$$W = \frac{3\hbar\varkappa}{mR_0} w_\alpha \Gamma \sum_{l=0}^{2K}{}' (2l+1) \left| C_{KKl0}^{KK} \right|^2 \times e^{-\gamma_l l(l+1)} \left| \int_0^1 e^{\beta P_2(\mu)} P_l(\mu) d\mu \right|^2 \sum_J w_J, \quad (4)$$

where K is the projection of the nuclear moment on the axis of symmetry; the prime for the sigma means that the summation is done only for even l; $C_{j_1 m_2 j_2 m_2}^{JM}$ are the Clebsch-Gordan coefficients; $\gamma_l = 2\kappa/\kappa_b^2 R_0$. Results of a comparison with an experiment are listed in Table II. The average value of w_α is twice as small as for even nuclei.

TABLE I

Nucleus	w_α
Ra^{222}	0.155
Ra^{224}	0.051
Ra^{226}	0.068
Ra^{228}	0.104
Th^{226}	0.057
Th^{228}	0.063
Th^{230}	0.094
Th^{232}	0.079
Th^{234}	0.201
U^{230}	0.113
U^{232}	0.094
U^{234}	0.088
U^{236}	0.134
U^{238}	0.110
Pu^{238}	0.090
Pu^{240}	0.096
Cm^{242}	0.094
Cm^{246}	0.107
Cm^{248}	0.189
Cf^{250}	0.095
	$\bar{w}_\alpha=0.10$

TABLE II

Nucleus	K	w_α
Ac^{227}	$3/2$	0.0308
Th^{229}	$5/2$	0.0459
U^{235}	$1/2$	0.0413
Np^{237}	$5/2$	0.0517
Np^{239}	$5/2$	0.0724
Pu^{239}	$5/2$	0.0387
Cm^{245}	$9/2$	0.0361
Bk^{249}	$7/2$	0.0560
		$\bar{w}_\alpha=0.047$

***In a recent article[5] it is stated that the 2^4-pole interaction of an alpha particle with a daughter nucleus was not taken into consideration in any of the earlier published articles; it is also asserted that the theoretical calculations for non-even nuclei were not brought to formulas comparable with experiment. Actually, reference 2 gives the derivations of simple formulas for the case of non-even nuclei and a comparison is made with experimental results. The 2^4-pole interaction of an alpha-particle with a daughter nucleus was considered in reference 3, published at an earlier date.**

†The daughter nucleus is always shown in the tables.

[1] V. G. Nosov, Dokl. Akad. Nauk SSSR **112**, 414 (1957), Soviet Phys. "Doklady" **2**, 48 (1957).

[2] V. G. Nosov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 226 (1957), Soviet Phys. JETP **6**, 176 (1958).

[3] V. G. Nosov, Izv. Akad. Nauk SSSR, Ser. Fiz. **21**, 1551 (1957) Columbia Tech. Transl. 1541 (1957).

[4] V. G. Nosov, Ядерные реакции при малых и средних энергиях, труды Всесоюзной конференции, (Nuclear Reactions at Low and Medium Energies, Transactions of the All-Union Conference), Academy of Sciences U.S.S.R., 1958, p. 589.

[5] Gol'din, Adel'son-Vel'skiĭ, Birzgal, Piliya, and Ter-Martirosyan, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 184 (1958), Soviet Phys. JETP **8**, 127 (1959).

Translated by Genevra Gerhart
314

BREAK-UP OF CHARGED PARTICLES BY A NUCLEAR COULOMB FIELD

M. Z. MAKSIMOV

Submitted to JETP editor January 14, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1582-1583 (May, 1959)

A charged particle with charge Z_a and mass number a, in flying past a target nucleus (Z, A), interacts with its Coulomb field. In some cases the energy of this interaction is sufficient to dissociate the incident particle (nuclei of deuterium, beryllium, singly ionized molecules of hydrogen, and so forth). The theory of such processes was first developed by Dancoff[1] for the break-up of fast deuterons and later was carried over without special changes to the break-up of beryllium nuclei.[2] It was assumed that the disturbance (the energy of the Coulomb interaction) is time dependent, and so they used the conclusions of perturbation theory for this case.

Although quantum-mechanical methods are used in the references cited, the intermediate calculations contain a number of approximations which can hardly be justified (see, for instance, the simplification made in computing the integral I_r and the further impulse integration in reference 1). Besides, the final results are complicated and cannot always be applied to concrete evaluation. In this connection it is interesting to note an easier way of computing cross sections of the aforementioned processes — by applying a classical analysis analogous to the derivation of the Thompson formulas for the ionization of atoms by impact.[3]

Let the complex particle (Z_a, a) be capable of decaying into the parts (Z_1, a_1) and (Z_2, a_2), and let ϵ_0 be the binding energy for this decay. Then, in interacting with the Coulomb field of the target nucleus, each of these parts will receive an additional momentum equal in order of magnitude to (see reference 3)

$$p_i \approx (2ZZ_i e^2 / vb) [1 + (ZZ_a e^2 / \mu_a v^2)^2]^{-1/2}; \quad (1)$$

where the index i refers to the break-up products; v is the speed of the initial particle (Z_a, a); b is its impact parameter relative to the target nucleus, and $\mu_a = aAM_p/(A + a)$ is their reduced mass. In such a case the cross section of the examined process obviously will be equal to

$$\sigma = \pi b_{max}^2, \quad (2)$$

where b_{max} is defined from the condition that the acquired energy of relative motion E′ of the break-up products will be equal to (or greater than) the binding energy ϵ_0 of this system, that is

$$E' = \frac{1}{2\mu}\left(\frac{a_1}{a}\,p_2 - \frac{a_2}{a}\,p_1\right)^2 \geqslant \varepsilon_0, \qquad \mu = \frac{a_1a_2}{a}M_p. \quad (3)$$

By substituting here p_1 and p_2 from formula (1), and solving the resulting equation for b^2 and then averaging it over all emission angles, we obtain the following expression for the cross section after some simple transformations:

$$\sigma = \frac{\pi(Ze^2)^2}{E_0\varepsilon_0}\,\frac{Z_1^2a_2^2+Z_2^2a_1^2}{a_1a_2}\left(1 - \frac{\varepsilon_0}{E_0}\,\frac{Z_a^2a_1a_2}{Z_1^2a_2^2+Z_2^2a_1^2}\left(\frac{A+a}{2A}\right)^2\right). \quad (4)$$

Here E_0 is the kinetic energy of the incident particle, which for relativistic speeds must be replaced in formula (4) by the quantity $E_0(1+E_0/2M_ac^2)(1+E_0/M_ac^2)^{-2}$. The formula obtained for the cross section is similar to the Dancoff results calculated by numerical integration for $E_0 \sim 200$ Mev and $A \sim 100$.

Since we are interested in the case $E_0 \gg \epsilon_0$ then, from relation (4) we find (expressing E_0 and ϵ_0 in Mev),

$$\sigma \approx 6.3\cdot 10^{-26}\,\frac{Z^2}{\varepsilon_0E_0}\,\frac{a_2^2Z_1^2+a_1^2Z_2^2}{a_1a_2}\,\mathrm{cm}^2. \quad (5)$$

As can be seen from the resulting formulas, the estimate of cross sections for the processes under consideration does not present any particular difficulty. Thus for instance, for deuterons ($\epsilon_0 = 2.18$) with an energy of ~ 200 Mev and for beryllium ($\epsilon_0 = 1.7$) with an energy of ~ 100 Mev, we have $1.4 \times 10^{-28} Z^2$ and $0.45 \times 10^{-28} Z^2$ cm^2 respectively. These magnitudes differ somewhat from those calculated in references 1 and 2 but the latter are also approximations and have not been checked by experiment.

In conclusion we note that the same formulas can be used to estimate the cross section of the Coulomb break-up of the molecules H_2^+, D_2^+ and others (supposing that $Z_1 \sim 1$; $Z_2 \sim 0$; $a_1 = a_2$; $\epsilon_0 \sim 2.5$ ev), for which with energies of 20 — 30 kev we obtain $\sigma \approx 10^{-18} Z^2$ cm^2. The author is grateful to Yu. V. Kursanov who pointed out the possibility of such processes taking place in ion sources with a high degree of ionization.

[1] S. Dancoff, Phys. Rev. **72**, 1017 (1947); A. Akhiezer and I. Pomeranchuk, Некоторые вопросы теории ядра, (Certain Problems in the Theory of the Nucleus), GITTL, M.-L. 1950, p. 128.

[2] V. I. Mamasakhlisov and G. A. Chilashvili, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 806 (1957), Soviet Phys. JETP **5**, 661 (1957).

[3] V. L. Granovskiĭ, Электрический ток в газе (Electrical Current in Gas), GITTL, M-L. 1952, p. 148.

Translated by Genevra Gerhart
315

A SIMPLE METHOD OF CALCULATING THE DEGREE OF IONIZATION AND THERMODYNAMIC FUNCTIONS OF A MULTIPLY IONIZED IDEAL GAS

Yu. P. RAĬZER

Submitted to JETP editor January 15, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1583-1585 (May, 1959)

THE thermodynamic functions of a gas at high temperatures, when the gas atoms are multiply ionized, are calculated on the basis of ionization equilibrium. For each pair of values of the temperature and density we must solve a nonlinear system consisting of a few nonlinear algebraic equations for ion concentrations; this requires a long calculation. Such calculations have thus far been carried out only for air[1] from 20,000° to 500,000° and from 10 to 10^{-3} times normal pressure. We here suggest a simple method for obtaining a fairly accurate estimate of the degree of ionization and of the thermodynamic functions of any gas.

We write a set of Saha equations for a gas consisting of atoms of a single element:

$$N_eN_{n+1}/N_n = (2g_{n+1}/g_n)(2\pi m_ekT/h^2)^{3/2}\exp(-I_{n+1}/kT),$$

$$N = \Sigma N_n, \qquad N_e = \Sigma nN_n; \qquad n = 0,\ 1,\ 2 \ldots Z. \quad (1)$$

Here N, N_n, N_e are the numbers of the original atoms, n-multiply ionized atoms and electrons per cm^3, and I_{n+1} is the (n+1)-th ionization potential. The statistical weight ratio g_{n+1}/g_n for the electronic states of the ions depends on the