. The eigenfunctions $E_{n,l,j}$ are given by

$$E_{n,l,j} = -u + (\hbar / w_1)\sqrt{2u/M}\,(n - 1/2),$$

and the solution of the radial part of the wave function is given by

$$R_{n,l,j}(r) = Nr^{-1}H_n[\alpha(r - r_1)]\exp[-\alpha^2(r - r_1)^2/2],$$

where $N = (\alpha/\pi^{1/2}2^n n!)^{1/2}$ is the normalization factor, $\alpha = \sqrt{2Mu/w_1^2\hbar^2}$ and H_n is a Hermite polynomial.

The matrix elements for dipole transitions which are important in the given energy region were obtained by numerical integration.

The excited level width Γ was determined as follows. A nucleon which has been excited through the absorption of a gamma quantum may return to the ground level immediately by emitting a gamma quantum of the same energy. This process is represented by the radiative width Γ_γ given by[6]

$$\Gamma_\gamma = 2\frac{L+1}{L}\frac{(2l+1)(2j+1)(2l'+1)}{[(2L+1)!!]^2}$$
$$\times\left[C(ll'L;\ 00)\,W\left(jlj'l';\ \frac{1}{2}L\right)\right]^2\hbar\omega\frac{e^2}{\hbar c}k^{2L}m_L^2,$$

where l, j pertain to the final state, l', j' pertain to the initial state, L is the emission multipole order (in our case $L = 1$), C is a Clebsch-Gordan coefficient, W is a Racah coefficient, and m_L is the radial part of the transition matrix element.

The nucleus can also return to the ground state by means of a cascade gamma transition, which can be calculated roughly.[7] The width for medium and heavy nuclei at $E_\gamma \sim 8 - 9$ Mev is found to be $0.2 - 1.0$ ev, which is considerably smaller than Γ_γ for a direct transition to the ground level and can thus be neglected.

However, it is also possible for the given nucleon to be scattered by another nucleon in the ground state; the width Γ_{sc} corresponds to this possibility. In order to determine Γ_{sc} approximately we use the picture of nucleon-nucleon scattering in the Fermi gas model; in order of magnitude it is found that[8]

$$\Gamma_{sc} \approx v_1\rho\sigma\hbar\{(E_1 - E_F)/E_F\}^2;\quad E_1 > E_F.$$

Here v_1 is the velocity of the excited nucleon, σ is the cross section for scattering by another nucleon, ρ is the density of nucleons which can efficiently scatter the excited nucleon. The last factor in the expression for Γ_{sc} takes the exclusion principle into account; all states below the Fermi energy E_F (in our case $E_F = 32$ Mev) are filled, with vacant states above this level.

The nucleon-nucleon scattering cross section σ was calculated using results obtained on the semiphenomenological theory with which Signell and Marshak[1] gave a good account of experimental findings. In the energy interval of interest here ($6 - 9$ Mev) the scattered phases in the cross section were calculated by extrapolating the results of this theory.

The total width of the excited level is thus $\Gamma = \Gamma_\gamma + \Gamma_{sc}$. The following results were obtained when the foregoing method was applied to resonance scattering of gamma rays on Ni^{58}, Cu^{63}, Pb^{208}, Bi^{209}, Sn^{118} and I^{127}.

In the case of the Ni^{58} nucleus the proton transition $1f_{7/2} \rightarrow 1g_{9/2}$ is important. A proton in the excited state $1g_{9/2}$ undergoes $1P_1$ scattering* on a neutron in the ground state $1f_{7/2}$. Experiments actually measure the integrated cross section

$$\overline{\sigma(\gamma,\gamma)} = \frac{1}{D}\int\sigma(\gamma,\gamma)\,dE_\gamma$$

averaged over the energy interval (integration is performed over the interval of interest, which in our case is of the order of 1 Mev). Taking this into account, at $E_\gamma = 7$ Mev we obtain $\overline{\sigma(\gamma,\gamma)} = 2.6$ mbn, which is in good agreement with experiment.

Cu^{63} differs from Ni^{58} by the addition of one proton in the $2p_{3/2}$ state and four neutrons, of which two are in the $2p_{3/2}$ state and two in the $1f_{5/2}$ state. An excited $1g_{9/2}$ proton can also be scattered by the latter neutrons, thus increasing Γ. The cross section corresponding to the proton transition $1f_{7/2} \rightarrow 1g_{9/2}$ is thus reduced, becoming 2.5 mbn. The experimental result is $\overline{\sigma(\gamma,\gamma)} \leq 1.8$ mbn.

The most efficient transition in Pb^{208} is $2f_{7/2} \rightarrow 2g_{9/2}$. The excited neutron is scattered by the $1h_{11/2}$ proton; the calculation gives $\overline{\sigma(\gamma,\gamma)} = 16$ mbn.

The calculation for Bi^{209} is practically the same as for Pb^{208} since only one proton in the $1h_{9/2}$ state is added, which is without appreciable effect. The cross section is the same as for Pb^{208} and the results for Pb^{208} and Bi^{209} are in good agreement with experiment. The efficient transition in Sn^{118} and I^{127} is $1f_{5/2} \rightarrow 1g_{7/2}$ (ΔE

*It should be noted that only P scattering is important. It can be shown by a direct calculation that S scattering, although possible, makes a negligible contribution since it corresponds to a very large value of Γ_{sc} (tens of kev).

~ 7 Mev). The width of the excited state is given by $\Gamma_\gamma \sim 325$ ev ($\Gamma_{SC} \sim 1$ ev). $\overline{\sigma(\gamma, \gamma)}$ for Sn^{118} is 12.5 mbn, which corresponds to the measured value. In I^{127} there is practically no change in the basic features of the $1f_{5/2} \rightarrow 1g_{7/2}$ transition. However, three protons are in the $1g_{7/2}$ state; there are fewer possibilities for a transition and the probability is diminished. The calculation of $\overline{\sigma(\gamma, \gamma)}$ gives 4.8 mbn, which can be compared with the experimental $\overline{\sigma(\gamma, \gamma)} < 3$ mbn. Thus, just as for the Cu^{63} nucleus, the calculation indicates that the cross section tends to decrease, although not so sharply as occurs experimentally.

The crude model that we are using must be regarded as only a first approximation, although it enables us to derive all of the basic features of gamma-ray scattering on nuclei in the subthreshold region. Relatively good agreement with experiment is obtained for both the cross section and its variation as we pass from a nucleus with closed shells to another which possesses extra nucleons.

Our theory has assumed a spherically symmetrical nucleus, although many nuclei are actually deformed. In most cases this deformation is small and its effect on gamma-ray scattering can be calculated by the method of Moszkowski,[9] who showed that for small deformations an energy level can be described by

$$E = E^{(0)} + E^{(1)}d + E^{(2)}d^2 + \dots$$

where $d = \frac{2}{3}\epsilon$, and ϵ is the ratio of the difference between the semiaxes of the ellipsoid describing the nucleus to the radius of the equivalent sphere; $E^{(1)}_{nljm}$ is defined by

$$E^{(1)}_{nljm} = -2E^{(0)}_{nljm} \frac{j(j+1) - 3m^2}{2j(2j+2)} f.$$

Here $E^{(0)}_{nljm}$ is the energy in the case of spherical symmetry and f is a coefficient which in our case of an oscillator type of potential equals one-half.

Confining ourselves to first order terms in the expansion of E with respect to powers of the deformation d and considering as a specific example the proton transition $1f_{5/2} \rightarrow 1g_{7/2}$ in I^{127} (assuming for simplicity that $\Delta m = 0$), we obtain $d = 0.02$ (the quadrupole moment of I^{127} is 46×10^{-26} cm^2 and $R = 1.2A^{1/3} \times 10^{-13}$ cm). Instead of a single resonance at $E_\gamma = 7$ Mev we obtain three less strong resonances (since the levels are now only doubly degenerate); one of these is 33.6 kev above, while the other two are 3.8 kev and 92.6 kev below, the previous resonance. The splitting is much larger than the resonance widths (~ 300 ev).

On the basis of the foregoing we note that the maximum of the subthreshold resonance need not necessarily coincide with the threshold of the (γ, n) and (γ, p) reactions but may be slightly shifted toward lower energies. This effect depends to a considerable degree on the specific distribution of the levels between which radiative transitions occur.

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[1] P. S. Signell and R. Marshak, Phys. Rev. **109**, 1229 (1958).

[2] E. Segrè (ed.) Experimental Nuclear Physics, Vol. 1, Wiley, New York, 1953; Russ. Transl. IIL, Moscow, 1955.

[3] E. G. Fuller and E. Hayward, Phys. Rev. **101**, 692 (1956).

[4] A. Schröder, Nuovo cimento **7**, 461 (1958).

[5] Feshbach, Porter and Weisskopf, Phys. Rev. **96**, 448 (1954).

[6] M. E. Rose, Multipole Fields, Wiley, New York, 1955; Russ. Transl. IIL, Moscow, 1957.

[7] E. Segrè (ed.) Experimental Nuclear Physics, Vol. 2, Wiley, New York, 1953; Russ. Transl. IIL, Moscow, 1955.

[8] Hayakawa, Kawai, and Kikuchi, Progr. Theoret. Phys. **13**, 415 (1955).

[9] S. A. Moszkowski, Phys. Rev. **99**, 803 (1955).

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