ULTRASONIC ABSORPTION IN METALS IN A MAGNETIC FIELD. I.

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Ultrasonic absorption in metals at low temperatures and in a strong magnetic field (Larmor frequency of conduction electrons much higher than the collision frequency) is studied theoretically. A case is considered of an arbitrary law of energy dispersion of the electrons and of an arbitrary orientation of the field **H** relative to the axes of the crystal and the direction of sound propagation. It is found that the sound absorption coefficient Γ can experience two types of periodic variation with respect to 1/H: oscillations and increments (changes of a given sign which commence periodically). An expression is obtained for the periodic part of Γ for an arbitrary form of the collision operator. It is established that the experimental investigation of the increments permits the determination of the Gaussian curvature of the Fermi surface at all its elliptic points; a study of the oscillations permits the complete reconstruction of the Fermi surface if it possesses a center of symmetry.

AN interesting effect has been established in the experiments of Bommel[1] and of Morse, Bohm, and Gavenda[2] — the oscillation of the ultrasonic absorption coefficient Γ in a metal upon variation in the magnitude of the magnetic field H. It was noted in Pippard's work[3] that the period of the oscillation is determined by the relation between the dimensions of the orbit of the electrons in the magnetic field and the sound wavelength. Starting from this representation, the authors of reference 2 estimated the mean limiting Fermi momentum for the substance studied (copper).

The theory of ultrasonic absorption in metals in the absence of a magnetic field was set up in the researches of Akhiezer,[4] and Akhiezer, Kaganov, and Lyubarskiĭ.[5] The aim of the present research was to discover what information can be obtained, relative to the shape of the Fermi surface in a metal, by studying the periodic part of the function Γ(H) for different orientations of the field **H** relative to the sound propagation vector **k**, and also to give order-of-magnitude estimates of the absorption coefficient Γ.*

It is also of interest to determine the asymptotic behavior of the coefficient Γ in strong magnetic fields, when the dimensions of the orbit of the electrons are much smaller than the sound wavelength, and the dependence of Γ(H) has a monotonic character. Here many different possibilities are encountered, and this question deserves special consideration.

1. The coefficient Γ can undergo periodic changes of two types: oscillations and periodic increments. The latter can exist only in the case in which the time average value of the velocity of the electron $\tilde{\mathbf{v}}$ in the direction of the sound wave is, generally speaking, different from zero. The distribution function of the electrons moving in the field of the sound wave, $F = F_0 + f$, where F_0 is the equilibrium distribution, f is a nonequilibrium addition of the form $f_0 \cos(\omega t - \mathbf{k}\cdot\mathbf{r})$. We shall see below that the absorption coefficient is determined by the quantity f_0^2. Obviously, f_0^2 is a maximum for those electrons which, after the time $T = 2\pi/\Omega$ of a revolution in the magnetic field, fall in the planes of equal phase of the sound wave. In this case, the frequency of variation of the sound-wave field, which acts on the electron in its motion,* is shown to be equal to the frequency Ω of rotation of the electron in the magnetic field H. For such electrons, the condition

$$\tilde{\alpha} = \mathbf{k}\tilde{\mathbf{v}} / \Omega = n \qquad (1.1)$$

must be satisfied (n is an integer or zero). The frequency $\Omega = eH/m^*c$, (where e is the electron charge and m^* the effective mass of the electron

*The ultrasonic absorption in a metal for certain cases of mutual orientation of the vectors **k** and **H** and for an isotropic quadratic energy dispersion law for the electrons has been studied by Steinberg.[6]

*Consideration of the motion of the electron in the static field of the sound wave, which lies at the basis of these qualitative considerations, is valid inasmuch as the velocity of sound $w \ll v$.

in the given trajectory) was introduced by I. M. Lifshitz, Azbel' and Kaganov.[7] The quantities $\tilde{\mathbf{v}}$ and m^* are functions of the quantities that are conserved in the presence of a magnetic field, namely the energy ϵ_0 and the projection of the quasi-momentum $\mathbf{p}$ in the direction of the field (the z axis).

In the absorption of sound, those electrons play a role for which the energy ϵ_0 is close to the value of the chemical potential μ_0. However, the dependence on p_z is very important. Because of it, for a given H, the condition (1.1) can be satisfied only for certain values of p_z. These values include, first, the solutions of the equation $\mathbf{k}\cdot\tilde{\mathbf{v}} = 0$ (if the Fermi surface is closed, this equation obviously always possesses at least one solution). Equation (1.1) is satisfied for $n = 0$ by these p_z for any value of the field, and if the field H (and also the frequency Ω) is sufficiently large, then (1.1) can have no other solutions.

If we decrease the field, then (1.1) ought to have a solution also at $n = \pm 1$ for H less than a certain H_1. A new group of electrons appears, whose distribution function differs markedly from the equilibrium distribution. Therefore, as the field H goes through the value H_1, the absorption coefficient Γ ought to increase. Further changes of this type in the coefficient Γ should occur when the field assumes the values $H_n = H_1/|n|$. Evidently, these considerations on the presence of periodic changes in the coefficient Γ apply, generally speaking, only to the case of a closed trajectory in the quasi-momentum space,[7] when the motion of the electron in the magnetic field is periodic. In the present paper we consider only this case.

The changes mentioned earlier can be of two types, depending on for which p_z (1.1) is first satisfied. The first type is connected with $|p_z|$ equal to p_z^0, the extremal values of $|p_z|$ on the Fermi surface. We shall see below that the corresponding changes are sufficiently smooth — sharp jumps take place in the derivatives of the coefficient Γ with respect to the magnetic field. By measuring the field H_1, it is possible to find the product $v_z^0 m^* = K^{-1/2}$, i.e., to determine the Gaussian curvature K of the Fermi surface at its elliptic point, to which the normal is directed along p_z. If the Fermi surface is nonconvex, then there can be several such points, to each of which, generally speaking, there corresponds a value of H_1. If the Fermi surface is open in the direction of $\mathbf{H}$, then the increments of the first type ought to be absent.

Increments of the second type, as we shall see below, represent sharp jumps. They are associated with such values of p_z for which the function $\tilde{\alpha}(p_z)$ has extrema, i.e., $d\tilde{\alpha}/dp_z = 0$. If the Fermi surface is an ellipsoid, then it is easy to see that this derivative does not vanish (it is equal to a constant). Therefore, the presence of jumps testifies to a sharp departure in the shape of the Fermi surface from ellipsoidal.

Clearly, these increments exist if the electron does not experience collisions within the period of rotation in the magnetic field. More precisely, for this result it is necessary that the inequality

$$\Omega t_0/2\pi \gg 1, \tag{1.2}$$

be satisfied, where $1/t_0$ is the collision frequency of the conduction electrons. We shall assume below that (1.2) holds. The frequency of the ultrasound can also be such that $\omega t_0 < 1$, and in our research just this limiting case was considered.

2. The oscillations of the coefficient Γ take place, generally speaking, for any relative orientation of the vectors $\mathbf{k}$ and $\mathbf{H}$. But it is easiest to observe them when these vectors are perpendicular, since the periodic increments are then absent. In fact, in this case $\tilde{\alpha} = 0$, inasmuch as $\tilde{v}_x = \tilde{v}_y = 0$ (the trajectories in the quasimomentum space are closed); we therefore consider the case in which $\mathbf{k} \perp \mathbf{H}$.

The trajectory of the electron at different points intersects the plane $\mathbf{k}\cdot\mathbf{r} = \text{const}$ at different angles. In particular, there are at least two points on it, and possibly more, in which the tangent to it lies in the plane $\mathbf{k}\cdot\mathbf{r} = \text{const}$. In it, the velocity component of the electron in the direction of the change of the field of the sound wave is equal to zero, and the electron spends more time close to the plane of equal phase, where such points lie, than to the other planes. Therefore, the quantity f_0^2 is determined fundamentally by the value of the field at these points, since the effect of the rapidly changing field on f_0^2 outside their immediate vicinity is shown to be very small. The latter confirmation is valid to a greater degree the larger the number of waves contained between these points becomes, since the effect on f_0^2 of the rapidly oscillating field between them becomes smaller.

Let us consider two such points — 1 and 2. The quantity f_0^2 reaches a relative extremum for a definite difference in the phase of the field of the sound wave at the given points. For a change in the constant magnetic field H, this difference returns to the previous value, when the number of sound waves between the points

$$a = (2\pi)^{-1} \int_{t_1}^{t_2} kv\,dt = A/2\pi \tag{2.1}$$

changes by an integer ($t_2 - t_1$ is the time of motion of the electron from point 1 to point 2). In this case f_0^2 again passes through an extremum each time.

Let us compute the integral (2.1). Let the vector $\mathbf{k}$ be directed along the ξ axis. The equations of motion of the electron in the magnetic field have the form

$$dp_\xi/dt = -(eH/c)\,v_\eta, \qquad dp_\eta/dt = (eH/c)\,v_\xi. \qquad (2.2)$$

Then

$$A = (ck/eH)\,(p_\eta^{(2)} - p_\eta^{(1)}), \qquad (2.3)$$

where $p_\eta^{(1)}$ and $p_\eta^{(2)}$ are the values of the projections of the quasimomentum on the η axis at points 1 and 2. Consequently, if $A \gg 1$, f_0^2 oscillates as a function of $1/H$ with period $\Delta(H^{-1}) = 2\pi e/ck \times |p_\eta^{(2)} - p_\eta^{(1)}|$. The period depends on p_z; therefore in the total experimentally-observed effect there will exist only oscillations corresponding to the distinct values of p_z, namely values for which $\Delta H^{-1}(p_z)$ has an extremum. If the plane $p_z = 0$ intersects the Fermi surface or surfaces, these values will necessarily include value $p_z = 0$. This follows from the fact that the Fermi surfaces either possess radial symmetry or occur in pairs that possess radial symmetry with respect to each other.

3. In the simplest case, when the Fermi surface has a center of symmetry, it is possible to point to an extraordinarily simple method which permits us to establish the surface from the period of oscillation of the coefficient Γ. This method can be shown to be suitable for experimenters, since it is connected in principle with the use of only a single specimen, the surface of which does not need careful treatment for the given purposes.

The method for establishing the Fermi surface lies in the following. Leaving the orientation of the field $\mathbf{H}$ unchanged relative to the crystallographic axes, we measure the periods of oscillation of $\Gamma(\mathbf{H})$ for various directions of the vector $\mathbf{k}$ in the plane perpendicular to $\mathbf{H}$. We assume initially that the oscillations are connected only with $p_z = 0$. Then, if the vector $\mathbf{k}$ is directed along the ξ axis we can determine the ordinates $p_\eta^{(i)}$ of the points at which the line $l(\mathbf{H})$ of intersection of the surface $\epsilon_0(\mathbf{p}) = \mu_0$ with the plane $p_z = \text{const}$ is parallel to the ξ axis. We draw the straight lines $p_\eta = p_\eta^{(i)}$ in the given plane, lines on which these points lie. Repeating such a procedure for different directions of the vector $\mathbf{k} \perp \mathbf{H}$, we obtain a family of lines in the plane $p_z = 0$. Evidently, the envelope of this family is indeed the line $l(\mathbf{H})$. Changing the direction of $\mathbf{H}$, we can determine the form of all possible central sections of the Fermi surface and thus establish it completely. We note that by determining the curve $l(\mathbf{H})$ for a given direction of $\mathbf{H}$, we can simultaneously establish a control on the validity of the determination of the line $l(\mathbf{H})$ for other orientations of the field. The possibility of such a control permits us to ascertain what oscillations correspond to $p_z = 0$, what (if there are any) correspond to other values of p_z, and also to determine whether the Fermi surface has a center of symmetry. The determination of the Gaussian curvature of this surface from the increments of the first type can serve as an additional means of control.

We note that if we succeed by experiment in obtaining a sufficiently strong inequality $A \gg 1$, then the use of films in place of bulk specimens can facilitate considerably the interpretation of the experimental data.

4. We now turn to the quantitative solution of the problem. Following Akhiezer,[4] we shall consider that in the field of a sound wave with $u_i = u_{i0} \exp[i(\omega t - \mathbf{k}\cdot\mathbf{r})]$ (u_i are the components of the displacement vector) the energy of the electron $\epsilon_0(\mathbf{p})$ is increased by an amount

$$\varepsilon' = \lambda_{ik} u_{ik}, \qquad (4.1)$$

proportional to the strain tensor u_{ik}. The distribution function of the conduction electrons $F = F_0[(\epsilon - \mu)/T] + f$ represents the sum of the equilibrium Fermi function (μ = chemical potential at the given point of the metal) and the nonequilibrium addition $f = -\chi \partial F_0/\partial\epsilon$. It can be shown that $\mu = \mu_0 + \mu'$, where μ_0 is the chemical potential in the undeformed metal,

$$\mu' = \bar{\lambda}_{ik} u_{ik} + u_{ii} N_0 h^3/2 \oint (d\Sigma/v) \qquad (4.2)$$

is a contribution which is determined both by the change in the energy of the electrons (the first component) and also by the change in their equilibrium density (second part) under the action of the deformation. Here N_0 is the density of electrons in the undeformed metal, the bar indicates averaging over the Fermi surface ($d\Sigma$ is an element of this surface, and v is the velocity of the electrons on it):

$$\bar{\varphi} = \oint \varphi\,(d\Sigma/v) / \oint (d\Sigma/v).$$

As shown in references 4 and 5, the temperature change in the field of the sound wave can be neglected in the kinetic equations.

The kinetic equation for the electrons in a magnetic field in the form assumed by I. M. Lifshitz, Azbel', and Kaganov[7] which is applicable to the present case, has the form

$$i\omega\chi + \Omega\partial\chi/\partial\tau - i\mathbf{v}\mathbf{k}\chi + \hat{W}\chi = \Lambda\dot{u} - e\mathscr{E}_\nu v_\nu \equiv Q. \quad (4.3)$$

Here $\tau = \Omega t$ is a dimensionless variable proportional to the time of motion of the electron in the magnetic field:

$$m^* = (2\pi)^{-1}\,\partial S/\partial\varepsilon,$$

where S is the area of the intersection of the surface $\epsilon_0(\mathbf{p}) = \text{const}$ with the plane $p_z = \text{const}$, and $\hat{W}$ is the collision operator. χ should be a periodic function of τ with period 2π. The right side of (4.3) is computed in reference 5:

$$\Lambda_{ik} = \lambda_{ik} - \bar{\lambda}_{ik} - \left[N_0 h^3/2 \oint (d\Sigma/v)\right]\delta_{ik};$$

the tensor notation in the expression $\Lambda_{ik}\dot{u}_{ik}$ is omitted here and below. Further,

$$\mathscr{E}_\nu = E'_\nu + [\dot{\mathbf{u}}\times\mathbf{H}]_\nu/c, \qquad \mathbf{E}' = \mathbf{E} + \nabla\mu'/e, \quad (4.4)$$

the field $\mathbf{E}$ is determined from the Maxwell equations

$$\operatorname{curl}\mathbf{E} = -(i\omega/c)\,\mathbf{H}', \quad \operatorname{curl}\mathbf{H}' = (4\pi/c)\,\mathbf{j}, \quad (4.5)$$

$$\operatorname{div}\mathbf{E} = -(8\pi e/h^3)\,\bar{\chi} \oint (d\Sigma/v), \quad (4.6)$$

whereas the field $\mathbf{G} = \dot{\mathbf{u}} \times \mathbf{H}/c$ arises because of the motion of the conductor in the magnetic field.

For good conductors, as noted in reference 5, (4.6) reduces to the equation

$$\bar{\chi} = 0, \quad (4.7)$$

i.e., to the condition of the vanishing of the volume density of charge. Two other equations for the components of $\mathbf{E}$ are obtained by substituting the expression for the components of the current density into (4.5):

$$j_\beta = -(2e/h^3)\int v_\beta\chi\,(d\Sigma/v),$$

where β denotes η or ξ.

The coefficient of sound absorption is

$$\Gamma = \sigma/2wE_{so}, \quad (4.8)$$

where $\sigma = T\dot{S}$, $\dot{S}$ is the rate of change of the entropy density, which is determined by the electronic collisions, E_{so} is the energy density of the sound wave, and w is the sound velocity.

Making use of the expression for the entropy density of a Fermi gas

$$S = -2h^{-3}\int [F\ln F + (1-F)\ln(1-F)]\,d^3\mathbf{p},$$

it is not difficult to show that

$$\sigma = h^{-3}\int |\chi\hat{W}\chi|\,d\Sigma/v. \quad (4.9)$$

In a number of interesting cases it is found possible to introduce the relaxation time, i.e., to assume that the collision operator is

$$\hat{W} = 1/t_0(\mathbf{p}). \quad (4.10)$$

Postponing the investigation of this question to Sec. 7, we now find a periodic solution (in τ) of (4.3) with account of (4.10). It is advantageous to write it down with the help of an operator R which acts on periodic functions of τ (with period 2π) in the following fashion:

$$\hat{R}\psi \equiv \Omega^{-1}\int_{-\infty}^{\tau}\exp\left[\int_{\tau}^{\tau'}(-i\alpha+\gamma)\,d\tau''\right]\psi(\tau')\,d\tau' = \Omega^{-1}\int_{\tau}^{\tau+2\pi}\exp\left[\int_{\tau}^{\tau'}(-i\alpha+\gamma)\,d\tau''\right]\psi(\tau')\,d\tau'/[\exp[2\pi(-i\tilde{\alpha}+\tilde{\gamma})]-1]$$

$$\equiv \hat{r}\psi/[\exp[2\pi(-i\tilde{\alpha}+\tilde{\gamma})]-1], \quad (4.11)$$

where $\alpha = \mathbf{k}\cdot\mathbf{v}/\Omega$, $\gamma = 1/\Omega t_0$; the sign ~ over a letter indicates averaging over the period 2π. The solution of Eq. (4.3) is

$$\chi(\tau) = \hat{R}Q. \quad (4.12)$$

We assume that $\omega t_0 \ll 1$. We further recall that, in accord with (1.2), $2\pi\gamma \ll 1$.

We now determine the components of the vector $\mathscr{E}_\nu$. It follows from (4.7) that

$$\mathscr{E}_\xi = \mathscr{E}_{0\xi} + \sum_\beta a_\beta\mathscr{E}_\beta, \quad (4.13)$$

where

$$\mathscr{E}_{0\xi} = \overline{R\Lambda\dot{u}}/e\overline{Rv_\xi}, \qquad a_\beta = -\overline{Rv_\beta}/\overline{Rv_\xi}. \quad (4.14)$$

Substituting the expression for j_β into (4.5), we arrive at the following equations:

$$E_\beta = E_{0\beta} + \sum_\nu b_{\beta\nu}\mathscr{E}_\nu, \quad (4.15)$$

where

$$E_{0\beta} = (8\pi iew^2/\omega c^2h^3)\int v_\beta R\Lambda\dot{u}\,d\Sigma/v,$$

$$b_{\beta\nu} = -(8\pi ie^2w^2/\omega c^2h^3)\int v_\beta Rv_\nu\,d\Sigma/v. \quad (4.16)$$

Their solution is

$$\mathscr{E}_\beta = \sum_{\beta'}(1-d)^{-1}_{\beta\beta'}(\mathscr{E}_{0\beta'} + G_{\beta'}), \quad (4.17)$$

where

$$d_{\beta\beta'} = b_{\beta\beta'} + b_{\beta\xi}a_{\beta'}, \qquad \mathscr{E}_{0\beta} = E_{0\beta} + b_{\beta\xi}\mathscr{E}_{0\xi} \quad (4.18)$$

We are interested in the case of large relaxation times ($t_0 \gtrsim 10^{-11}$ sec) and ultrasonic frequencies of the order of $10^8 - 10^9$ sec^{-1}. Under these conditions, it is usually true that $|d_{\beta\beta'}| \gg 1$. For example, for the case in which $\mathbf{H} \perp \mathbf{k}$ and $|\alpha| \gg 1$, it is possible to obtain the estimate

$$|d_{\beta\beta'}| \sim 4\pi\sigma_0 w^2/\omega c^2\alpha, \qquad (4.19)$$

the validity of which will be demonstrated in what follows. Here σ_0 is the conductivity of the metal in the absence of a magnetic field. Assuming $\sigma_0 \sim 10^{20}$ sec^{-1}, $\omega \sim 10^9$ sec^{-1}, $\alpha \sim 10$, we obtain $|d_{\beta\beta'}| \sim 10$. Such estimates are easily obtained in other cases. If the inequality $|d_{\beta\beta'}| \gg 1$ holds, then

$$\mathscr{E}_\beta = -\sum_{\beta'} d^{-1}_{\beta\beta'} (\mathscr{E}_{0\beta'} + G_{\beta'}). \qquad (4.20)$$

This case is of fundamental interest to us. However, for very high ultrasonic frequencies, there is an opposite limiting case $|d_{\beta\beta'}| \ll 1$. Then

$$\mathscr{E}_\beta = \mathscr{E}_{0\beta} + G_\beta. \qquad (4.21)$$

Evidently, the field $\mathbf{G}$ associated with the motion of the conductor in the magnetic field differs from zero only if the directions of the vectors $\mathbf{u}$ and $\mathbf{H}$ do not coincide, and its presence can affect the form of the coefficient Γ only if it is not directed along $\mathbf{k}$. It is of interest to compare the order of magnitude of $\Lambda\dot{u}$ and $e\mathbf{v}(1-d)^{-1}\mathbf{G}$ which enter into the right-hand side of the kinetic equation. For example, in the case in which the estimate (4.19) is valid, their ratio is of the order of $4\pi\sigma_0 w^2/\omega c^2 \ll 1$, and the right-hand side of the kinetic equation can be represented with sufficient accuracy in the form

$$Q = \Lambda\dot{u} - e\mathbf{E}'\mathbf{v}. \qquad (4.22)$$

Inasmuch as we are interested in the case of not too strong magnetic fields, in which the dimensions of the electronic orbit are greater than or of the order of the acoustic wavelength, we shall assume in what follows that the right side of the kinetic equation has the form (4.22). We emphasize, however, that the field $\mathbf{G}$ can be shown to be important in the determination of the absorption coefficient in the case of a sufficiently strong magnetic field when $A \ll 1$.

On the other hand, components containing E_β on the right side of the kinetic equation are small in comparison with the term Λu, roughly speaking, when $|d_{\beta\beta'}| \ll 1$. In the most interesting case, $|d_{\beta\beta'}| \ll 1$, the terms are generally comparable, and it is necessary to take them all into account in the determination of the absorption coefficient.

5. We begin with the determination of the coefficient Γ when the vectors $\mathbf{k}$ and $\mathbf{H}$ are mutually perpendicular. If $|\alpha| \gg 1$ for most values of τ, then the integral $\hat{r}\Lambda\dot{u}$ can easily be computed by the method of stationary phase, and is equal to

$$\hat{r}\Lambda u = \Omega^{-1} e^{iA(\tau)} [\Lambda_1 \sqrt{2\pi/\alpha_1'}\, e^{-i(A_1+\pi/4)} + \Lambda_2 \sqrt{2\pi/|\alpha_2'|}\, e^{-i(A_2-\pi/4)}]\, \dot{u}, \qquad (5.1)$$

$$A(\tau) = (ck/eH)[p_\eta(\tau) - p_\eta(0)] = \int_0^\tau \alpha\, d\tau'. \qquad (5.2)$$

The prime here denotes differentiation with respect to τ; the indices 1 and 2 indicate that the values of the function are taken at points of stationary phase, where $\mathbf{k}\cdot\mathbf{v}(\tau) = 0$. In this case, it is assumed that $\alpha_1' > 0$, $\alpha_2' < 0$. For simplicity, we have assumed that such points are two in number; however, the contribution can easily be generalized to a case in which there are more than two.

In the calculation of the integral (5.1), we have taken into account only the first component on the right side of (4.4), because, if $|d_{\beta\beta'}| \gg 1$, $\gamma \ll 1$, then the contribution from it is larger than from the second component. This is readily verified by computing the quantities entering into (4.20) and making use of the integration formula (5.1). In this fashion it is easy to obtain also the estimate (4.19).

The integrand in (4.9) is

$$t_0^{-1}|\chi|^2 = (2\pi)^{-1} t_0 [|\Lambda_1\dot{u}|^2/\alpha_1' + |\Lambda_2\dot{u}|^2/|\alpha_2'| - 2(\Lambda_1\dot{u}^*)(\Lambda_2\dot{u}) \sin A/\sqrt{|\alpha_1'\alpha_2'|}], \qquad (5.3)$$

where $A = A_2 - A_1$. It is clear that σ represents the sum of the part of σ_0 which depends smoothly on H and the oscillating component $\Delta\sigma$.

Substituting (5.3) into (4.9) and taking into account the identity $d\Sigma/v = m^* d\tau\, dp_z$, it is possible to represent the expression for σ_0 in the following form:

$$\sigma_0 = h^{-3} \int \Omega t_0 \delta(\mathbf{kv}) |\Lambda\dot{u}|^2 d\Sigma/v \sim N_0\mu_0\Omega t_0\omega^3 \mathbf{u}_0^2/vw. \qquad (5.4)$$

For the calculation of $\Delta\sigma$ it is again necessary to make use of the method of steepest descents:

$$\Delta\sigma = -\frac{2eH}{ckh^3}\int t_0 (\Lambda_1\dot{u}^*)(\Lambda_2\dot{u}) \sin A\, |v_{\xi 1}' v_{\xi 2}'|^{-1/2} dp_z = \sigma' \sin\left[\frac{ck}{eH}(p^0_{\eta 2} - p^0_{\eta 1}) \pm \frac{\pi}{4}\right], \qquad (5.5)$$

where

$$\sigma' \sim \sigma_0 (\Omega w/\omega v)^{1/2}. \qquad (5.6)$$

The superscript 0 denotes that the corresponding quantities are taken at the saddle point where the first derivative vanishes:

$$d(p_{\eta 2} - p_{\eta 1})/dp_z = 0, \tag{5.7}$$

and the sign of the phase in (5.5) coincides with the sign of the second derivative at this point.* If there are other values of p_z near $p_z = 0$ for which (5.7) holds, $\Delta\sigma$ is the sum of the several components of type (5.5).

Let us estimate the absorption coefficient Γ. Substituting (5.4) in (4.9) and taking it into account that $E_{SO} = \rho\omega^2 u_0^2/2$ (ρ = density of the crystal) and $\Lambda_{ik} \sim \mu_0$, we find that the nonoscillating part is

$$\Gamma_0 \sim N_0\mu_0\Omega t_0\omega / \rho v w^2. \tag{5.8}$$

It is proportional to the magnetic field and the ultrasonic frequency. If we investigate the dependence of Γ_0 on the frequency at a given maximum or minimum, i.e., for a fixed value of $\alpha = \omega v/\Omega w$, then we find that the coefficient Γ_0 is proportional to the square of the frequency. According to (5.6),† $\Delta\Gamma \sim \Gamma_0\sqrt{\Omega w/\omega v}$.

6. We now turn to the investigation of the increment of the coefficient Γ. To estimate the relative value of the effect, we determine the absorption coefficient in such a field H that for most electrons, the quantity $|\tilde{\alpha}|$ is close to unity but is always less than it.

We again consider a case in which (4.22) is valid with sufficient accuracy. It is easy to verify that the important region in the integral (4.9) is that close to $p_z = 0$ and to the other p_z (if they exist) which cause $\tilde{\alpha}$ to vanish. For simplicity, we assume that the value $p_z = 0$ is alone in this relation. We expand $\tilde{\alpha}$ near it up to the linear term $\tilde{\alpha} = \tilde{\alpha}' p_z$, and integrate over infinite limits, obtaining

$$\sigma = \frac{1}{2\pi h^3}\int \frac{m^*}{\Omega^2 t_0}\frac{|r\tilde{Q}|^2 dp_z}{\tilde{\alpha}'^2 p_z^2 + \gamma^2} = \frac{m^*|r\tilde{Q}|^2}{2h^3 k(\partial\tilde{v}/\partial p_z)} \tag{6.1}$$

(the values of all the quantities are taken at $p_z = 0$). Then

$$\Gamma_0 \sim N_0\mu_0\omega / \rho v w^2. \tag{6.2}$$

This estimate remains valid even when $\tilde{\alpha}(p_z) = 0$ has several solutions.

If $2\pi\gamma \ll 1$, then the derivative $d\Gamma/dH$ must undergo a sharp jump near $H = H_1$. Under such conditions, near $H = H_n \equiv H_1/|n|$, the higher-order derivatives of the coefficient of absorption with respect to the magnetic field ought to undergo jumps. In practice, it is easiest to discover jumps in the first derivative [a discontinuity in the function $\Gamma(H)$], and we shall consider in detail the case $|n| = 1$. To estimate the value of the jump, we determine the increment experienced by the coefficient Γ when the field reaches the value H_1. We write

$$v_i(\tau) = \tilde{v}_i + \Delta v_i(\tau).$$

It is not difficult to show (see, for example, reference 9) that near $p_z = p_z^0$ on the Fermi surface, $\Delta v_i(\tau)$ generally has the form

$$\Delta v_i(\tau) = \eta_i v_z^0 \sqrt{|\Delta p_z/p_z^0|}\cos(\tau + \varphi_i) + \cdots, \tag{6.3}$$

where $\Delta p_z = p_z - p_z^0$. The next term of the expansion is linear in $|\Delta p_z/p_z^0|$, and may already contain the cosine of twice the angle, etc. Similarly, $\Lambda = \tilde{\Lambda} + \Delta\Lambda$, wherein $\Delta\Lambda$ close to p_z^0 is determined by an expansion of the type (6.3).

If $|\Delta\alpha| < 1$, χ has the order of $|\Lambda/\Omega\alpha|$ and is a smooth function of p_z for all p_z except those for which $\tilde{\alpha}(p_{zn}) = n$. For $p_z = p_{zn}$, χ has a sharp maximum [the width is of the order of $\gamma/(\partial\tilde{\alpha}/\partial p_z)_n$]. The presence of such maxima points up the character of the dependence of $\Gamma(H)$. Let us determine the form of the function χ close to such a maximum. Let $|p_{zn} - p_z^0| \ll p_z^0$. Then

$$p_{zn} - p_z^0 = (n - \alpha^0)/(\partial\tilde{\alpha}/\partial p_z)^0 \tag{6.4}$$

(the superscript 0 means that the corresponding quantities are evaluated at $p_z = p_z^0$). Close to $p_z = p_{zn}$, we have

$$\chi(\tau) = \int_{\tau}^{\tau+2\pi} \exp\left[\int_{\tau}^{\tau'}(-in - i\Delta\alpha - i\delta\tilde{\alpha} + \tilde{\gamma})\,d\tau''\right] \times (Q^0 + \Delta Q + \delta\tilde{Q})\,d\tau'/2\pi\Omega(-i\delta\tilde{\alpha} + \tilde{\gamma}). \tag{6.5}$$

Here,

$$\delta\tilde{\alpha} = (\partial\tilde{\alpha}/\partial p_z)^0(p_{zn} - p_z),$$

while $\delta\tilde{Q}$ is determined in similar fashion. If $|\Delta\alpha| \ll 1$, we can expand the exponent in (6.5) in powers of $-i\Delta\alpha$. In this case, a sharp maximum is absent from the zero-order term in $\sqrt{|\Delta p_z/p_z^0|}$. In computing the terms of higher order in $\sqrt{|\Delta p_z/p_z^0|}$, we neglect $\tilde{\gamma}$ in the exponent in (6.5).

Is is seen that when $|n| = 1$, the term of first order actually has a maximum for $p_z = p_{z1}$. However, if $|n| > 1$, this term vanishes for $p_z = p_{zn}$ because of the orthogonality of the trigonometric functions. Therefore, when $|n| = 2$, sharp maxi-

*If the line $l(H)$ is not convex, cases for which $\alpha(\tau_i) = \alpha'(\tau_i) = 0$ are possible for certain directions of the vector $k \perp H$. The expression for the periods of the corresponding oscillations remains in these cases as before, but the phase and the dependence of the amplitude on the field change slightly.

†We note that a much weaker inequality than (1.2), $\Omega t_0 \gtrsim 1$, is actually necessary for the existence of the oscillation. If $\Omega t_0 \sim 1$, the periods of oscillation are determined by (5.5) as before.

ma can exist only for terms beginning with the second order, etc. We now consider in detail the case $|n| = 1$. We see that if $|p_{z1} - p_z^0| \ll p_z^0$, then $|\chi|^2/t_0$, the integrand of (4.9), can be represented in the form of a sum of two components. One of them is a smooth function of p_z close to p_z^0. The second, is very small in the range of integration in (4.9) if the difference $H - H_1$ is of one sign, and has a sharp maximum as a function of p_z in this interval if the difference $H - H_1$ is of another sign. Close to $p_z = p_{z1}$, it has the form

$$b\,|\,p_z - p_{z1}| \,/\, (\delta\widetilde{\alpha}^2 + \widetilde{\gamma}^2). \tag{6.6}$$

The integral of the first component is a smooth function of H. Equation (6.1) represents an estimate of its contribution to the absorption coefficient Γ. The contribution from the second component is essentially different from zero only for a definite sign of the difference $H - H_1$. In the given case it also represents the increment in the coefficient Γ. In order to determine it, it suffices to integrate (6.6). As a result we get

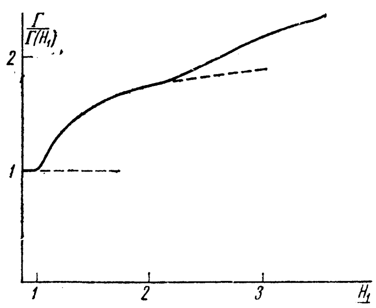
$$\delta_1\Gamma = \pm\Gamma_1\,|\,H_1 - H\,|\,/\,H_1, \tag{6.7}$$

where

$$\Gamma_1 \sim N_0\mu_0\omega\,/\,\rho v w^2. \tag{6.8}$$

Equation (6.7) gives the dependence on the field in the interval $\gamma \ll |H_1 - H|/H_1 \ll 1$ with sufficient accuracy. It differs from zero only for $H > H_1$ or for $H < H_1$, depending whether the derivative $\mathbf{k}\cdot(\partial\widetilde{\mathbf{v}}/\partial p_z)^0$ is negative or positive, respectively. For an increase of the field the sign of (6.7) is opposite to the sign of this derivative. We see that $d\Gamma/dH$ experiences sharp jumps close to $H = H_1$; if the field changes by an amount of the order γH_1, this derivative changes by an amount of the order Γ/H_1. It is clear that if $|n| = 2$, $\delta_2\Gamma \sim (H_2 - H)^2/H_2^2$, and the second derivative $d^2\Gamma/dH^2$ can have a sharp jump close to $H = H_2$, etc.

The graph [constructed for one of the cases in which the vectors $\mathbf{k}$ and $\mathbf{H}$ are parallel, $\epsilon_0 = \mathbf{p}^2/2m^*$, $2\pi\gamma(H_1) = 0.1$] permits us to visualize the form of the function $\Gamma(H^{-1})$. The dashed lines on the graph extrapolate the dependence $\Gamma(H^{-1})$ under the assumption that the derivatives of the absorption coefficient with respect to the field do not experience sharp discontinuities at the corresponding points.



In this same way, it is not difficult to estimate the increment of the second type. For example, for $|n| = 1$, $|\Delta\alpha| \lesssim 1$, it is equal in order of magnitude to

$$\delta\Gamma \sim N_0\mu_0 t_0^{1/2}\Delta v^2\omega^{3/2}\,/\,\rho v^{5/2} w^{5/2}. \tag{6.9}$$

The coefficient Γ experiences this increment if the field changes by an amount of the order γH_1, i.e., it has the character of a sharp jump. Upon a subsequent change in the field in the same direction, the value of p_{z1} will depart from the extremum point of the function $\widetilde{\alpha}(p_z)$, which, generally speaking, ought to lead to a decrease in the absorption coefficient.

In this case discontinuities can occur also for $|n| > 1$.

We have considered two basic types of increment. If the Fermi surface has a complicated structure (for example, if it is self-intersecting), then, in addition to those described, there can be other types.

7. In conclusion we turn our attention to the possibility of introducing a relaxation time. The collision operator has the form

$$\hat{W}f = f(\zeta)\int M(\zeta, \zeta')m^{*\prime}d\varepsilon' dp_z' d\tau' - \int N(\zeta, \zeta')\, f(\zeta')\, m^{*\prime} d\varepsilon' dp_z' d\tau', \tag{7.1}$$

where ζ denotes the set of variables ϵ, p_z, and τ. It is evident that the relaxation time can be introduced in all cases when $|\alpha| \gg 1$. Then $f(\epsilon, p_z, \tau)$ represents a rapidly oscillating function of τ, the second term in (7.1) is much less than the first, and

$$\hat{W} = t_0^{-1} = \int M(\zeta, \zeta')\, m^{*\prime} d\varepsilon' dp_z' d\tau'. \tag{7.2}$$

Thus, we can conclude the presence of an oscillation of the coefficient Γ for an arbitrary character of the collision.

It is useful to note one other case, in which a relaxation time exists. It can also be introduced if, for certain p_z, $|\chi|$ has a sharp maximum that exceeds markedly the average value of this function, and the coefficient Γ or its increment are determined fundamentally by the behavior of the function χ close to this maximum. Such is the case when the expression (6.1) is valid for increments of both the first and the second type. Then again the first term in (7.1) is much larger than the second, which contains the average of the function χ. The situation here is somewhat reminiscent, as shown by Azbel' and Kaner,[11] of the anomalous skin effect. There, too, the possibility of the introduction of a relaxation time is brought about by the presence of a maximum of the distribution function of the electrons whose velocities are almost parallel to the surface of the metal.

In the remaining cases, the assumption on the collision operator of the form (4.10) permits us to estimate the order of magnitude of the absorption coefficient.

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