INTERACTION BETWEEN 660-Mev PROTONS AND ATOMIC NUCLEI AND THE NUCLEAR INTERNAL MOMENTUM DISTRIBUTION OF NUCLEONS

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Submitted to JETP December 20, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1631-1649 (June, 1959)

We investigated the angular distributions and (by magnetic analysis) the energy spectra of secondary particles (mainly protons with energies $\gtrsim$ 60 Mev) emitted at angles of 7, 12.2, 18, 24, and 30° in reactions between 660-Mev protons and the nuclei of Be, C, Cu, and U. The differential cross sections for the emission of such secondary charged particles increase with decreasing angle. In order of decreasing energy, the various spectral regions of all the investigated elements correspond respectively to diffractional scattering of protons on nuclei (in the small-angle region), single quasi-elastic nucleon collisions, π-meson production on bound nucleons, and intranuclear cascade.

The experimental energy spectra for single quasi-elastic proton-nucleus scattering are compared with the spectra computed in the impulse approximation under various assumptions regarding the momentum distributions of the nucleons in the nuclei. The Be and C data are consistent with a Gaussian nucleon-momentum distribution with a 1/e value at an energy of approximately 20 Mev.

1. INTRODUCTION

INFORMATION on the momentum distributions of nucleons in the ground states of nuclei are gathered from the energy and angular distributions of the products of nuclear reactions induced by high-energy particles. Chew and Goldberger,[1] in their analysis of the data of Hadley and York[2] on the capture of protons from carbon nuclei by 90-Mev neutrons, have shown that the energy and angle distributions of the deuterons correspond to a momentum distribution of the form $\alpha/\pi(\alpha^2 + s^2)^2$ with $\alpha^2/2M = 18$ Mev, rather than a distribution corresponding to a model of a Fermi gas made up of non-interacting particles. Selove[3] found, in the case of neutron capture from carbon nuclei by 95 Mev protons, that the experimental data can be satisfactorily explained by approximating the internuclear momentum distribution of the neutrons by means of a sum of two Gaussian distributions of the form $\exp\{-s^2/s_1^2\} + 0.15 \exp\{-s^2/s_2^2\}$, where $s^2/2M$ is the energy of the bound neutron, $s_1^2/2M = 7$ and $s_2^2/2M = 50$ Mev. It is essential to assume the presence of a considerable admixture of high-momentum components in the wave function of the ground states of a light nucleus in order to explain qualitatively the direct knock-out of deuterons from nuclei of lithium, beryllium, carbon, and oxygen by 670 Mev protons,[4] as well as to explain the strong smearing on the high-energy side of the spectra of positive and negative pions produced in p + C collisions at the same energy.[5] According to Henley,[6] a Gaussian momentum distribution, corresponding to a mean kinetic energy of 19.3 Mev, agrees best of all with such characteristics of pion production in p + C collisions at 340 Mev (reference 7), as the threshold value of the proton energy, the excitation curve, and the pion energy and angular distributions.

The character of the momentum distribution of the nucleons in the ground state of the nuclei manifests itself most directly in quasi-elastic scattering of fast protons in light nuclei. This process was investigated experimentally at 340 Mev by Cladis, Hess, and Moyer,[8] who found that, within the framework of the impulse approximation, the energy spectra of the protons that are scattered quasi-elastically in carbon and oxygen nuclei satisfy a Gaussian nucleon-momentum distribution, with a value of 1/e at an energy of 16 ± 3 Mev. Subsequently Wilcox and Moyer,[9] by measuring the energy spectrum of the protons in quasi-elastic p-p collisions at 340 Mev, registered by coincidence from both scattered protons, have shown that the Gaussian distribution of the proton momenta inside the beryllium nuclei, with a value 1/e near

20 Mev, gives an acceptable agreement with experiment. Observations of the angular correlations in quasi-elastic p-p collisions at 925 Mev, principally in light nuclei contained in photoemulsions, have led McEwen, Dixon, and Duke[10] to the conclusion that the proton momentum distribution can be approximated by a Gaussian distribution with $1/e$ at 11 ± 3 Mev, but somewhat better agreement with experiment is obtained by the sum of two Gaussian distributions of the form $\exp\{-s^2/s_1^2\} + 0.05 \exp\{-s^2/s_2^2\}$, with $s_1^2/2M = 7$ and $s_2^2/2M = 40$ Mev. Finally, it follows from experiments on the nuclear photoeffect that a Gaussian distribution with $1/e$ at 19 Mev is a good approximation of the momentum distribution of the quasideuteron groups in carbon and oxygen nuclei.[11]

We can thus conclude from the aggregate of data on high energy nuclear reactions of various types that, unlike the Fermi model with its characteristic sharp upper boundary of nucleon momentum distribution, models that use distributions that smear out towards the higher momenta yield acceptable agreement with experiment.

It is the purpose of this article to discuss the principal results of research on the interaction between 660-Mev protons and nuclei of beryllium, carbon, copper, and uranium. The experiments consisted above all of measuring, by the magnetic deflection method, the energy spectra of secondary particles, particularly quasi-elastically scattered protons. Since the condition of applicability of the impulse approximation should be better satisfied with increasing energy of the incident proton, one would expect that the regions of the spectra measured at large angles, corresponding to the higher energies, would contain essentially protons emitted as the result of paired proton-nucleon collisions in the nuclei. The spectra obtained for quasi-elastically scattered protons were compared with the spectra which were calculated with allowance for relativistic kinematics and which satisfied various assumptions regarding the nucleon momentum distributions in the nuclei. In addition, we measured the angular distributions of the secondary charged particles emitted in the investigated collisions.

The experiments were performed with the six-meter synchrocyclotron of the Joint Institute for Nuclear Research.

2. CALCULATION OF THE MOMENTUM DISTRIBUTION IN QUASI-ELASTIC PROTON-NUCLEON SCATTERING

In the case of light nuclei we can assume that a fast incident nucleon is scattered once by an individual nucleon of the nucleus. If the time of collision between two nucleons is much less than the "time of reversal" of the particles in the nucleus, we can assume, in the spirit of the impulse approximation, that the state of the nucleus does not change noticeably during the collision. Furthermore, the role of interaction between the struck nucleon and the remaining nucleons of the nucleus reduces to the "creation" of a momentum distribution at the nucleon that participates in the collision. Therefore collisions between the incident nucleon and the nucleons of the nucleus can be considered as collisions with free particles, the only difference being that the particles of the nucleus move during the time of collision. The amplitude of such "quasi-elastic" collisions with the nucleus is obtained by summing the amplitudes of scattering by the individual nucleons of the nucleus. In the range of large angles, where interference of the waves scattered by the individual nucleons can be neglected, the problem of scattering by the nucleus reduces essentially to the problem of scattering by a classical ensemble of independent particles that have a specified momentum distribution. The distribution function, in turn, is determined by the square of the modulus of the Fourier component of the wave function of the nucleus. We shall consider below the energy distribution of the nucleons, scattered "quasi-elastically" at a specified angle, in the indicated approximation, with allowance for the relativistic kinematics.

The differential cross section $d\sigma_c = \sigma_c(E_c, \cos\vartheta_c)\, d\omega_c$ of elastic collision of two free nucleons in their center of mass system (c.m.s.), and the distribution function $f(\mathbf{s})(d\mathbf{s})$ of the intranuclear particles by momentum, will be assumed known. The momenta of the incident and scattered nucleons will be denoted in the laboratory system by $\mathbf{p}$ and $\mathbf{q}$, while $\mathbf{s}$ will denote the momentum of the intranuclear nucleon. The energy of a particle with momentum $\mathbf{p}$ will be denoted E_p, etc. The corresponding quantities in the c.m.s. will be identified by a subscript "c", while 4-dimensional momenta will be denoted by $\hat{p}$, $\hat{q}$, etc.

To find the number of collisions that lead to a specified momentum $\mathbf{q}$ (more accurately, to momenta concentrated in the element $(d\mathbf{q})$ about the momentum $\mathbf{q}$), we assume that the incident nucleons, like the target nucleons, are distributed in accordance with a certain law. We introduce the distribution functions $f(\mathbf{r}, \mathbf{p}, t)(d\mathbf{r})(d\mathbf{p})$ and $f_n(\mathbf{r}, \mathbf{s}, t)(d\mathbf{r})(d\mathbf{s})$, which determine the probable numbers of the incident particles and the particles of the nucleus within the volume elements $(d\mathbf{r})(d\mathbf{p})$ and $(d\mathbf{r})(d\mathbf{s})$ in phase space. The functions f and f_n are scalars under the Lorentz transformation.[12] Let us consider collisions of particles with momenta **p** and **s**. Changing over to the system where the particles with momentum **p** are at rest, the number of collisions per unit volume per unit time in this system, i.e., per unit 4-dimensional volume, can be written

$$dN = f^0 f_n^0 (d\mathbf{p}^0)(d\mathbf{s}^0)\, d\sigma v, \tag{1}$$

where $d\sigma$ is the invariant differential cross section[13] and v is the relative velocity. The superscript 0 denotes that the corresponding quantities are taken in the system where the incident particles are at rest. Since v and $(d\mathbf{p}^0)(d\mathbf{s}^0)$ are invariant, we express them in terms of the velocities*

$$v = [|\mathbf{v}_p - \mathbf{v}_s|^2 - [\mathbf{v}_p \times \mathbf{v}_s]^2 / c^2]^{1/2} [1 - \mathbf{v}_p \mathbf{v}_s / c^2]^{-1},$$

$$(d\mathbf{p}^0)(d\mathbf{s}^0) = (d\mathbf{p})(d\mathbf{s})(1 - \mathbf{v}_p \mathbf{v}_s / c^2), \tag{2}$$

where $\mathbf{v}_s = c^2\mathbf{s}/E_s$ and $\mathbf{v}_p = c^2\mathbf{p}/E_p$, and integrate over $(d\mathbf{p})$ and $(d\mathbf{s})$ to obtain an expression for the total number of collisions per unit 4-dimensional volume, required to obtain the specified result:

$$N = \int (d\mathbf{p})(d\mathbf{s}) f(\mathbf{r}, \mathbf{p}, t) f_n(\mathbf{r}, \mathbf{s}, t) \times [|\mathbf{v}_p - \mathbf{v}_s|^2 - [\mathbf{v}_p \times \mathbf{v}_s]^2 / c^2]^{1/2} d\sigma. \tag{3}$$

If the incident nucleons form a linear monochromatic beam, then $f(\mathbf{r}, \mathbf{p}, t)$ should be chosen in the form

$$f(\mathbf{r}, \mathbf{p}, t) = \frac{\rho^0}{\sqrt{1 - \beta_p^2}} \delta(\mathbf{p} - \mathbf{p}_0),$$

where ρ^0 is an invariant that represents the density of the incident particle in the rest system. Then†

$$N = \frac{\rho^0}{\sqrt{1 - \beta_p^2}} \times \int (d\mathbf{s}) f_n(\mathbf{r}, \mathbf{s}, t) [|\mathbf{v}_p - \mathbf{v}_s|^2 - [\mathbf{v}_p \times \mathbf{v}_s]^2 / c^2]^{1/2} d\sigma. \tag{4}$$

The scattering cross section $d\sigma(\mathbf{q}_c)$ in the specified momentum interval $(d\mathbf{q}_c)$ in the c.m.s. can be written

$$d\sigma(\mathbf{q}_c) = \sigma_c \frac{\delta(q_c - p_c)}{p_c^2} (d\mathbf{q}_c) = \sigma_c \left| \frac{\partial E_c'}{\partial q_c} \right| \frac{\delta(E_c' - E_c)}{p_c^2} (d\mathbf{q}_c), \tag{5}$$

where E_c and E_c' are the total energies in the c.m.s. before and after the collision, while $\sigma_c = d\sigma_c/d\omega$. The law of momentum conservation is assumed to hold in (5). Making use of the fact that if the masses of the colliding particles are equal we have $\partial E_c'/\partial q_c = 2c^2 q_c / E_{cq}$, and also making use of the invariance of $(d\mathbf{q}_c)/E_{cq} = (d\mathbf{q})/E_q$ and of the fact that $d\sigma(\mathbf{q}_c) = d\sigma(\mathbf{q})$, we obtain the scattering cross section in the specified momentum interval $(d\mathbf{q})$ in the laboratory system

$$d\sigma(\mathbf{q}) = \sigma_c \frac{2c^2}{p_c} \delta(E_c' - E_c) \frac{(d\mathbf{q})}{E_q}, \tag{6}$$

where the quantities pertaining to the center of mass system are assumed to be expressed in terms of the corresponding quantities in the laboratory system.

Inserting (6) into (5), putting $f_n = \rho_n f(\mathbf{s})$ and dividing N by $\rho^0 \rho_n^0 v_p$, we obtain the effective cross section for scattering on one of the nucleons of the nucleus.

$$d\sigma_{\text{eff}} = \frac{2Mc^2}{p} \frac{(d\mathbf{q})}{E_q} \int (d\mathbf{s}) f(\mathbf{s}) [|\mathbf{v}_p - \mathbf{v}_s|^2 - [\mathbf{v}_p \times \mathbf{v}_s]^2 / c^2]^{1/2} \sigma_c \times \delta(E_c' - E_c) / p_c \sqrt{1 - \beta_p^2} = N / \rho^0 \rho_n^0 v_p. \tag{7}$$

In calculating the integral contained in (7), it is convenient to introduce in s-space a polar system of coordinates (s, θ, Φ) with a polar axis in the direction of momentum transfer $\boldsymbol{\kappa} = \mathbf{q} - \mathbf{p}$. Let us first transform a δ-function. Since $E_c = 2E_{cp} = 2(E_p - \mathbf{v} \cdot \mathbf{p})(1 - v^2/c^2)^{-1/2}$, where **v** is the velocity of the center of mass of the colliding nucleons (and analogously for E_c'), we get

$$\delta(E_c' - E_c) = \tfrac{1}{2}\sqrt{1 - v^2/c^2}\, \delta(E_q - \mathbf{v}\mathbf{q} - E_p + \mathbf{v}\mathbf{p}) = (1/2c)\, \delta(\hat{\kappa}\hat{P}) \sqrt{\hat{P}^2}, \tag{8}$$

where $\hat{P}$ and $\hat{\kappa}$ are the 4-vectors of total momentum and momentum transfer, and the scalar product of the two vectors $\hat{a}$ and $\hat{b}$ is defined as $\hat{a}\hat{b} = -(a_4 b_4 + \mathbf{ab}) = a_0 b_0 - \mathbf{ab}$. When integrating over the polar angle θ, we make use of the well-known formula

$$\delta(f(\theta)) = \sum_{\theta_0} \delta(\theta - \theta_0) / |\partial f(\theta)/\partial\theta|_{\theta = \theta_0},$$

where the summation is carried out over all roots of equation $f(\theta_0) = 0$. In this case the role of $f(\theta)$ is played by the function $\hat{\kappa}\hat{P} = (E_q - E_p) \times (E_s + E_p)/c^2 - (\mathbf{p} + \mathbf{s})\boldsymbol{\kappa}$. Setting this equal to zero, we find

$$\cos\theta_0 = [(1/c^2)(E_q - E_p)(E_s + E_p) - \mathbf{p}\boldsymbol{\kappa}] / s|\boldsymbol{\kappa}|. \tag{9}$$

The condition $|\cos\theta_0| \le 1$ leads, in conjunction

*The connection between $(d\mathbf{s}^0)(d\mathbf{p}^0)$ and $(d\mathbf{p})(d\mathbf{s})$ follows from the fact that $(d\mathbf{p})$ can be considered as the fourth component of the infinitesimal 4-vector $\mathcal{E} = ((d\mathbf{p})\mathbf{v}_p, ic(d\mathbf{p}))$.

†A formula analogous to (4) was obtained earlier by Chernikov.[14]

with (9), to the result that the permissible values of s are bounded from below by a minimum value $s = s_0$:

$$s_0 = \frac{1}{\varkappa^2 - (E_q - E_p)^2/c^2}\left|\varkappa\left\{\frac{E_p(E_q-E_p)}{c^2} - \varkappa \mathbf{p}\right\} + \frac{E_q - E_p}{c}\left\{\left[\frac{E_p(E_q-E_p)}{c^2} - \mathbf{p}\varkappa\right]^2 - \left[\frac{(E_q-E_p)^2}{c^2} - \varkappa^2\right]M^2c^2\right\}^{1/2}\right|. \quad (10)$$

The square root within the absolute sign is always real, since $\hat{\kappa}$ is either a space-like or a null vector. The derivative $|\partial f(\theta)/\partial\theta|_{\theta=\theta_0}$ has a value $|\partial(\hat{\kappa}\hat{P})/\partial\theta|_{\theta=\theta_0} = \kappa s \sin\theta_0$. Thus

$$\delta(E_c' - E_c) = \frac{1}{2}\frac{\sqrt{\hat{P}^2}}{c}\frac{\delta(\theta-\theta_0)}{\varkappa s \sin\theta_0}. \quad (11)$$

Inserting (11) into (7) and considering that

$$p_c = \sqrt{\frac{(\hat{P}\hat{p})^2 - \hat{P}^2\hat{p}^2}{\hat{P}^2}} = \frac{1}{\sqrt{\hat{P}^2}}\frac{(Mc)^2}{\sqrt{(1-\beta_p^2)(1-\beta_s^2)}} \times \frac{1}{c}\left[|\mathbf{v}_p - \mathbf{v}_s|^2 - [\mathbf{v}_p \times \mathbf{v}_s]^2/c^2\right]^{1/2} \quad (12)$$

(this equality can be readily obtained by using the general form of the Lorentz transformation and expressing p_c in terms of the laboratory quantities), we obtain after integrating over θ

$$d\sigma_{\text{eff}} = \frac{c^2}{p\varkappa}\frac{(d\mathbf{q})}{E_q}\int\limits_{s_0}^{\infty}\int\limits_0^{2\pi} d\Phi \frac{s\,ds f(s)}{E_s}\sigma_c(E_c, \cos\vartheta_c)(\hat{P})^2. \quad (13)$$

A summation of all the nucleons of the nucleus yields

$$\frac{d^2\sigma}{d\omega\,dq} = \frac{1}{p|\mathbf{q}-\mathbf{p}|}\frac{q^2}{\sqrt{c^2q^2 + M^2c^4}} \times \sum_{i=1}^{A}\int\limits_{s_0(q)}^{\infty}\int\limits_0^{2\pi} d\Phi \frac{s\,ds f(s)}{\sqrt{c^2s^2 + M^2c^4}} E_c^2\sigma_c^i(E_c, \vartheta_c). \quad (14)$$

The quantities E_c and $\cos\vartheta_c$ are readily expressed in terms of the laboratory system and equal, respectively

$$E_c = c\sqrt{\hat{P}^2} = c\left[2\left(M^2c^2 + E_pE_s/c^2 - \mathbf{ps}\right)\right]^{1/2},$$

$$\cos\vartheta_c = 1 + \frac{E_s(E_q - E_p)/c^2 - (\varkappa \mathbf{s}) - (E_pE_q/c^2 - \mathbf{pq}) + M^2c^2}{E_sE_p/c^2 - \mathbf{sp} - M^2c^2} = 1 + \frac{2[M^2c^2 - (E_pE_q/c^2) + \mathbf{pq}]}{E_c^2/2c^2 - 2M^2c^2}, \quad (15)$$

where

$$\varkappa\mathbf{s} = \varkappa s\cos\theta_0, \quad \mathbf{pq} = pq\cos\vartheta, \quad \mathbf{p}\varkappa = p\varkappa\cos\theta_1,$$

$$\cos(\widehat{\mathbf{ps}}) = \cos\theta_0\cos\theta_1 + \sin\theta_0\sin\theta_1\cos\Phi.$$

In the nonrelativistic limit

$$s_0 \to q(q - p\cos\vartheta)/|\mathbf{q} - \mathbf{p}|.$$

Assuming that σ_c depends only on the momentum transfer, we obtain an equation similar to that previously derived by Wolff[15]

$$\frac{d^2\sigma}{d\omega\,dq} = \frac{8\pi q^2}{p|\mathbf{q}-\mathbf{p}|}\sum_{i=1}^{A}\sigma_c^i(\mathbf{q}-\mathbf{p})\int\limits_{s_0}^{\infty} sf(s)\,ds. \quad (16)$$

3. THE EXPERIMENT

The energy spectra of the secondary particles emitted in p + Be, p + C, p + Cu, and p + U collisions, were measured in the approximate energy interval from 100 to 700 Mev, at angles of 7, 12.2, 18, 24, and 30°. The magnetic analyzer used was the same as employed in a previous investigation of momentum spectra of the products of the $pp \to np\pi^+$, $pp \to pp\pi^0$, and $pp \to d\pi^+$ reactions (reference 16) and in experiments on the direct knock-out of deuterons from light nuclei by 670-Mev protons.[4] The papers referred to contain detailed information on the analyzer, its placement relative to the primary beam of the protons, the concrete shield of the synchrocyclotron, and the procedure used to process the measurement data and the corrections introduced. In the present experiment, the changeover from one angle of observation to another was made by moving the target R (using the notation of Fig. 1 in reference 16) along the primary proton beam, which passed near the edge of the pole pieces of the analyzer magnet. In all cases, the particles entered and left at right angles to the boundary of

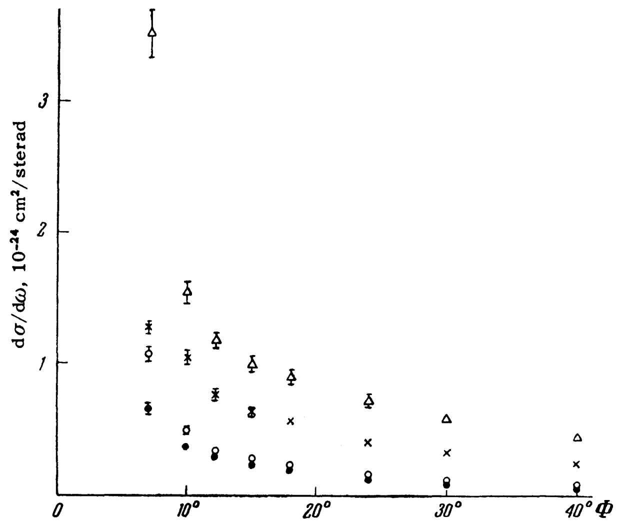

FIG. 1. Angular distribution of secondary charged particles with range > 3.1 g/cm² of tolane, emitted in collisions of 660-Mev protons with Be, C, Cu, and U. The experimental points correspond to: • – beryllium ο – carbon, x – copper, Δ – uranium.

TABLE I

Angle of observation Φ, degrees	7	12.2	18	24	30
Distance from target to the edge of the pole piece, cm	410	218	138	94	64
Deflection angle, degrees	18.3	24	29.5	19.5	25.7
Radius of curvature ρ of the central trajectory, cm	360	284.8	222	350	265
Angular divergence of secondary beam in the horizontal plane, degrees	0.10	0.14	0.17	0.20	0.22
Calculated resolution ΔHρ/Hρ, %	1.9	1.5	1.2	2.6	1.9

the magnetic field. The data of Table I show how changes in the angle of observation affected the distance between the target and the forward edge of the pole, the angle of deflection of the particles in the magnetic field, the radius of curvature of the central trajectory, the angular divergence of the separated beam of secondary particles in the horizontal plane, and the resolution $\Delta H\rho/H\rho$ of the instrument, calculated with allowance for geometrical factors only ($\Delta H\rho$ is the width of the peak at half height).

The energy of the primary protons directly in front of the target was 661 Mev; the mean squared spread in the beam proton energies amounted to approximately 1%. The particles at the analyzer exit were detected by a telescope consisting of four scintillation counters (total thickness 3.1 g/cm^2 of tolane) connected for coincidence. The threshold of proton registration of the telescope was 60 Mev. The registration efficiency for particles in the investigated range of energies was practically 100%. The entire path of the particles in the analyzer, all the way to the telescope, was in vacuum. Targets 2.5 cm high and 1 cm wide were made of chemically pure beryllium, carbon, copper, and uranium and their thicknesses were 3.26, 1.09, 3.45 and 7.62 g/cm^2 respectively.

To find the absolute values of the differential cross sections in the spectrum, $d^2\sigma/d\omega\,dE$, we measured the differential cross sections $d\sigma/d\omega$ for the emission of secondary charged particles in the investigated collisions, within the angle interval from 7 to 40°. In these measurements, made at angles of 24, 30, and 40° with an angular resolution of ±0.8° (the resolution was ±0.4° at smaller angles), the threshold for registration was the same as in the spectrum measurements. From the observed yield of charged particles we calculated the contribution of charged pions, estimated from data on the production of $\pi^{\pm}$ mesons in p + C collisions at 670 Mev,[5] under the assumption that the cross section of the latter process increases as $A^{2/3}$. The contribution of charged pions to the total yield of secondary charged particles did not exceed 7%. The area under the curves of the energy spectra of the secondary particles was normalized to the corresponding experimental values of $d\sigma/d\omega$. Furthermore, the spectra were linearly extrapolated from the last experimental point to the threshold value of the proton-registration energy. The accuracy of this method of estimating $d^2\sigma/d\omega\,dE$ amounts to approximately 20%.

The experimentally-obtained spectrum $F(E)$ is related to the true spectrum $G(E)$ by the following integral equation

$$F(E) = \int_0^\infty G(E')\,K(E, E')\,dE', \tag{17}$$

where $K(E, E')$ is the experimental curve of the resolution of the instrument. The function $K(E, E')$ was determined, for each angle of observation, by measuring the form of the peak corresponding to protons from elastic p-p scattering. The peaks were observed under the same conditions as the observations of $F(E)$. The curves obtained for the resolving power fitted best the function

$$K(E, E') = \frac{1}{2a}\exp\{-|E - E'|/a\}, \tag{18}$$

where E is the energy at the center of the peak of elastic p-p scattering. The solution of Eq. (17) can be written, with sufficient accuracy, in the form

$$G(E) \approx F(E) - a^2 d^2F(E)/dE^2, \tag{19}$$

since $E \gg a$ and since the values of a corresponding to the given angle of observation remain essentially constant over the section of the spectrum occupied by the maximum of the quasi-elastic scattering. Table II gives the values of E, a and $\Delta E/E$, corresponding to the angles under

TABLE II

Angle of observation Φ, degrees	7	12.2	18	24	30
E, Mev	647	621	578	521	455
a, Mev	14.4	9.3	6.3	15.0	10.3
$\Delta E/E$, %	4.7	2.9	1.6	4.4	4.0

Φ, degrees	dσ/dω, 10^{-24} cm²/sterad			
	Be	C	Cu	U
7	0.660±0,033	1.100±0.055	1.168±0.058	3.650±0.182
10	0,376±0.019	0.492±0.015	1.052±0.053	1,554±0.077
12.2	0,298±0,015	0;336±0,017	0,765±0.039	1.180±0,059
15	0,227±0.011	0.276±0.014	0.642±0.032	0,998±0.050
18	0.195±0.010	0.230±0,011	0.556±0.028	0,900±0,045
24	0.125±0,006	0.158±0.008	0,400±0.020	0,716±0.036
30	0,089±0.004	0.114±0,006	0.335±0.017	0.592±0.030
40	0,054±0.003	0,074±0.004	0,228±0.011	0,443±0.022

consideration; here ΔE is the total width at half the height of the peak of elastic p-p scattering. It should be noted that in the region of the maximum of quasi-elastic scattering, measured at an angle of 30°, the magnitude of the correction for the instrumental spectrum errors, $a^2b^2F(E)/dE^2$, amounted to no more than 2 or 3%.

4. EXPERIMENTAL RESULTS

A. Data on angular distribution. The per-nucleus values of the differential cross sections for the emission of secondary charged particles, registered under the condition of the present experiments, are listed in Table III as functions of the emission angle Φ in the laboratory system, and are also shown in Fig. 1. The total errors, comprising statistical errors in the measured number of secondary particles and errors in the determined intensity of the primary proton beam, are also indicated.

In the range from 40° to 18°, where diffraction scattering is negligibly small (see below, Figs. 3 – 7), the values of $d\sigma/d\omega$ increase smoothly with diminishing emission angle for all the investigated nuclei. The dependence of $d\sigma/d\omega$ on A can be approximated here by a function such as kA^r. The results of such a treatment of the experimental data is shown in Fig. 2. Over the interval from beryllium to copper, the values of $d\sigma/d\omega$ increase approximately in the following manner (in units of 10^{-26} cm²/sterad):

$$(d\sigma/d\omega)_{40°} \approx 1.20\,A^{0.71}, \qquad (d\sigma/d\omega)_{30°} \approx 2.12A^{0.67},$$

$$(d\sigma/d\omega)_{24°} \approx 3.62\,A^{0.58}, \qquad (d\sigma/d\omega)_{18°} \approx 6.12A^{0.53}.$$

These results indicate that as the angle decreases, $d\sigma/d\omega$ becomes less dependent on A. Judging from the data obtained for uranium, the dependence of $d\sigma/d\omega$ on A becomes weaker toward the end of the periodic table. Naturally, it is impossible to use these results to estimate the differential cross sections of the investigated nuclear interactions and to establish their dependence on A, since the average number of secondary particles emitted in each collision

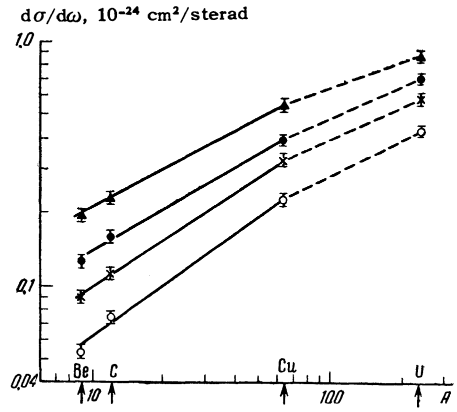

FIG. 2. Differential cross sections for the emission of secondary charged particles with range greater than 3.1 g/cm² of tolane, produced by 660-Mev incident protons, as a function of the mass number. The experimental points correspond to the following angles (laboratory system): ▲ – 18°, ● – 24°, x – 30°, o – 40°.

and registered under the conditions of the present experiments is still unknown.

At angles smaller than 18° the differential cross section changes rapidly, and in a manner that is not the same for all investigated nuclei, owing to the considerable contribution of diffraction scattering to the yield of secondary particles.

B. Energy spectra. By measuring the number of secondary particles at the output of the spectrometer as a function of the intensity of the magnetic field we were able, after introducing the necessary corrections and after normalizing the results to unit monitor count, to plot the relative momentum of the secondary particles at equal intervals of $H\rho$. The transformation from the momentum spectrum to the energy spectrum is made under the assumption that the secondary particles are protons. This assumption calls for a further check in that region of the spectrum which corresponds to the cascade protons, since these still contain a certain relatively small admixture of deuterons, emitted when the secondary nucleons pick up nucleons from the surface of the nucleus.[17] Apparently such an indirect pick up process can also lead to the emission of a small number of fast tritium nuclei. According to the

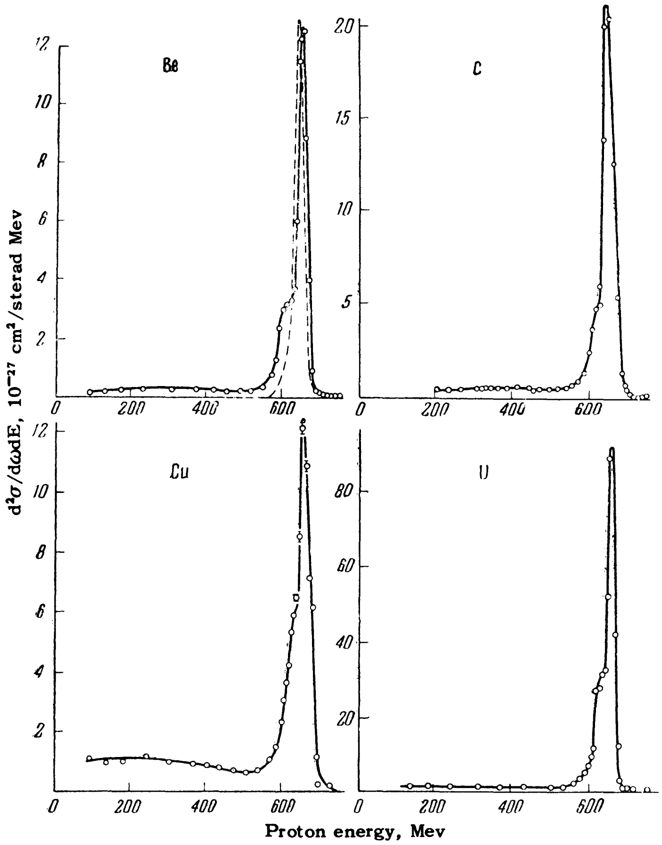

FIG. 3. Energy spectra of secondary charged particles from p + Be, p + C, p + Cu, and p + U collisions at 660 Mev. The dotted lines indicate the resolution curves. The angle of observation is 7°.

data of Hess and Moyer,[18] in the bombardment of carbon, copper, and uranium by 300-Mev protons the yields of the secondary protons, deuterons, and tritium nuclei at angles of 26, 40, and 60°, are related approximately as 1000 : 50 : 2, and the energy spectrum of the deuterons, diminishing with energy approximately as E^{-3}, extends almost to 150 Mev. Bearing in mind the relative insensitivity of the intranuclear cascade process to the energy of the incident nucleon, it can be assumed that as the energy is increased from 300 to 670 Mev, the ratio between the yields of the secondary protons, deuterons, and tritium nuclei remains essentially unchanged.

The secondary charged particles observed at small angles may include also deuterons from the reaction $pp \to d\pi^+$ and $pn \to d\pi^0$, possibly taking place on bound nucleons. At 660 Mev the contribution of these reactions to the total cross sections of the free pp and np collisions is respectively ~ 7.5 and ~ 4%. One can expect, however, the yield of deuterons from the nuclei to be greatly reduced in view of the fact that the mean free path of the deuterons in nuclear matter is small compared with dimensions of the nucleus.

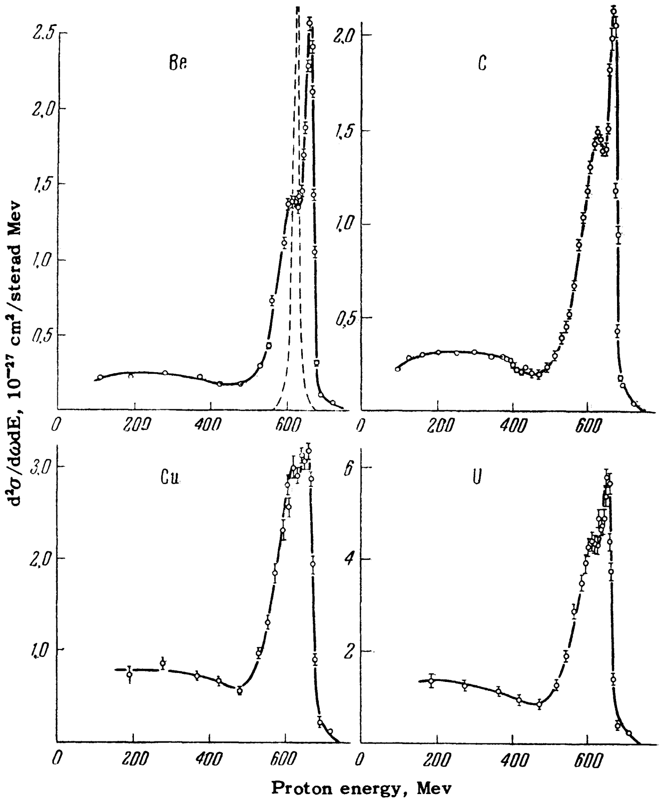

FIG. 4. The same as in Fig. 3. Angle of observation 12.2°.

The energy spectra of secondary protons from p + Be, p + C, p + Cu, and p + U collisions are shown in Figs. 3 — 7 in the form of solid curves, drawn to fit the experimental points; the vertical line at each point denotes the statistical measurement errors. The dotted lines in the same figures show the observed peaks of elastic p-p scattering. The height of these peaks, which represent the resolution curves of the spectrometer, bears no relation to the ordinate scale.

In all the nuclei investigated at 7°, the high-energy portions of the spectrum consist of intense peaks, essentially due to protons that are diffraction-scattered by the nuclei. These peaks are superimposed on the right-side slopes of the maxima of the quasi-elastic proton scattering. The shift of the diffraction-scattering peaks towards energies higher than those of the elastic p-p scattering peaks is due to the different energies of the corresponding recoil nuclei. It can be seen that at 7° the yield of secondary protons with energies E > 500 Mev is due essentially to diffraction scattering. A much smaller contribution to the total yield of secondary protons is made by diffraction scattering at 12.2°, particularly in the case of p + Cu collisions. This property of the spectra at 12.2° is compatible with the known values of the nuclear radii. Thus, if we use the nuclear radii as determined from the cross sec-

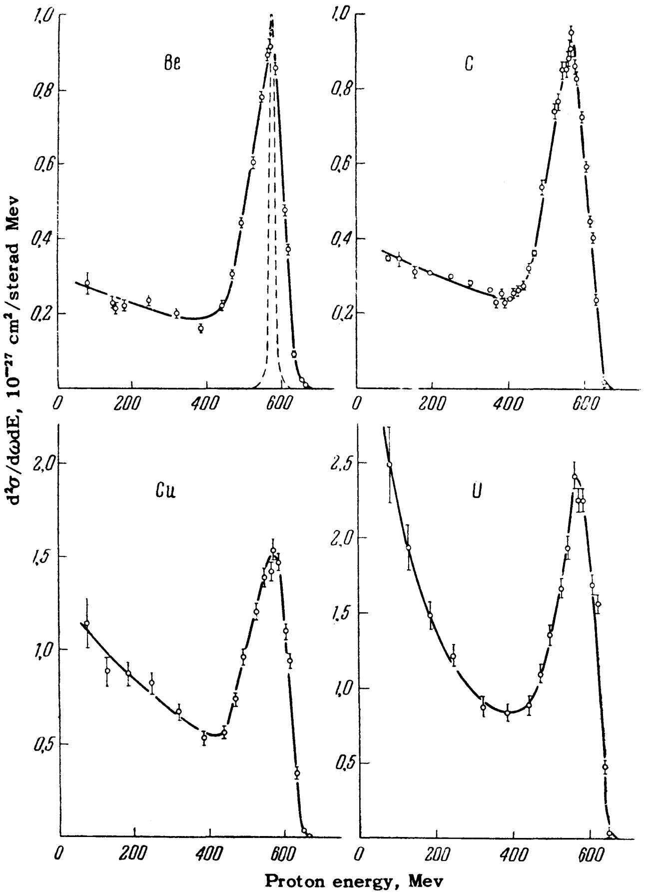

FIG. 5. The same as in Fig. 3. Angle of observation 18°.

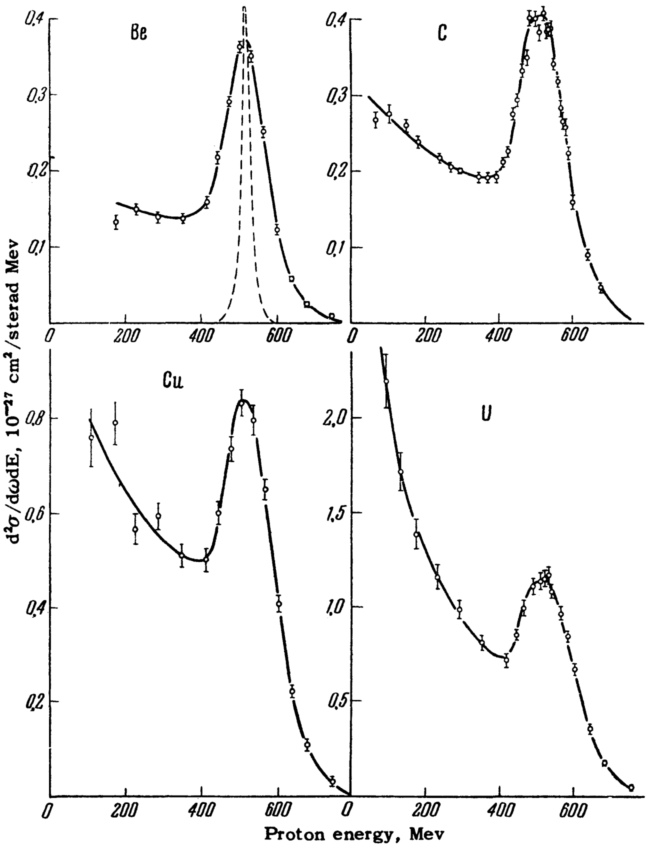

FIG. 6. The same as Fig. 3. Angle of observation 24°.

tions of absorption of 1400-Mev neutrons[19] and 860-Mev[20] and 900-Mev[21] protons, and insert the value $R = 1.26\ A^{1/3} \times 10^{-13}$ cm into the expression for the diffraction-scattering angular distribution

$$d\sigma / d\omega = k^2 (R')^4 \left[J_1\left(2kR' \sin\frac{\theta}{2}\right) \Big/ 2kR' \sin\frac{\theta}{2}\right]^2 \quad (20)$$

(where θ is the scattering angle, $k = 2\pi/\lambda$, λ is the wavelength of the incident proton, R is the radius of the scattering nucleus, $R' = R + 1/k$, and J_1 is the Bessel function of the first kind), we find that near 12.2° there should be a fourth maximum for uranium, while in the case of copper this scattering angle corresponds approximately to the position of the second minimum.

A characteristic feature of the spectra measured at 18° is the absence of typical diffraction peaks and a sharp cutoff of the quasi-elastic scattering maxima near the upper boundary of the energy region that is allowed by the conservation laws. At 24° and 30°, the spectra of all the investigated elements display in the high-energy region pronounced maxima corresponding to single quasi-elastic collisions between the protons and the nucleons in the nuclei. Assuming that the quasi-elastic scattering maxima have a symmetrical form, it is possible to estimate the differential cross section for quasi-elastic scattering by measuring the areas under the curves and comparing their values with the sum $Z d\sigma_{pp}/d\omega$ + $(A - Z) d\sigma_{pn}/d\omega$, where $d\sigma_{pp}/d\omega$ and $d\sigma_{pn}/d\omega$ are the corresponding differential cross sections of elastic pp and pn scattering.* It was found that the quasi-elastic scattering in beryllium, carbon, copper, and uranium at 30° amounts to ~60, ~53, ~22, and ~9% of the free scattering, respectively. The important fact is that the proton, after the first collision with one of the nucleons of the nucleus, can be emitted with a noticeable probability even from the heaviest nuclei, without experiencing any other interaction. This fact can be considered as an indication that such nucleon collisions occur predominantly in the surface layer of the nucleus. In connection with this, it must be noted that in the bombardment of aluminum and uranium by 460- and 1840-Mev protons, the calculations of the intranuclear cascades, carried out by Metropolis et al.,[23] predict,

*In these calculations the values of $d\sigma_{pn}/d\omega$ for 670 Mev were assumed equal to the corresponding values of the differential cross sections of elastic np scattering obtained by Kazarinov and Simonov at 580 Mev.[22]

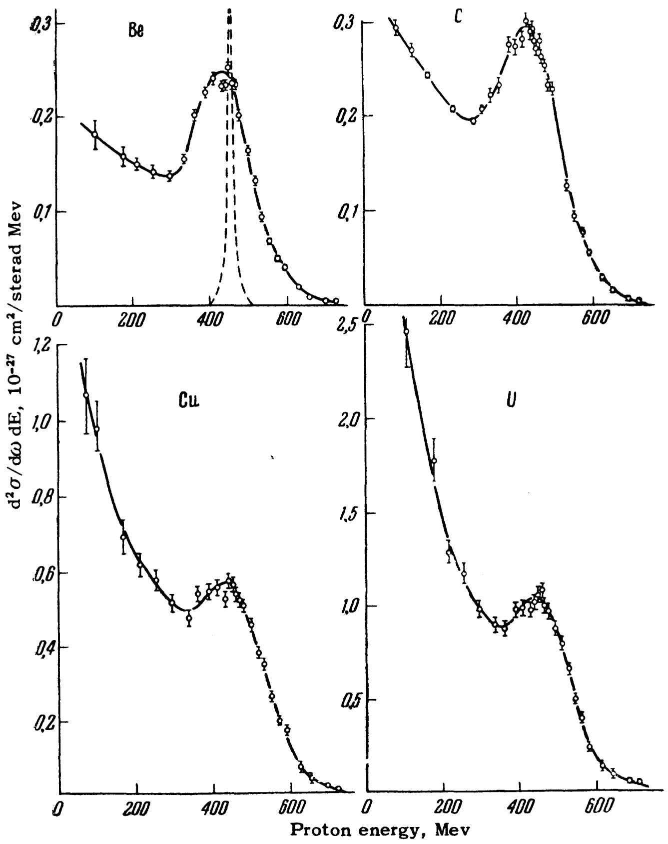

FIG. 7. The same as Fig. 3. Angle of observation 30°.

in the approximation employing a nuclear model with a sharp boundary, a secondary-proton energy spectrum without the maximum due to protons that have experienced only one paired collision. The use of a model of nucleus with diffuse boundary in cascade calculations is expected to increase the relative frequency of proton emission after the first collision, and thus match more accurately the experimental spectrum of the secondary protons.

The strong smearing of the quasi-elastic scattering maxima as compared with the elastic pp-scattering peak is a direct indication of the existence of a momentum distribution of the nucleons in the nuclei. The quasi-elastic scattering maxima are shifted somewhat towards the energies lower than those of the elastic pp scattering peaks. An analogous decrease in the mean proton energy in quasi-elastic scattering was observed also at 340 Mev and was attributed to nucleon collisions in the potential well, to energy losses by excitation of the residual nuclei, and to an increase in the scattering cross sections at relatively small collision energies.[8]

The continuous spectra observed below the quasi-elastic scattering maxima are due to inelastic processes. In the case of p + Be and p + C collisions, the continuous spectra are very similar in form to the spectra of the recoil protons in the reactions $pp \rightarrow np\pi^+$ and $pp \rightarrow pp\pi^0$ (see Figs. 2 and 3 in reference 16). We conclude therefore that when protons of a given energy collide with light nuclei, the continuous spectra of the secondary protons correspond essentially to processes of pion production in single proton-nucleon collisions, and not to an intranuclear cascade process. To the contrary, in the case of p + Cu and p + U collisions the continuous spectra occur essentially as a result of the cascade process. This is proved by the fact that the number of particles in the continuous spectrum increases the more steeply with diminishing energy, the greater the size of the nucleus, and consequently the more intense the development of the cascade process in the nucleus. This feature of continuous spectra becomes more pronounced with increasing angle of observation.

5. COMPARISON OF EXPERIMENTAL AND CALCULATED ENERGY SPECTRA OF QUASI-ELASTIC SCATTERING

For comparison we used spectra of quasi-elastic scattering in beryllium and copper, measured at 30°, since the interference effects should have the least influence in this case. Tests were made on momentum distributions of two types:

a) Gaussian distribution

$$f(s) \sim \exp\{-s^2/s_0^2\}$$

at four values of s_0 corresponding to energies of 16, 18, 20, and 22 Mev;

b) Three-parameter sum of two Gaussian distributions

$$f(s) \sim \exp\{-s^2/s_1^2\} + \alpha \exp\{-s^2/s_2^2\}$$

with $s_1^2/2M = 16$ and $s_2^2/2M = 50$ Mev, and also the similar Selove distribution with $s_1^2/2M = 7$, $s_2^2/2M = 50$ Mev, and $\alpha = 0.15$. The parameter α was chosen to obtain the best agreement between the calculations and the experimental data at the end of the descending branch (towards the higher energies) of the quasi-elastic scattering maximum.

The values of E_c and $\cos\vartheta_c$ calculated from (15) for specified values of s and E_q and the integrals contained in the expression (14) were calculated with the "Ural" electronic computer of the Joint Institute for Nuclear Research. The differential cross sections of the pp and pn scattering, corresponding to each pair of values E_c and $\cos\vartheta_c$, were found from graphs of $d\sigma_{pp}/d\omega$

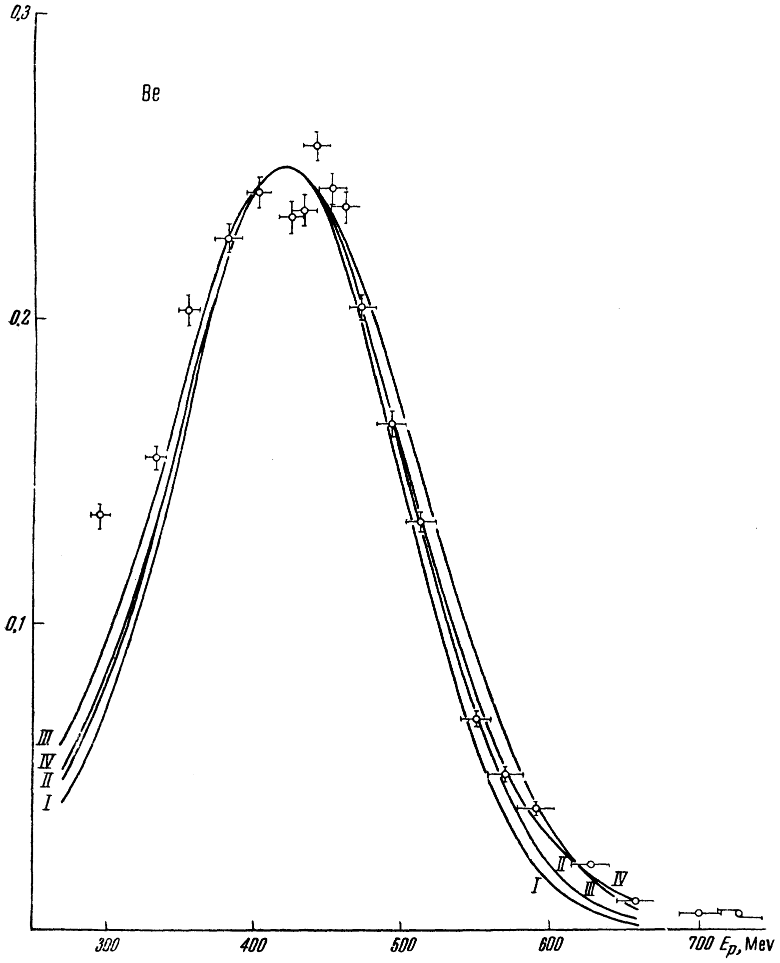

FIG. 8. Energy spectrum of quasi-elastically scattered protons: theoretical curves and experimental data for beryllium at an incident-proton energy of 660 Mev. Angle of observation 30°; I, II, III – Gaussian distribution with values of 1/e at 16, 18, and 22 Mev respectively; IV – sum of two gaussian distributions of the form $\exp\{-s^2/s_1^2\} + \alpha \exp\{-s^2/s_2^2\}$ with $s_1^2/2M = 16$ and $s_2^2/2M = 50$ Mev, $\alpha = 0.06$.

$d\sigma_{pn}/d\omega$, obtained at various energies, plotted for scattering angles in the range $20° \leq \vartheta_c \leq 160°$. Graphical interpolation was used to obtain intermediate values of $d\sigma_{pp}/d\omega$ and $d\sigma_{pn}/d\omega$ from the experimental data on the differential cross sections of pp and pn scattering as gathered by Hess.[24] The values of $d\sigma_{pn}/d\omega$ were estimated in the same manner for energies greater than 580 Mev, using the measured differential cross sections of elastic np scattering at 580 Mev[22] and quasi-elastic pn scattering at 970 Mev.[25]

The final results of the calculations are obtained in the form of a sum of momentum distributions of the emitted protons

$$Zd^2\sigma_{pp}/d\omega\, dq + (A - Z)\, d^2\sigma_{pn}/d\omega\, dq,$$

a sum depending only on q and readily transformed into the energy spectra of protons that have experienced single quasi-elastic collisions. Some of the calculated quasi-elastic scattering energy spectra in beryllium and carbon are shown in Figs. 8 and 9 together with the experimental data. The horizontal lines at the experimental points represent the energy resolution, which has deteriorated from 3.8% at 300 Mev to 4.1% at 660 Mev. Since the calculations do not take into account the presence of the potential well and the effect of excitation of residual nuclei, the theoretical curves had to be shifted towards the lower energies in order to superimpose the theoretical maximum on the quasi-elastic maximum. This shift amounted to approximately 30 Mev for these spectra.

As can be seen from Figs. 8 and 9, at energies less than the energy of maximum quasi-elastic scattering, the theoretical curves lie lower than the experimental points. This discrepancy can be attributed to a superposition of a smeared spectrum of multiple proton scattering in the nuclei on the left branch of the quasi-elastic maximum. The comparison of the theoretical curves with the experimental data was made in the high-energy portion of the spectrum, where the effect of multiple nuclear scattering should be insignificant. The observed spectra agree in first approximation with a Gaussian momentum distribution with 1/e at approximately 20 Mev. It follows therefore that the mean-square value of the nucleon momentum in beryllium and carbon

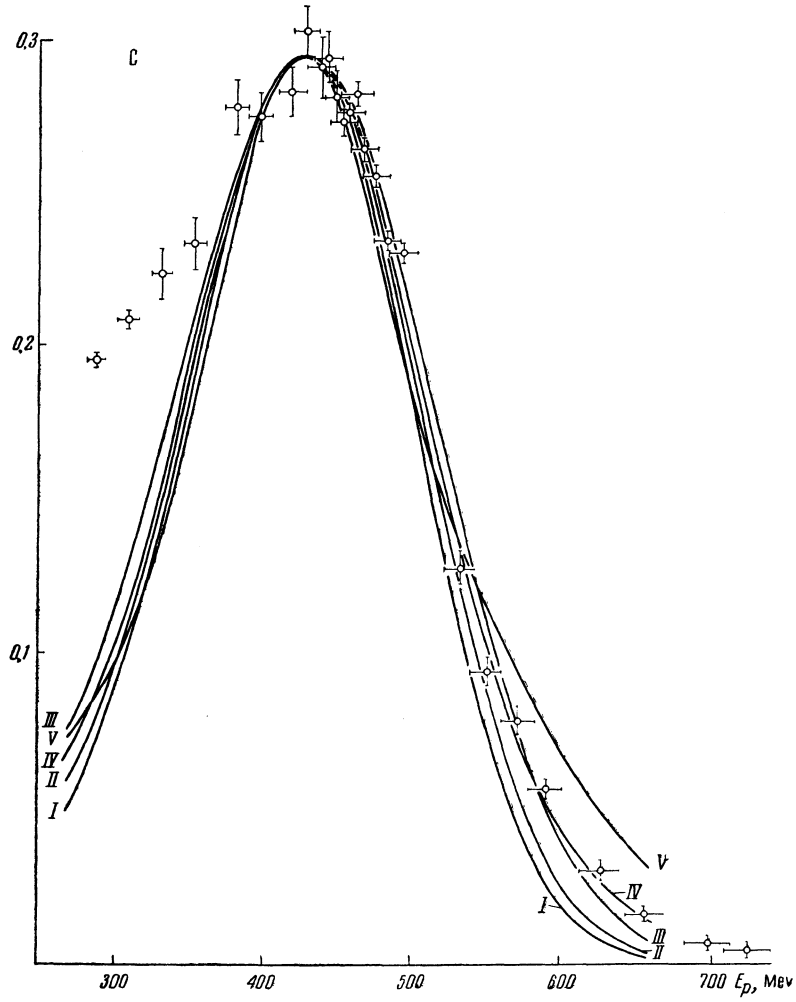

FIG. 9. Same as in Fig. 8 but for carbon. The values used for the sum of the two Gaussian distributions are

$s_1^2/2M = 16$, $s_2^2/2M = 50$ Mev,

$\alpha = 0.09$ (*IV*) and $s_1^2/2M = 7$,

$s_2^2/2M = 50$ Mev, $\alpha = 0{,}15$ (*V*)

nuclei corresponds to an energy of approximately 30 Mev. Apparently a somewhat better agreement with experiment at the uppermost edge of the carbon spectrum is given by the three-parameter sum of two Gaussian distributions with $s_1^2/2M = 16$, $s_2^2/2M = 50$ Mev and $\alpha = 0.09$. However, the experimental data are not sufficiently accurate to favor, with any degree of assurance, this distribution over a simple Gaussian distribution. As to the Selove distribution,[3] it contains too many high momenta, and therefore causes the cross sections away from the maximum of quasi-elastic scattering to be too high.

The results obtained are evidence that a theory, based on the concept of two-particle collisions of fast nucleons in the nuclei, can account in general outline for the shape of the high-energy branch of the quasi-elastic maximum. Nevertheless, this circumstance must not be overestimated, since the result of comparison of theory with experiment is determined fully by the shape of the high-energy portion of the spectrum itself, corresponding to collisions of incident protons with high-momentum nucleons, i.e., with nucleons that interact greatly with other particles during the instant of collision. It is clear, however, that the conditions of applicability of the impulse approximation are satisfied worst of all for such collisions. It should be noted in connection with this that the scattering of the protons by tight two-nucleon groups and possibly also by α particles, which exist in nuclei in the form of substructural formations, undoubtedly leads to a certain distortion of the high-energy portion of the quasi-elastic scattering spectrum. On the other hand, the background observed in the region of energies greater than that which the proton can possibly have after the first quasi-elastic scattering may be partially due to the deuterons knocked out from the nuclei by collisions between protons and tight two-nucleon groups.

These remarks do not lessen the importance of the basic fact that an explanation of the observed quasi-elastic scattering spectra calls for distributions that extend to large momenta. Were the nucleons inside the nucleus to have a Fermi momentum distribution with an upper limit corresponding to $E_F \approx 30$ Mev (the degenerate-gas model), no quasi-elastically scattered protons with energies greater than 525 Mev should be observed at all at 30° under the conditions of the present experiments. The high-momentum states of the nucleons in the nuclei are apparently due to the strong two-nucleon interaction via a short-range potential. These interactions are usually experienced by the nucleons as they move independently in the central field of the nucleus.

6. CONCLUSION

The energy spectra obtained for the secondary charged particles, essentially protons, give an idea of the relative role of various processes that occur upon interaction of 660-Mev protons with light and heavy nuclei. Such processes include diffraction scattering of protons by nuclei and quasi-elastic scattering by individual nucleons, the intranuclear cascade process, and the production of pions in nucleon collisions. Undoubtedly, a relatively small contribution to the secondary-proton yield is also made by collisions between protons and tight two-nucleon groups, as evidenced by the presence of a direct knock-out of deuterons from light nuclei by protons, and by the strong smearing of the pion spectra on the high-energy side.

The momentum distribution obtained for the nucleons in nuclei of beryllium and carbon, namely a Gaussian distribution with $1/e$ at approximately 20 Mev, is in satisfactory agreement with the results obtained at 340 Mev by the Berkeley group.[8,9] Thus, an analysis of the quasi-elastic scattering of protons in light nuclei, over a broad range of collision energies, leads approximately to the same momentum distribution of the nucleons. This fact denotes, apparently, that an analysis of the quasi-elastic scattering in beryllium and carbon nuclei, based on the impulse approximation, is sufficiently well founded. It must be emphasized that at the present measurement accuracy, the spectra of quasi-elastic scattering in beryllium and carbon can be obtained from the same nucleon momentum distribution. As to the spectra of quasi-elastic scattering in copper and uranium, no definite conclusions can be drawn regarding the momentum distribution of the nucleons in heavy nuclei, owing to the strong influence of multiple nuclear scattering in these substances.

The authors thank R. N. Fedorova and I. V. Popova for programming and performance of the calculations, and also S. M. Bilen'kiĭ, N. P. Klepikov, L. M. Soroko, and N. A. Chernikov, for useful discussions.

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Translated by J. G. Adashko
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