ON THE DEPENDENCE OF THE ANGLE BETWEEN THE DIRECTION OF MOTION OF SHOWER PARTICLES AND THE AXIS OF THE SHOWER ON THE DISTANCE FROM THE AXIS

V. V. GUZHAVIN and I. P. IVANENKO

Moscow State University

Submitted to JETP editor November 24, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1509-1512 (May, 1959)

The dependence of the mean angle between the motion of particles and the axis of a shower on the distance from the axis is calculated. The case considered is that of an electron-photon shower averaged over the depth, with no account taken of ionization losses. The results of the calculation are compared with experiment.

PAPERS on the three-dimensional cascade theory of showers have dealt with either the spatial distribution functions of the particles, integrated over the angle variables, or the angular distribution functions, integrated over a plane perpendicular to the axis of the shower.[1] At the present time, however, in connection with the study of high-energy electron-photon showers in photographic plates, and also in the study of the soft component in the cores of extensive atmospheric showers one needs a knowledge of the complete spatial and angular distribution function of the particles. For the correct analysis of the experimental data it is particularly important to know the angular distribution function of the particles at a prescribed distance from the axis of the shower. This problem is extremely complicated mathematically, and here there is clearly no hope of getting an analytic solution. Therefore it is necessary to look for convenient approximate methods for solving the problem. It seems to us that in this case the method of moments can be very useful. Another possible approach is the numerical solution of the problem with high-speed electronic computing machines. But because of certain peculiarities of the electromagnetic cascade process — the energy spectrum of the particles goes as E^{-s} and the multiplicity increases rapidly with the depth — the solution of problems of this type encounters grave difficulties, and there has so far not been a single electronic-computer calculation of the three-dimensional development of a shower.

In the present paper we take the first step of calculating the average angle $\overline{\theta}_x(E, x)$ between the axis of the shower and the motion of particles having an energy E and present at a given distance x from the axis. A knowledge of the function $\overline{\theta}_x(E, x)$ given by the cascade theory will make it possible to proceed with more assurance in taking into account the effects of nuclear processes in the development of a given shower, if the nature of the primary particle that produced the shower is not evident. We shall consider the "equilibrium" value of $\theta_x(E, x)$, i.e., the average obtained by integration over the depth of development of the shower. Since this function is primarily of physical interest for $x \ll 1$, we neglect ionization losses. It can be shown that in an electromagnetic cascade shower the number of particles with energy $E < 10^8$ ev is small out to distances $\tilde{x} = Ex/E_s \lesssim 0.1$ (here x is expressed in terms of t as a unit, and $E_s = 21$ Mev).

Let the position of the particle in the plane perpendicular to the axis of the shower be given by the coordinates x and y. The direction of motion of the particle is given by the angles θ_x and θ_y in two mutually perpendicular planes with their intersection parallel to the axis of the shower. Let $P(E_0, E, x, y, \theta_x, \theta_y)\,dE dx dy d\theta_x d\theta_y$ be the number of electrons with energy in the range dE, position in the area dxdy, and motion in the range of angles $d\theta_x d\theta_y$ in a shower produced by a primary particle of energy E_0; and let $\Gamma(E_0, E, x, y, \theta_x, \theta_y)\,dE dx dy d\theta_x d\theta_y$ be the analogous number of photons. We treat the scattering as multiple, in the Landau approximation.[2] The calculation is made in the small-angle approximation, i.e., $\cos\theta$ is replaced by 1 and $\sin\theta$ by θ. We neglect the backward current of particles through the boundary of the absorber at $t = 0$.[3]

For the case in which an electron of energy E_0 is incident vertically on the boundary of the layer of material at $t = 0$, we integrate the fundamental equations of three-dimensional cascade theory with respect to t from 0 to ∞ and with respect to y and θ_y from $-\infty$ to ∞. We multiply the equations so obtained by θ_x^n and integrate them with respect to θ_x from $-\infty$ to ∞ and with respect to x from $-\infty$ to x_0. We then have:

$$\int_{-\infty}^{\infty} P(E_0, E, x_0, \theta_x)\,\theta_x^{n+1} d\theta_x = L_1 [P_{0n}(E_0, E, x_0), \Gamma_{0n}(E_0, E, x_0)] + \frac{E_s^2}{4E^2} n(n-1) P_{0,\,n-2}(E_0, E, x_0) - \delta(E_0 - E)\,\delta_{n0};$$

$$\int_{-\infty}^{\infty} \Gamma(E_0, E, x_0, \theta_x)\,\theta_x^{n+1} d\theta_x = L_2 [P_{0n}(E_0, E, x_0), \Gamma_{0n}(E_0, E, x_0)]. \qquad (1)$$

Here L_1 and L_2 are integral operators that give the effects of radiative deceleration and pair production; $\delta(E_0 - E)$ is the δ function; and δ_{n0} is the Kronecker symbol. We have introduced the notations

$$P_{0n}(E_0, E, x_0) = \int_{-\infty}^{x_0} dx \int_{-\infty}^{\infty} P(E_0, E, x, \theta_x)\,\theta_x^n d\theta_x;$$

$$\Gamma_{0n}(E_0, E, x_0) = \int_{-\infty}^{x_0} dx \int_{-\infty}^{\infty} \Gamma(E_0, E, x, \theta_x)\,\theta_x^n d\theta_x, \qquad (2)$$

where

$$P(E_0, E, x, \theta_x) = \iint_{-\infty}^{\infty} P(E_0, E, x, y, \theta_x, \theta_y)\,dy\,d\theta_y;$$

$$\Gamma(E_0, E, x, \theta_x) = \iint_{-\infty}^{\infty} \Gamma(E_0, E, x, y, \theta_x, \theta_y)\,dy\,d\theta_y. \qquad (3)$$

The quantities in which we are interested are

$$\overline{\theta_x^P}(E, x_0) = \int_{-\infty}^{\infty} P(E_0, E, x_0, \theta_x)\,\theta_x d\theta_x \Big/ \int_{-\infty}^{\infty} P(E_0, E, x_0, \theta_x)\,d\theta_x;$$

$$\overline{\theta_x^\Gamma}(E, x_0) = \int_{-\infty}^{\infty} \Gamma(E_0, E, x_0, \theta_x)\,\theta_x d\theta_x \Big/ \int_{-\infty}^{\infty} \Gamma(E_0, E, x_0, \theta_x)\,d\theta_x. \qquad (4)$$

In order to find them, we must examine the system (1) for $n_4 = 0$. Then, using the explicit expressions for the operators L_1 and L_2,[4] we have

$$\int_{-\infty}^{\infty} P(E_0, E, x_0, \theta_x)\,\theta_x d\theta_x = 2\int_0^1 \Gamma_{00}\left(E_0, \frac{E}{u}, x_0\right) \psi_0(u)\,\frac{du}{u} - \int_0^1 \left[P_{00}(E_0, E, x_0) - \frac{1}{1-v} P_{00}\left(E_0, \frac{E}{1-v}, x_0\right)\right] \varphi_0(v)\,dv - \delta(E_0 - E);$$

$$\int_{-\infty}^{\infty} \Gamma(E_0, E, x_0, \theta_x)\,\theta_x d\theta_x = \int_0^1 P_{00}\left(E_0, \frac{E}{v}, x_0\right) \varphi_0(v)\,\frac{dv}{v} - \sigma_0 \Gamma_{00}(E_0, E, x_0). \qquad (5)$$

Here

$$P_{00}(E_0, E, x_0) = \int_{-\infty}^{x_0} P(E_0, E, x)\,dx,$$

$$\Gamma_{00}(E_0, E, x_0) = \int_{-\infty}^{x_0} \Gamma(E_0, E, x)\,dx.$$

For $P(E_0, E, x)$ and $\Gamma(E_0, E, x)$ we can obtain the following expressions:

$$P(E_0, E, x) = 2\int_x^{\infty} \frac{P(E_0, E, r)\,r\,dr}{\sqrt{r^2 - x^2}},$$

$$\Gamma(E_0, E, x) = 2\int_x^{\infty} \frac{\Gamma(E_0, E, r)\,r\,dr}{\sqrt{r^2 - x^2}}. \qquad (6)$$

The functions $P(E_0, E, r)$ and $\Gamma(E_0, E, r)$ can be represented in the form

$$P(E_0, E, r) = P_{\mathrm{long}}(E_0, E)\,P_r(Er/E_s),$$

$$\Gamma(E_0, E, r) = \Gamma_{\mathrm{long}}(E_0, E)\,\Gamma_r(Er/E_s), \qquad (7)$$

where

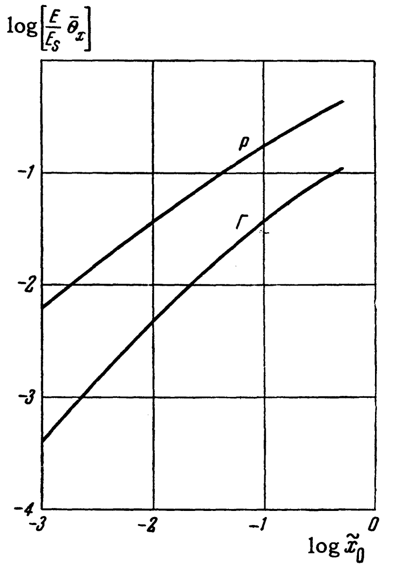
$$P_{\mathrm{long}}(E_0, E) = P(E_0, E)\,E^2/E_s^2,$$

$$\Gamma_{\mathrm{long}}(E_0, E) = \Gamma(E_0, E)\,E^2/E_s^2. \qquad (8)$$

$P(E_0, E)$ and $\Gamma(E_0, E)$ are the equilibrium spectra of electrons and photons, respectively. The functions P_r and Γ_r are normalized in the following way:

$$\int_0^{\infty} P_r(\tilde{x})\,\tilde{x}\,d\tilde{x} = \int_0^{\infty} \Gamma_r(\tilde{x})\,\tilde{x}\,d\tilde{x} = 1. \qquad (9)$$

For the numerical calculations of $\overline{\theta}_r(E, x_0)$ we obtained values of the functions $P_r(x)$ and $\Gamma_r(x)$ by the method of moments explained in reference 5. By substituting all the required values in Eq. (5) and carrying out the rather lengthy calculation one can get convenient formulas for the computation of $\overline{\theta_x^P}(E, x_0)$ and $\overline{\theta_x^\Gamma}(E, x_0)$; these contain single integrations and two double integrations that can be computed relatively easily. The results of the computation are shown in the diagram. It is interesting to compare the values of the mean angle so obtained with experiment. N. L. Grigorov and M. A. Kondrat'eva have obtained the corresponding experimental values in a study of an electron shower caused by a primary electron or photon or energy $E_0 \approx 10^{13}$ ev. The table shows the experimental and calculated values of the mean angle for various distances from the axis. It can be seen from the table that the calculated values of $\overline{\theta}_x$ agree well with the experimental values. We must take into account the fact that the authors mentioned measured the angles $\overline{\theta}$, not the projections $\overline{\theta}_x$. Knowing the angular distribution of the shower particles at a given distance from the axis, we can



Dependence of the mean angle $\bar{\theta}_x$ (in radians) made by paths of particles with the axis of the shower on the distance from the axis. Abscissas are values of the logarithm of the distance $\tilde{x}_0 = Ex_0/E_s$. Ordinates are values of the logarithm of the mean angle with the axis for electrons (curve P) and photons (curve Γ), in both cases multiplied by E/E_s.

show that $\bar{\theta}_x \approx \bar{\theta}_{exp}/1.6$. Furthermore the experimental errors in these measurements of $\bar{\theta}$ were 20 to 30 percent. The agreement of the calculated and experimental values of $\bar{\theta}_x$ enables us to con-

r/t	0.01	0.04	0.1	0.2
$\bar{\theta}^\circ_{x\,\mathrm{calc}}$	0.7	2.1	4.6	7.9
$\bar{\theta}^\circ_{\mathrm{exp}}$	1.2	4.2	9	11.5
$\bar{\theta}^\circ_{x\,\mathrm{exp}} \approx \bar{\theta}^\circ_{\mathrm{exp}}/1.6$	0.75	2.6	5.6	7.2

clude with assurance that the shower studied by Grigorov and Kondrat'eva was a pure electron-photon shower. We can at any rate be sure that the part played by nuclear processes in the development of this shower was a small one.

[1] K. Kamata and I. Nischimura, Suppl. to Prog. Theor. Phys. **6**, 93 (1958).

[2] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 1007 (1940).

[3] Blocker, Kennery, and Panofsky, Phys. Rev. **79**, 419 (1950).

[4] S. Z. Belen'kii, Лавинные процессы в космических лучах, (Cascade Processes in Cosmic Rays), 1948.

[5] V. V. Guzavin and I. P. Ivanenko, Suppl. Nuovo cimento **8**, 749 (1958).

Translated by W. H. Furry
295