Letters to the Editor

CONCERNING RYBUSHKO'S WORK, "ON THE EQUATIONS OF MOTION OF ROTATING MASSES IN THE GENERAL THEORY OF RELATIVITY"

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In a paper by Ryabushko,[1] an attempt was made to get the equations of motion of a system of rotating bodies by Infeld's method, that is by introducing a Dirac δ function into the energy-momentum tensor. For this, Ryabushko uses the well-known formula of Newtonian mechanics by which the velocity of an arbitrary point of a rigid body a is $\mathbf{v}'_a = \mathbf{v}_a + [\boldsymbol{\omega}_a \times \mathbf{r}_a]$, where $\mathbf{v}_a$ is the velocity of the center of mass, $\boldsymbol{\omega}_a$ the angular velocity of the body, and $\mathbf{r}_a = \mathbf{r} - \mathbf{a}$ is the position vector drawn from the center of mass of the body to the point whose velocity is being determined. Ryabushko proposes to replace the expression $[\boldsymbol{\omega}_a \times \mathbf{r}_a]$ in the case when the body a is restricted to a point by the expression $\frac{1}{2}[\boldsymbol{\sigma}_a \times \nabla\delta_a]$, where $\boldsymbol{\sigma}_a$ is a pseudovector and the operator $\nabla = e_i\partial/\partial x_i$ acts on the delta function δ_a. The velocity expression thus obtained is substituted into the energy-momentum tensor [Eq. (3.4)].

However, the pseudovector $\boldsymbol{\sigma}_a$ is a combination of the spatial components of the antisymmetric angular momentum 4-tensor $S^{\alpha\beta}$. Consequently, Ryabushko supposes that the remaining components S^{0i} ($i = 1, 2, 3$) are identically zero. This hypothesis, according to Corinaldesi and Papapetrou,[2] can be realized for a given particle a by the introduction of the center of mass coordinates of the particle. If we have a central body of large mass whose field is given by a Schwarzschild solution, for the motion of particles of infinitesimal mass in this field we can introduce the centers of mass of these particles relative to the stationary coordinate system. However, if we have only n particles without a central body, it is impossible to introduce uniquely their centers of mass (the components of the center of mass of one particle are not components of any 4-vector). In this case, in the absence of a privileged coordinate system, one must use the full angular momentum tensor $S^{\alpha\beta}$, where $S^{0i} \neq 0$. Consequently, the energy-momentum tensor (3.4) in Ryabushko's paper cannot be correct. We shall show that even Ryabushko's equations of motion cannot be true.

In reference 1 there is obtained a correction to the equations of motion in general relativity for the case of rotating masses, which in the two-body case has the form

$$D_a^i = -\frac{fm_a m_b}{c^2}\{(\dot a^s - \dot b^s)[\sigma_a\nabla(1/r_{ab}), a^s]^i - 2(\dot a^s - \dot b^s)$$
$$\times[\sigma_b\nabla(1/r_{ab}), a^s]^i + (2\dot a^s - \dot b^s)[\sigma_b\nabla(1/r_{ab}), a^i]^s + (2\dot b^s - \dot a^s)$$
$$\times[\sigma_a\nabla(1/r_{ab}), a^i]^s + \text{quadratic terms in } \sigma. \qquad (1)$$

σ_a and σ_b are the Newtonian proper angular momenta of the bodies. The Latin indices i, k, s . . . take on the values 1, 2, 3 and refer only to the spatial coordinates. The relativistic equations of the progressive motion of the a-th body is described by the form

$$m_a \ddot{a}^i = (fm_a m_b / r_{ab})_{,a^i} + F_a^i + D_a^i, \qquad (2)$$

where F_a^i is the correction to the Newtonian force in the second approximation of general relativity not counting the rotation of the bodies, as given by Einstein, Infeld, Fock, and others.

According to Fock (reference 2, p. 359) that correction to the Lagrangian in the problem of two rotating bodies which depends on the rotations of the bodies and influences their motions has the form

$$L_\omega = fc^{-2}[m_a\omega_{si}^{(b)}I_{sj}^{(b)}(3\dot b_i - 4\dot a_i)$$
$$- m_b\omega_{si}^{(a)}I_{sj}^{(a)}(3\dot a_i - 4\dot b_i)](a_j - b_j)/|\mathbf{a} - \mathbf{b}|^3. \qquad (3)$$

$\omega_{si}^{(a)}$ is the antisymmetric angular velocity tensor of the a-th body, and $I_{sj}^{(a)}$ is the symmetric moment of inertia tensor of the same body. The correction to the equations of motion is given by the expression

$$\partial L_\omega/\partial a_l - (d/dt)\,\partial L_\omega/\partial \dot a_l = c^{-2}[fm_a\omega_{sk}^{(b)}I_{sj}^{(b)}(3\dot b_k - 4\dot a_k)$$
$$- fm_b\omega_{sk}^{(a)}I_{sj}^{(a)}(3\dot a_k - 4\dot b_k)][\delta_{lj}/|\mathbf{a}-\mathbf{b}|^3$$
$$- 3(a_l - b_l)(a_j + b_j)/|\mathbf{a}-\mathbf{b}|^5]$$
$$+ c^{-2}(4fm_a\omega_{sl}^{(b)}I_{sj}^{(b)} + 3fm_b\omega_{sl}^{(a)}I_{sj}^{(a)})[(\dot a_j - \dot b_j)/|\mathbf{a}-\mathbf{b}|^3$$
$$- 3(a_j - b_j)(a_l - b_l)(\dot a_l - \dot b_l)/|\mathbf{a}-\mathbf{b}|^5]. \qquad (4)$$

If each body has a spherically symmetric mass distribution, we can put

$$I_{sj}^{(a)} = I^{(a)}\delta_{sj}. \qquad (5)$$

Formula (1) can be written in this case as

$$
\begin{aligned}
D_a^i = (f m_a m_b / c^2) \{ [(2/m_a) I^{(a)} \omega_{si}^{(a)} (3\dot{b}^s - 2\dot{a}^s) \\
+ (2/m_b) I^{(b)} \omega_{si}^{(b)} (4\dot{a}^s - 3\dot{b}^s)] \, |\mathbf{a} - \mathbf{b}|^{-5} \\
+ (2/m_a) I^{(a)} [3\omega_{ji}^{(a)} (a_j - b_j)(a_s - b_s)(\dot{a}_s - \dot{b}_s) \\
+ 3\omega_{js}^{(a)} (a_j - b_j)(a_i - b_i)(2\dot{b}_s - \dot{a}_s)] \, |\mathbf{a} - \mathbf{b}|^{-5} \\
+ (2/m_b) I^{(b)} [-6\omega_{ji}^{(b)} (a_j - b_j)(a_s - b_s)(\dot{a}_s - \dot{b}_s) \\
+ 3\omega_{js}^{(b)} (2\dot{a}_s - \dot{b}_s)(a_j - b_j)(a_i - b_i)] \, |\mathbf{a} - \mathbf{b}|^{-5} \}. \qquad (6)
\end{aligned}
$$

As is easily verified, formulas (4) and (6) under conditions (5) only coincide if $\sigma_a = 0$, $b_s = 0$ (a case considered by Lense and Thirring, Das, and the author, giving the motion of a satellite of small mass around a rotating central body). In the general case (4) and (6) do not coincide. Consequently, Ryabushko's assertion in reference 1 that for spherically symmetric bodies the first members of (1) coincide with the results of Fock[3] is therefore incorrect.

Formula (1) cannot be correct even under the following considerations: D_a^i from (1) and (6) can be got from Lagrange's equation

$$D_a^i = \partial L'_\omega / \partial a_i - (d/dt) \partial L'_\omega / \partial \dot{a}_i \qquad (7)$$

with a correction to the Lagrangian because of the rotation of the bodies

$$
\begin{aligned}
L'_\omega = f c^{-2} [m_a \omega_{si}^{(b)} I_{sj}^{(b)} (2\dot{b}_i - 4\dot{a}_i) \\
+ m_b \omega_{si}^{(a)} I_{sj}^{(a)} (2\dot{a}_i - 4\dot{b}_i)] (a_j - b_j) / |\mathbf{a} - \mathbf{b}|^3 \qquad (8)
\end{aligned}
$$

using condition (5). It is easily verified that, under the interchange of a and b, L'_W goes over to $-L'_W$. This fact contradicts the requirement that the correction to the Lagrangian, just like the full Lagrangian, must be invariant under the interchange of the two bodies; Fock's correction to the Lagrangian obviously satisfies this requirement.

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ON THE DEPTH OF THE POTENTIAL WELL FOR Λ PARTICLES IN HEAVY HYPERNUCLEI

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THE energy of the Λ particles in heavy hypernuclei, together with the binding energies of the light hypernuclei, imposes certain restrictions on the Λ-nucleon potential. The Λ particles in heavy hypernuclei can be regarded as moving in a square potential well whose depth is determined by the interaction of the Λ particle with the nucleons.[1,2] The author and Lyul'ka[3,4] considered Λ-nucleon potentials derived from meson theory. In order to avoid singularities at small distances, the momenta of the virtual mesons had to be cut off. These potentials yield the correct dependence of the binding energy B_Λ on the number of particles in light hypernuclei. They lead to a stronger interaction of the Λ-nucleon pair in the singlet state, which is in agreement with the value zero for the spin of ${}_\Lambda H^4$, as estimated from the ratio of the number of mesonic and non-mesonic decays. In the present note we make an estimate of the potential energy of the interaction of Λ particles in nuclear matter on the basis of the potentials obtained in references 3 and 4. In estimating the potential energy the nucleons in the nuclear matter were regarded as an incompressible degenerate Fermi gas. We carried out calculations for two values of the nuclear matter density, given by the radii $R = 1.2 A^{1/3} \times 10^{-13}$ and $R = 1.4 A^{1/3} \times 10^{-13}$. The actual density lies apparently somewhere between these limits.[5]

In the table we given the values of the potential energy of Λ particles in nuclear matter, U^{1K}, $U^{2\pi}$, $U^{K\pi}$, and U^{2K}, for Λ-nucleon potentials corresponding to the exchange of a single K, two π, a K and a π, and two K mesons, respectively. We also list the total potential energy U (all values are in Mev). In computing these values we assumed two types of coupling between the K mesons and the baryons: the scalar and the pseudoscalar coupling. The coupling between the baryons and the π mesons was assumed to be pseudovector with the coupling constant $f^2 = 0.08$. We used a rectangular cut-off with $k_m = 6\mu_\pi$. The resulting potential energy U