Letters to the Editor

THE PROBLEM OF DETERMINING THE DIELECTRIC PERMITTIVITY AND MAGNETIC PERMEABILITY TENSORS OF A MEDIUM

G. S. KRINCHIK and M. V. CHETKIN

Moscow State University

Submitted to JETP editor December 29, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1924-1925 (June, 1959)

In the study of gyrotropic media, both in optical and in radiofrequency investigations,[1,2] it is important to know whether the gyrotropy of the medium is connected with its dielectric permittivity or with its magnetic permeability.

By solving Maxwell's equations in a medium described by tensors ϵ and μ with the following nonvanishing components:

$$\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon,\quad \varepsilon_{xy}=-\varepsilon_{yx}=-i\varepsilon M,\quad \varepsilon_{zz}=\varepsilon_0,$$
$$\mu_{xx}=\mu_{yy}=\mu,\quad \mu_{xy}=-\mu_{xy}=-i\mu M',\quad \mu_{zz}=\mu_0,$$

it is not difficult to obtain the equation that determines the index of refraction n^* of such a medium:

$$[\mu_0\gamma^{*2}+\mu(\alpha^{*2}+\beta^{*2})][\varepsilon_0\gamma^{*2}+\varepsilon(\alpha^{*2}+\beta^{*2})]n^{*4}$$
$$-[\varepsilon^2\mu\mu_0(1-M^2)(\alpha^{*2}+\beta^{*2})$$
$$+\varepsilon_0\varepsilon\mu^2(1-M'^2)(\alpha^{*2}+\beta^{*2})+2\varepsilon_0\mu_0\varepsilon\mu(1+MM')\gamma^{*2}]n^{*2}$$
$$+\varepsilon_0\mu_0\varepsilon^2\mu^2(1-M^2)(1-M'^2)=0, \qquad (1)$$

where α^*, β^*, and γ^* are the direction cosines of the wave normal. The Faraday effect and the polar and meridional Kerr effects (in all three cases the magnetization vector lies in the plane of incidence of the light) do not permit a separation of the effects of the parameters M and M'; for to the first order in M and M', (1) gives the following equation for the index of refraction of circularly polarized waves:

$$n_{\pm}^{*2}=n^2[1\pm\gamma^*(M+M')],$$

where $n=\sqrt{\epsilon_0\mu_0}$ is the index of refraction of the unmagnetized medium.

A different result is obtained for the case of transverse magnetization (magnetization vector perpendicular to the plane of incidence of the light). In this case $\gamma^*=0$, and we get from (1)

$$n_1^{*2}=\varepsilon\mu_0(1-M^2), \qquad (2a)$$

$$n_2^{*2}=\varepsilon_0\mu(1-M'^2). \qquad (2b)$$

It can be shown that n_1^* relates to a wave whose electric vector is parallel to the plane of incidence (p-wave), n_2^* to a wave whose electric vector is perpendicular to the plane of incidence (s-wave). Thus we have obtained generalized formulas for the Cotton-Mouton effect.*

A similar separation of M and M' is obtained also in the case of reflection of light with transverse magnetization. The condition of continuity of the tangential components of the electric and magnetic field intensities, together with the condition div $\mathbf{B}=0$, leads in the approximation of small M and M' to the following reflection coefficients:

$$\frac{R_p}{A_p}=\frac{\alpha n-\alpha^*\mu_0}{\alpha n+\alpha^*\mu_0}-2iM\frac{\alpha\beta\mu_0}{(\alpha n+\alpha^*\mu_0)^2}, \qquad (3a)$$

$$\frac{R_s}{A_s}=\frac{\alpha n-\alpha^*\varepsilon_0}{\alpha n+\alpha^*\varepsilon_0}-2iM'\frac{\alpha\beta\varepsilon_0}{(\alpha n+\alpha^*\varepsilon_0)^2}, \qquad (3b)$$

where $\alpha=\cos\varphi$, $\beta=\sin\varphi$; φ is the angle of incidence of the light. From (3b) we obtain a formula for the relative change of intensity of the reflected light for a gyromagnetic medium:

$$\delta=\frac{I-I_0}{I_0}=\frac{2\sin 2\varphi}{(1-\varepsilon_1)^2+\varepsilon_2^2}[M_1'\varepsilon_2-M_2'(1-\varepsilon_1)],$$

where $\epsilon=\epsilon_0=\epsilon_1-i\epsilon_2$, $\mu=\mu_0=1$; $M'=M_1'-iM_2'$. The analogous formula for the p-wave was obtained in reference 4; there, however, it was incorrectly supposed that the formula held also for a gyromagnetic medium. Thus in the case of a gyroelectric medium, the transverse effects are determined by formulas (2a) and (3a); in the case of a gyromagnetic, by formulas (2b) and (3b). If the medium is bi-gyrotropic, i.e., described simultaneously by the tensors ϵ and μ, then (2) and (3) can be used to separate the effects of the gyroelectric and gyromagnetic parts.

The existing experimental material on measurement of the transverse effects allows the following conclusions to be drawn. Metallic ferromagnetics at optical frequencies possess gyroelectric properties, since the effect determined by formula (3a) differs from zero.[2] Ferrites at superhigh frequencies possess gyromagnetic properties, since the effect determined by formula (2b) differs from zero.[5] It is clear that the Hall effect should lead to gyroelectricity of the medium; this is apparently the origin of the gyrotropic properties of germanium in the same frequency range.[6] In this connection one should look for bi-gyrotropy in ferrites and metallic ferromagnetics that have a large Hall effect.

*We observe that formulas similar to (1) and (2) obtained by Sokolov[3] are erroneous, in consequence of the fact that the components of the expression curl $\mu \mathbf{H} - \mu$ curl $\mathbf{H}$ are quantities of the first order in M'.

[1] P. Epstein, Usp. Fiz. Nauk **65**, 283 (1958) [translated from Revs. Modern Phys. **28**, 3 (1956)]. M. A. Gintsburg, Dokl. Akad. Nauk SSSR **95**, 489 (1954).

[2] G. S. Krinchik, Izv. Akad. Nauk SSSR, Ser. Fiz. **21**, 1293 (1957), Columbia Tech. Transl. p. 1279. G. S. Krinchik and R. D. Nuralieva, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1022 (1959), Soviet Phys. JETP **9**, 724 (1959).

[3] A. V. Sokolov, Физика металлов и металловедение (Physics of Metals and Metal Research) **3**, 210 (1956).

[4] G. S. Krinchik, Физика металлов и металловедение (Physics of Metals and Metal Research) **7**, 181 (1959).

[5] M. T. Weiss and A. G. Fox, Phys. Rev. **88**, 146 (1952).

[6] N. N. Neprimerov, Izv. Akad. Nauk SSSR, Ser. Fiz., **21**, 1288 (1957), Columbia Tech. Transl. p. 1275.

Translated by W. F. Brown, Jr.
381

ON THE OVERHAUSER EFFECT IN SATURATION OF FORBIDDEN RESONANCE

L. L. BUISHVILI

Tbilisi State University

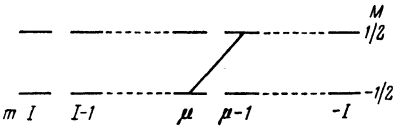
Submitted to JETP editor December 30, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1926-1927 (June, 1959)

KHUTSISHVILI[1] has considered the stationary Overhauser effect in saturation of an allowed transition in paramagnetic-resonance spectrum.

In the present paper we consider the stationary Overhauser effect, but in saturation of a forbidden transition in the paramagnetic-resonance spectrum (we note that Jeffries[2] has considered dynamic polarization of the nuclei, obtained in saturation of a forbidden transition, but for the case when the relaxation time of the nuclei is considerably longer than the relaxation time of the electrons).

Let us consider a system consisting of an electron shell with an effective spin S of one-half and a nucleus with spin I placed in an external magnetic field H. Considering the external field to be sufficiently strong, we neglect (in the calculation of the level population) the energies of the spin-spin interaction and the Zeeman energy of the nucleus. In such an approximation, we obtain $2I + 1$ pairs of levels, the difference in the energies of the components of each pair being $g\beta H$. The level scheme is shown in the diagram (M and m are the projections of the spins of the electron and nucleus on the external field).



In the case of axial symmetry of the intra-crystalline electric field and in the case of an external field H parallel to the symmetry axis (the z axis), we obtain transitions that satisfy the selection rules $\Delta M = -\Delta m = \pm 1$ if the alternating field is parallel to H. For other directions of H relative to z we obtain also other forbidden transitions, in particular, transitions that satisfy the selection rule $\Delta M = \Delta m = \pm 1$.

We assume henceforth that only vertical relaxation (transitions $\Delta M = \pm 1$, $\Delta m = 0$) and relaxation due to the hyperfine interaction (transitions $\Delta M = -\Delta M = \pm 1$) are present.

For brevity we denote by μ the state corresponding to $M = -\frac{1}{2}$, $m = \mu$, and by μ' the state with $M = \frac{1}{2}$, $m = \mu$. We can write for relaxation transitions

$$W(\mu, \mu') = We^{-\delta}, \quad W(\mu', \mu) = We^{\delta},$$

$$W(\mu, \mu - 1') = \lambda We^{-\delta}, \quad W(\mu - 1', \mu) = \lambda We^{\delta},$$

where $2\delta = g\beta H/kT$, W is a certain function of the temperature and of the external field, and λ is a function of T, H, and μ.

Let the forbidden resonance $\mu \rightleftarrows \mu - 1'$ be saturated. We denote by $W(\mu)$ the probability of this transition, caused by an alternating field, per unit time. We introduce a resonance saturation parameter $s(\mu)$ in accordance with the formula

$$N(\mu) - N(\mu - 1') = \frac{N}{2I+1} \tanh \delta \, [1 - s(\mu)].$$

We can obtain the following expression for $s(\mu)$ and the parameters that characterize the degree of orientation of the nuclei:

$$s(\mu) = \frac{W(\mu)\,[(I + 1 - \mu)\,e^{-\delta} + (I + \mu)\,e^{\delta}]}{W(\mu)\,[(I + 1 - \mu)\,e^{-\delta} + (I + \mu)\,e^{\delta}] + \lambda W (2I + 1)},$$

$$f_1(\mu) = -s(\mu)\,\frac{[I + (I + 1) - \mu(\mu - 1)]\sinh\delta}{I\,[(I + 1 - \mu)\,e^{-\delta} + (I + \mu)\,e^{\delta}]},$$

$$f_2(\mu) = -s(\mu)\,\frac{(2\mu - 1)\,[I(I + 1) - \mu(\mu - 1)]\sinh\delta}{I(2I - 1)\,[(I + 1 - \mu)\,e^{-\delta} + (I + \mu)\,e^{\delta}]}.$$