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# *STRUCTURE OF A MAGNETOHYDRODYNAMIC SHOCK WAVE IN A PARTIALLY IONIZED GAS*

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A magnetohydrodynamic shock wave in a partially ionized gas consists of a thin plasma discontinuity and a transition zone. The equations for the transition zone are solved approximately for certain special cases. The charge-exchange effect does not significantly influence the general character of the motion but decreases its scale. As long as the wave can be considered stationary within the transition zone the magnitude of energy dissipation is independent of the degree of ionization.

THE structure of magnetohydrodynamic shock waves in a plasma of either infinite or finite isotropic conductivity has been studied a number of times.[1-3] Viscosity and Joule heat losses determine the width of the shock front, which can be of the order of a few mean free paths or considerably smaller, depending on the shock strength and conductivity. In cases of practical interest the gas is usually not fully ionized. In interstellar gas 0.1% ionization is reached in neutral hydrogen regions and 90% in ionized hydrogen regions (helium atoms are usually neutral). Different degrees of ionization can exist in stellar atomospheres. The gas is also usually not fully ionized in laboratory experiments. It is therefore of interest to examine the structure of a wave in the presence of neutral atoms. Such a study may, in particular, help to determine when the magnetic field can serve as a "damper" which reduces shock wave dissipation.

Let a plane wave be excited by the motion of a "piston" in the direction perpendicular to a field $H_0$. In the absence of all interaction between ions and neutral atoms two waves would be excited by the piston — a magnetohydrodynamic wave in the plasma and an ordinary neutral gas wave. When the different kinds of particles ahead of the front have the same temperature the plasma is compressed less than the neutral gas because of magnetic pressure and the presence of electrons (the molecular weight being reduced to one-half). Therefore with identical velocity of the gases in the laboratory system the wave front will move faster in the plasma than in the neutral gas and will thus propagate in an undisturbed gas.

At temperatures below 100,000° and with not too low ionization ($\geq 10\%$) the mean free path of ions is hundreds or thousands of times smaller than the mean free path of neutral atoms. Therefore the presence of neutral atoms has no effect within the plasma wave front even when we neglect the reduction of the front width due to Joule losses, and the pressure jump is determined by the ordinary shock adiabate. The properties of the neutral gas do not become discontinuous at the shock front but will change gradually behind the front through interaction with the ions while their velocities are unequal. The present paper is a calculation of the structure of the region in which the parameters vary relatively smoothly. Joule losses and plasma viscosity are important in the much thinner plasma wave front and can be neglected in the region of present interest, which is typically hundreds of times larger. The cross section for the transfer of momentum from electrons to ions is large because of electrostatic interaction; therefore ions entrain electrons. Moreover, the inequality of their mean velocities in a plane wave would result in charge separation, which is impossible if we disregard high-frequency oscillations. Therefore the plasma can be regarded as a single whole. We shall assume for simplicity that the ion wave does not induce additional ionization. This assumption is permissible in an interstellar gas, where ionization is determined by the stellar radiation field and not by the kinetic temperature.

Under the foregoing assumptions, the equations of steady-state motion in a coordinate system moving with the shock front are given by

$$\rho_i v_i = M_i, \qquad \rho_n v_n = M_n, \qquad H\rho_i^{-1} = \text{const}, \tag{1}$$

$$\rho_i v_i \frac{dv_i}{dx} + \frac{dp_i}{dx} + \frac{1}{8\pi}\frac{dH^2}{dx} - (v_n - v_i)\, n_i n_n \mu v_{\mathbf{rel}}\sigma = 0, \tag{2}$$

$$\rho_n v_n \frac{dv_n}{dx} + \frac{dp_n}{dx} + (v_n - v_i)\, n_i n_n \mu v_{\mathrm{rel}} \sigma = 0, \tag{3}$$

where $M_i$ and $M_n$ are constants equal to the mass flows of the respective gases, $v_i$ and $v_n$ are the mean velocity of the plasma and atoms, and $v_{rel}$ is the mean relative velocity of ions and atoms which determines the collision frequency. We shall hereinafter consider not too weak waves, in which the ion velocity jump is greater than the thermal velocity; then $v_{rel} \approx v_n - v_i$. This assumption permits us to consider only the part of the wave in which the velocity difference has still not been reduced to the thermal velocity. $\mu = m_n m_i/(m_n + m_i) = m/2$ is the reduced mass (with $m_n = m_i$ for simplicity) and $\sigma$ is the cross section for momentum transfer in collisions of ions with neutral atoms. $p_n$ and $p_i$ are determined primarily by compression and heating resulting from ion-atom collisions.

In each collision an ion and an atom acquire on the average the energy $\frac{1}{2}m(v_n - v_i)^2/4$. Therefore the heat increment (per $cm^3$/sec) of the ion and neutral gases is

$$q_n = q_i = {}^1/_8 n_i n_n \sigma m (v_n - v_i)^3. \tag{4}$$

The pressure is given by

$$dp_i = \gamma \frac{p_i}{\rho_i} d\rho_i + (\gamma - 1)\, q_i dt,$$
$$dp_n = \gamma \frac{p_n}{\rho_n} d\rho_n + (\gamma - 1)\, q_n dt, \tag{5}$$

where $dt = v_i^{-1} dx$ and $v_n^{-1} dx$, respectively.

Since neither the velocity nor the pressure remains constant behind the front our parameter of wave strength will be the velocity U of the front with respect to the quiet gas. We now introduce the dimensionless variables

$$w_i = v_i / U, \quad w_n = v_n / U, \quad \Pi_i = p_i / M_i U, \quad \Pi_n = p_n / M_n U,$$
$$Q = H_0^2 / 8\pi M_i U, \qquad I = n_{i0} / n_{n0}, \qquad \xi = n_{n0} \sigma x,$$

where $n_{n0}$ and $n_{i0}$ are the atom and ion concentration ahead of the front. Equations (2), (3), and (5) become

$$\frac{dw_i}{d\xi} + Q \frac{d}{d\xi}\left(\frac{1}{w_i^2}\right) + \frac{d\Pi_i}{d\xi} - \frac{1}{2}\frac{(w_n - w_i)^2}{w_i w_n} = 0, \tag{6}$$

$$\frac{dw_n}{d\xi} + \frac{d\Pi_n}{d\xi} + I \frac{(w_n - w_i)^2}{2 w_i w_n} = 0, \tag{7}$$

$$\frac{d\Pi_i}{d\xi} + \gamma \Pi_i \frac{d \ln w_i}{d\xi} - \frac{\gamma - 1}{8} \frac{(w_n - w_i)^3}{w_i^2 w_n} = 0, \tag{8}$$

$$\frac{d\Pi_n}{d\xi} + \gamma \Pi_n \frac{d \ln w_n}{d\xi} - \frac{\gamma - 1}{8} I \frac{(w_n - w_i)^3}{w_i w_n^2} = 0. \tag{9}$$

At $\xi = 0$ (subscript 1) the functions are determined by the parameters of the plasma discontinuity:

$$w_{i1} = \rho_{i0} / \rho_{i1} = 1/\alpha, \quad \Pi_{i1} = p_{i1} / M_i U,$$
$$w_{n1} = 1, \quad \Pi_{n1} = \Pi_{n0} = p_{n0} / M_n U.$$

Compression and heating in a magnetohydrodynamic wave have previously been calculated,[4,5] but must now be expressed in our dimensionless quantities. From the shock adiabate for a monatomic gas we obtain

$$Q\alpha^2 + 5\alpha (Q + \Pi_{i0} + 0.2) - 4 = 0, \tag{10}$$
$$\Pi_{i1} - \Pi_{i0} = (\alpha - 1)\,[1/\alpha - Q(\alpha + 1)]. \tag{11}$$

Equations (6) — (9) can be integrated numerically but the basic parameters of the solution can be obtained in the rough approximation $\Pi_i = \Pi_n = 0$. Subsequent estimates indicate that this approximation is adequate in many cases since, although the wave velocity may considerably exceed the velocity of sound, heating of the gas is insignificant in the presence of a sufficiently strong field.[6] This is evident, specifically, from the adiabat for the gas as a whole.

Multiplying (6) by I, adding to (7) and integrating, we obtain the momentum integral for the entire gas:

$$w_i + Q / w_i^2 + \Pi_i + w_n / I + \Pi_n / I$$
$$= w_{i1} + Q / w_{i1}^2 + \Pi_{i1} + w_{n1} / I + \Pi_{n1} / I$$
$$= 1/\alpha + Q\alpha^2 + 1/I + \Pi_{i1} + \Pi_{n1} / I. \tag{12}$$

Since $\alpha$ and Q are related by (10) and we have assumed $\Pi_{n1} = \Pi_{i1} = 0$, (12) gives us $w_i$ in terms of $w_n$ and I as well as the solution for a rarefaction wave. $\xi$ for a pair of values of $w_n$ and $w_i$ is obtained from (7) by means of a quadrature:

$$\xi = -\frac{2}{I} \int_1^{w_n} \frac{w_n w_i}{(w_n - w_i)^2} dw_n. \tag{13}$$

The result of the calculation for Q = 0.25 and I = 1 is shown in Fig. 1. This example corresponds to the average conditions expected in the rarefied gas of our galaxy[7] — $\rho_{n0} \approx \rho_{i0} \approx 10^{-26}$ g/cm$^3$, v = 100 km/sec, H ≈ 5 × $10^{-6}$ oersteds, U ≈ 200 km/sec. U is taken to be twice as large as v,

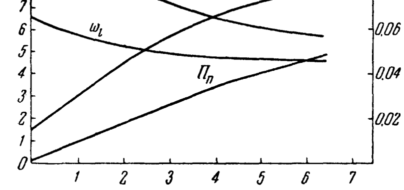


FIG. 1. Q = 0.25, I = 1.

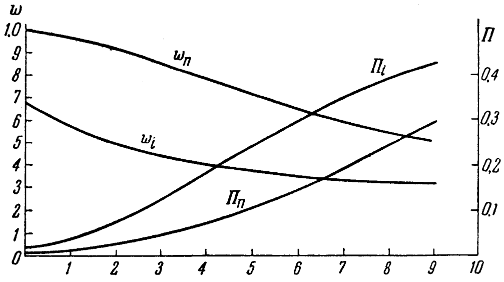


FIG. 2. Q = 0.25, I = 0.3.

since, as is evident from Fig. 1, the final gas compression is close to 2 ($w \approx 0.5$).

In order to calculate the pressures $\Pi_i$ and $\Pi_n$ we assume in first approximation that pressure has little effect on the motion. Substituting the values of $w_i$ and $w_n$ calculated above into (8) and (9) and performing a numerical integration, we obtain the curves of $\Pi_i$ and $\Pi_n$ in Fig. 1. We have $\Pi_{n,i} \ll w_{n,i}$; therefore, as is evident from (12), the first approximation is adequate and the motion actually depends only slightly on the gas pressure.

In order to elucidate the dependence of the solution on the parameters similar calculations were carried out for the following cases: Q = 0.25, I = 0.3; Q = 0.25, I = 3; Q = 0.15, I = 1 (Figs. 2–4). The general character of the solution is the same in all cases, but in a weak field and with strong ionization $w_n$ varies considerably in the distance $\xi = 1$, so that the ordinary viscosity of the neutral gas may now be important. With weak ionization $\Pi_i$ has a considerable effect on the motion since with low ionization the field moderates the compression to a lesser extent ($w_i \approx 0.3$) and supersonic motion results in considerable heating of the gas as a whole.

We now calculate the irreversible dissipation of energy in the wave. Let the wave compress and heat the gas, after which it expands adiabatically to its initial density in a long time interval. The internal energy of a mass unit in the wave is given for both the plasma and gas by

$$\frac{1}{\gamma-1}\frac{p}{\rho} = \frac{1}{\gamma-1}\Pi w U^2$$

with the proper subscripts. During the subsequent adiabatic expansion this energy varies proportionally to $w^{\gamma-1}$. Thus the irreversible energy dissipation amounts to $U^2(\gamma-1)^{-1}(\Pi_i w_i^\gamma + \Pi_n w_n^\gamma) - \epsilon_0$, where $\epsilon_0$ is the internal energy ahead of the front. This quantity can be easily computed by means of Figs. 1–4. In all of the cases considered except that of a strongly ionized gas (I = 3) most of the dissipation occurs behind the plasma discontinuity rather than at the discontinuity. For I = 3 the dissipation at the discontinuity is approximately equal to that behind it.

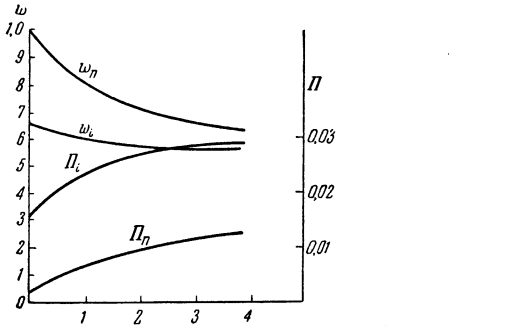


FIG. 3. Q = 0.25, I = 3.

The irreversible energy dissipation, the final total pressure and final velocity can also be determined by the shock adiabate method[5] if we assume that there is sufficient friction between the gases and that the magnetic pressure thus also acts on the neutral gas. The figures show that for this purpose the separation of the planes at which jumps of the various quantities are measured must exceed 5 or 10 mean free paths in the neutral gas. The calculation can be preformed by means of (10) and (11) with Q and $\Pi_i$ replaced by $Q'$ and $\Pi'$ as follows:

$$Q' = \frac{H_0^2}{8\pi(M_i + M_n)U} = Q\frac{I}{1+I},$$

$$\Pi' = \frac{p_n + p_i}{(M_n + M_i)U} = \frac{\Pi_n + I\Pi_i}{1+I}.$$

The results agree with Figs. 1–4 to within 10–20%. Where the influence of $\Pi_i$ on motion can no longer be neglected Fig. 1, as would be expected, gives systematically larger values than the shock adiabate. Since Q is smaller for the entire gas than for the plasma alone the final compression must be greater than at the plasma discontinuity. This accounts for the fact that the curve of $w_i$ continues to descend and does not approach the curve of $w_n$.

We have thus far only considered the usual elastic interaction between ions and atoms, which is sufficient if these do not belong to the same element and their relative velocities are not too large

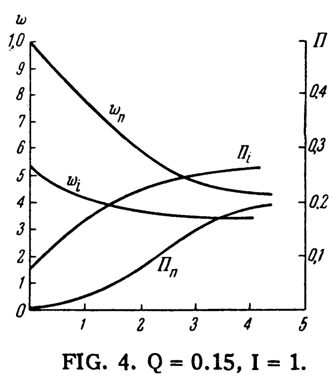


FIG. 4. Q = 0.15, I = 1.

(below 1000 km/sec, for example). When these conditions are not fulfilled charge exchange becomes much more significant; an electron is transfered from an atom to an ion without essential change in the motion of either particle. The charge-exchange cross sections for identical particles is larger than the gaskinetic cross section by a factor of a few tens. The width of the transition zone will be correspondingly reduced but will still exceed that of the plasma wave front (when Joule losses are taken into account), and the division of the wave into a plasma discontinuity and a transition zone remains valid. Therefore the fundamental equations will not be changed greatly if the charge-exchange cross section $\sigma_i$ is introduced into the definition $\xi = n_{n0}\sigma x$. The principal difference will lie in the fact that after charge exchange the velocities of the particles will remain practically unchanged; this is equivalent to 180° scattering. Therefore the momentum change of the gases in one charge exchange will be given by $m(v_n - v_i)$ rather than by $\frac{1}{2}m(v_n - v_i)$ as previously; this means that the last terms in (2) and (3) are doubled.

The quantities $q_n$ and $q_i$ in (4) are also changed. The neutral atom resulting from charge exchange retains the thermal component of ion velocity, so that charge exchange is a mechanism for heat transfer. The resultant ion possesses the velocity $v_n - v_i$ with respect to the plasma and its associated field. The energy of relative motion is slowly transformed into thermal energy through collisions with other ions if the mean free ion path is smaller than the radius of gyration. When the mean free ion path is greater than the radius of gyration, during a single rotation this energy is transformed into the energy of spiral motion and then, as a result of collisions, into thermal energy. We can therefore write

$$q_i = n_i n_n \sigma_1 [{}^1/_2 m (v_n - v_i)^2 - {}^3/_2 k (T_i - T_n)] (v_n - v_i),$$
$$q_n = {}^3/_2 n_i n_n \sigma_1 k (T_i - T_n)(v_n - v_i). \qquad (14)$$

Since there are practically no collisions between neutral atoms during the time between charge exchanges the distribution of the neutral gas cannot be Maxwellian. However, since atoms are produced from ions, which can exchange energy at not too high temperatures, the distribution of atomic velocities will not be extremely non-Maxwellian. Deviations can result from 1) the continuous rise of the plasma "temperature" in conjunction with different atom "ages" and 2) the departure of the ion velocity distribution from the Maxwellian because of the limited interaction time. This is especially important for relatively fast particles since the elastic ion-scattering cross section for ions decreases as $v^{-4}$. Therefore the thermal velocities of individual particles can hardly exceed $v_n - v_i$. On the other hand, $v_n - v_i$ decreases continually, thus complicating the picture to an even greater degree.

Returning to the basic equations, we find that the first equation in (5) will retain its form with a different meaning for $q_i$. The second equation is changed since there is no adiabatic heating in the absence of collisions between atoms. The atom concentration increases because there is an increase in the concentration of the ions out of which the former are produced, but the atoms themselves are subject to no forces. Therefore in the second equation of (5) only the second term remains, with the suitable meaning of $q_n$. Equations (6) and (7) remain unchanged with $\xi = 2n_{n0}\sigma_i x$, which means that in a single charge exchange twice as much momentum is transferred as previously.

It would not be meaningful to solve the complete system of equations under the given simplifying assumptions. We shall consider only the heat supplied to both gases:

$$q = q_i + q_n = {}^1/_2 n_i n_n \sigma_1 m (v_n - v_i)^3,$$

which is twice as large as previously. Both the rate of heating and the rate of momentum transfer have been doubled. Therefore the entire process occurs twice as rapidly (taking the mean free time as the unit) but ultimately results in the same change of total internal gas energy as previously. This also follows, of course, from the fact that the shock adiabat must give the same solution for the sum of the gases independently of their interaction mechanism.

Summarizing, it can be stated that when pressure plays an insignificant part the velocity distribution will be that shown in the figures but with the new definition of $\xi$. The values of $\Pi_i$ and $\Pi_n$ will be somewhat changed but the general character of the curves and their sum will be conserved. The curves of $\Pi_n$ and $\Pi_i$ will be separated by a distance corresponding to the mean time for charge exchange, which serves as the mechanism for transferring energy from ions to atoms. At high gas velocities the ion path before elastic scattering by an ion can become greater than its path before charge exchange, and Joule losses will be small because of the high temperature. In this case a plasma wave front will not be formed; the plasma and atoms will move together with the field at some mean velocity. Momentum transfer will still occur as a result of charge exchange and ion acceleration by the field. The entropy will increase because the field twists the trajectories of ions passing from the quiet gas through the front as neutral atoms, after which collisions occur, and also because of the Joule losses.

Since the energy dissipation of the wave in a partially ionized gas is given by the shock adiabate with Q for the entire gas, it does not depend on the degree of ionization when the wave can be regarded as stationary within the transition region. With extremely low ionization the transition region may become so wide that this condition is not fulfilled. However, when charge exchange occurs the transition zone is less wide and the possibility of nonstationary conditions plays a smaller part.

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