SOVIET PHYSICS JETP VOLUME 37 (10), NUMBER 1 JANUARY, 1960

ON HYDRODYNAMIC BOUNDARY CONDITIONS FOR EVAPORATION AND CONDENSATION

R. Ya. KUCHEROV and L. É. RIKENGLAZ

Submitted to JETP editor January 15, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 125-126 (July, 1959)

The boundary conditions have been found for hydrodynamic equations in the presence of evaporation and condensation. For small evaporation rates the temperature jump and the deviation of the vapor pressure from the equilibrium value are shown to be of the order of the ratio between the speed of the vapor flow v and the mean speed of heat transfer c. It is shown that the expressions commonly used in the literature for the flow of materials and heat in the presence of evaporation and condensation contain an error.

IN the solution of hydrodynamic problems involving evaporation and condensation, the temperature T(0) and pressure p(0) of the vapor above the liquid (or solid) phase must be given as boundary conditions. Generally speaking, the vapor temperature is not the same as the temperature of the liquid surface, T_0, nor is its pressure equal to the saturated vapor pressure p_0 at the temperature T_0. This effect is analogous to the temperature jump at the boundary between a gas and a solid wall in the presence of heat flow.[1]

In the present work, the value of T(0) and p(0) are found on the basis of kinetic theory. For this purpose it is assumed that the vapor is an ideal gas, that the vapor molecules striking the liquid surface are completely captured, and that the molecules emitted from the liquid have the Maxwellian distribution corresponding to the temperature T_0 and the pressure p_0. The latter assumption is satisfactorily fulfilled in the case of monatomic gases,[2] which are the only ones we shall consider.

To simplify the calculations we limit ourselves to the case in which $v/c \ll 1$, and shall consider a one-dimensional flow of vapor at the plane liquid surface x = 0. (The x axis is directed out of the liquid into the vapor). To describe the state of the vapor we shall introduce the distribution function f(x, ξ), where ξ is the velocity of the molecules in the laboratory coordinate system.

Denoting the current density of the vapor molecules and the energy flux density by τ(x) and Q(x) respectively, we have, immediately outside the liquid surface x = 0,

$$\tau(0) = \tau_0 + \int\limits_{-\infty}^{\infty}\int d\xi_z d\xi_y \int\limits_{-\infty}^{0} \xi_x f(0, \boldsymbol{\xi})\, d\xi_x,$$

$$Q(0) = Q_0 + \frac{m}{2}\int\limits_{-\infty}^{\infty}\int d\xi_z d\xi_y \int\limits_{-\infty}^{0} \xi_x \xi^2 f(0, \boldsymbol{\xi})\, d\xi_x, \qquad (1)$$

where τ_0 and Q_0 are the molecular current density and the energy flux density for the vapor molecules emitted from the liquid.

Making the assumptions given above,

$$\tau_0 = p_0 / \sqrt{2\pi mkT_0}, \qquad Q_0 = 2kT_0\tau_0. \qquad (2)$$

To calculate τ(0) and Q(0) we use the distribution function in a "13 moment" approximation:[3]

$$f(x, \boldsymbol{\xi}) = f_0\Big[1 + \sum_{ij}{}' \frac{p_{ij}}{2pRT}\, u_i u_j - \sum_i \frac{S_i u_i}{2pRT}\Big(1 - \frac{u^2}{5RT}\Big)\Big],$$
$$i, j = x, y, z, \qquad (3)$$

where

$$f_0 = n\,(m / 2\pi kT)^{3/2} \exp(-mu^2 / 2kT),$$
$$\mathbf{u} = \boldsymbol{\xi} - \mathbf{v}, \qquad R = k/m, \qquad (4)$$

n is the density of molecules, p_{ij} is the viscous stress tensor, **v** is the velocity of the local center of inertia, and $\mathbf{S} = m\int \mathbf{u}u^2 f\,d\mathbf{u}$ is twice the heat flow, related to the total energy flow by the formula

$$\mathbf{Q} = {}^1\!/_2\mathbf{S} + \tau c_p T,$$

where c_p is the heat capacity at constant pressure, equal to $^5/_2$k.

Substituting (2) and (3) into (1) and carrying out the integration to the accuracy of the terms linear in v/c, we obtain

$$\tau(0) = 2\{p_0 / \sqrt{2\pi mkT_0} - p(0) / \sqrt{2\pi mkT(0)}\}, \qquad (5)$$
$$Q(0) = 2\{p_0\sqrt{2kT_0 / \pi m} - p(0)\sqrt{2kT(0) / \pi m}\}. \qquad (6)$$

From this it can be seen that the flux of particles and energy, when $v/c \ll 1$, is equal to twice the difference between the corresponding Maxwellian currents directed out of and into the surface, and not to this difference itself, as it is usually taken to be (cf, for instance, references 2 and 4).

The appearance of the factor 2 is related to the fact that in determining the flux falling on the surface, the Maxwellian distribution function for a gas at rest has always been used as the zeroth-order approximation, whereas the proper form appears to be the Maxwellian distribution for a moving gas [Eq. (4)].

Solving Eqs. (5) and (6) for $p(0)$ and $T(0)$, we find the following expressions for the temperature and pressure of the vapor at the surface:

$$T(0)/T_0 = 1 - Q(0)/2Q_0 + \tau(0)/2\tau_0, \quad (7)$$
$$p(0)/p_0 = 1 - Q(0)/4Q_0 - \tau(0)/4\tau_0. \quad (8)$$

When $S/p\tau \gg 1$, (7) reduces to the well-known formula for the temperature jump at a non-absorbent surface.[3] In this case, it is known that the temperature jump is of the order of λ/a, the ratio between the free path and a characteristic dimension for the problem.

In the other limiting case, $S/p\tau \ll 1$, where convective heat transfer is much larger than molecular transfer, we have

$$T(0)/T_0 = 1 - \tau(0)/8\tau_0,$$
$$p(0)/p_0 = 1 - 9\tau(0)/16\tau_0, \quad (9)$$

i.e., the temperature jump and the deviation of the vapor pressure from its equilibrium value are of order v/c.

Equations (5) and (6), or (7) and (8), can serve as boundary conditions for hydrodynamic equations involving evaporation and condensation.

The authors express their gratitude to M. I. Kaganov for profitable discussions, and to Yu. A. Shuander for his interest in the work.

---

[1] L. D. Landau and E. M. Lifshitz, Меxаника сплошных сред (Mechanics of Continuous Media), M. 1944.

[2] G. Wyllie, Proc. Roy. Soc. (London) **A197**, 383 (1949).

[3] H. Grad, Coll. Механика (Mechanics) **4**, IIL, M. 1952, p. 71.

[4] M. S. Plesset, J. Chem. Phys. **20**, 790 (1952).

Translated by D. C. West
19