POLARIZATION TENSORS IN THE BORN APPROXIMATION

L. D. PUZIKOV and Ya. A. SMORODINSKIĬ

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It has been repeatedly remarked by various writers that the polarization of elastically-scattered nucleons, calculated in first Born approximation, is zero. It is also of interest to examine in the Born approximation the polarization of particles with higher spins.

The formal reason for the appearance of new selection rules for the polarized states is the Hermitian nature of the scattering amplitude in the Born approximation. If, in the unitarity relation for elastic scattering (cf. reference 1)

$$M(\mathbf{n}_2, \mathbf{n}_1) - M^+(\mathbf{n}_1, \mathbf{n}_2) = \frac{ik}{2\pi}\int M^+(\mathbf{n}_2, \mathbf{n})\, M(\mathbf{n}_1, \mathbf{n})\, d\mathbf{n}$$

we neglect the right member, as a quantity quadratic in the scattering amplitude, we get

$$M(\mathbf{n}_2, \mathbf{n}_1) = M^+(\mathbf{n}_1, \mathbf{n}_2). \qquad (1)$$

If the scattering is described by a (Hermitian) Hamiltonian, this relation also follows from the fact that the scattering amplitude is the matrix element of the Hamiltonian calculated with plane waves.

Similarly, when besides elastic scattering there is also possible the reaction

$$a + a' \to b + b', \qquad (2)$$

the unitarity relations for the scattering and reaction amplitudes given in reference 3, after neglect of terms quadratic in the amplitudes, provide the relations

$$\begin{aligned} M_{aa}(\mathbf{n}_2, \mathbf{n}_1) &= M_{aa}^+(\mathbf{n}_1, \mathbf{n}_2), \\ k_a M_{ba}(\mathbf{n}_2, \mathbf{n}_1) &= k_b M_{ab}^+(\mathbf{n}_1, \mathbf{n}_2), \\ M_{bb}(\mathbf{n}_2, \mathbf{n}_1) &= M_{bb}^+(\mathbf{n}_1, \mathbf{n}_2). \end{aligned} \qquad (1a)$$

Here M_{aa} and M_{bb} are the amplitudes for the elastic scatterings $a + a' \to a + a'$ and $b + b' \to b + b'$, respectively; M_{ba} is the amplitude for the reaction (2), and M_{ab} is the amplitude for the inverse reaction.

Following references 2 and 3, we shall characterize the spin states of the particles before and after the reaction by the polarization tensors $\rho^{(0)}_{JMJ'M'}$ (before the reaction) and $\rho_{KNK'N'}$ (after the reaction). The values of JM and J′M′ show the rank and projection component of the polarization tensors for particles a and a′, respectively; the values of KN and K′N′ show the rank and projection component of the polarization tensors for particles b and b′. As has been shown in the papers referred to,

$$\rho_{KNK'N'} = \frac{1}{\sigma_{ba}} \sum_{JMJ'M'} K^{JMJ'M'}_{KNK'N'}(\mathbf{n}_2, \mathbf{n}_1)\, \rho^{(0)}_{JMJ'M'},$$

where

$$\sigma_{ba} = \mathrm{Sp}\,\{M_{ba}\,\rho^{(0)} M_{ba}^+\};$$

$$K^{JMJ'M'}_{KNK'N'}(\mathbf{n}_2, \mathbf{n}_1) = \mathrm{Sp}\,\{M_{ba}(\mathbf{n}_2, \mathbf{n}_1)\theta_{JM}\theta_{J'M'} \times M_{ba}^+(\mathbf{n}_2, \mathbf{n}_1)\, T^+_{KN} T^+_{K'N'}\},$$

and also there follows from time reversibility the following relation for the coefficients K:

$$k_a^2 K^{JMJ'M'}_{KNK'N'}(\mathbf{n}_2, \mathbf{n}_1) = (-1)^{J+M+J'+M'+K+N+K'+N'} k_b^2 K^{K-N\,K'-N'}_{J-M\,J'-M'}(\mathbf{n}_1, \mathbf{n}_2). \qquad (3)$$

The operators θ_{JM}, $\theta_{J'M'}$, T_{KN}, $T_{K'N'}$ are irreducible tensor operators in the spin spaces of the respective particles a, a′, b, b′.

From the condition (1a) of the Hermitian nature of the scattering amplitude and the Hermitian properties of the tensor operators ($T^+_{qK} = (-1)^K T_{q-K}$) it is not hard to get another relation for the coefficients K:

$$k_a^2 K^{JMJ'M'}_{KNK'N'}(\mathbf{n}_2, \mathbf{n}_1) = (-1)^{M+M'+N+N'} k_b^2 K^{K-N\,K'-N'}_{J-M\,J'-M'}(\mathbf{n}_1, \mathbf{n}_2). \qquad (4)$$

Comparing the relations (3) and (4) we see that $K^{JMJ'M'}_{KNK'N'} = 0$ if the sum $J + J' + K + K'$ is odd.

We have obtained the selection rule for the polarized states in the reaction (2). It is not hard to get from the first and third of the relations (1a) a similar selection rule for the elastic channels of the reaction.

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